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Numerical Study of Energy Dissipation of Pooled Stepped Spillways

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Abstract

Water transferring to the dam downstream creates high levels of kinetic energy. Stepped spillways are amongst the most effective spillways in reducing the kinetic energy of the flow moving towards the downstream. The geometry of the steps in stepped spillways can affect the reduction of kinetic energy of the flow transferring to the downstream. Recently pooled configurations have been more prevalent than smooth ones because this kind of spillways could have more dissipation. Therefore, in this study the effect of different number of pooled steps and discharge on flow pattern especially energy dissipation was investigated. The VOF method was used to simulate the flow surface and the k- ε (RNG) turbulence model was used for flow turbulence simulation. Comparing the results obtained from the numerical simulation with the experimental data indicated an acceptable level of consistency. Comparing the obtained results showed that decreasing the number of the steps of pooled stepped spillways reduced flow velocity and increased the relative energy dissipation at the end of the spillway. Decreasing the number of steps increased the turbulent kinetic energy value. Also, the maximum turbulent kinetic energy was obtained near the step's pool. Moreover the results indicated that the value of turbulent kinetic energy increased along the spillway.

Keywords: Three-Dimensional Simulation; Stepped Spillway; Energy Dissipation; Flow Patterns; Turbulent Kinetic Energy.

1. Introduction

One of the objectives of designing spillways is to transfer water flow to the dam downstream. Transferring water flow to the downstream creates high levels of kinetic energy. This energy can damage the spillway downstream if it is not somehow reduced. Stepped spillways which have been used for 3500 years [1] can reduce this energy and the dimensions of the downstream stilling basin [1-4]. The flow passing over stepped spillways create three types of flow pattern: nappe, transition, and skimming flows [5]. The formation of these three patterns on the stepped spillways depends on the height and length of the steps as well as the flow discharge [1, 6-8] Recently, construction of numerous stepped spillway on gravity dams [9] and this fact that stepped spillway perform desirably in energy dissipation has motivated researchers to experimentally study the flow characteristics on stepped spillway. The amount of energy dissipation has always been focused upon in most of the researches carried out on stepped spillways [10-12]. Chinnarasri and Wongwises [13] examined the flow patterns and energy dissipation and came to the conclusion that the geometry of the steps affects energy dissipation. Furthermore, energy dissipation increases as the number of the steps increases for a specific discharge with a constant step height. Comparing the relative energy dissipation over a stepped spillway with different steps height along the spillway by Felder and Chanson [14] indicated that energy dissipation is insignificant over stepped spillways with uniform step height in comparison with stepped spillways with non-uniform step heights. Felder et al. [7, 8] Felder and Chanson [15], Thorwarth [16] and Kokpinar [17] conducted experimental studies on flat and pooled stepped spillways. Comparing the results demonstrated that the number of the

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steps in different slopes changes the spillway's performance. In such a manner that energy dissipation is less in the pooled stepped spillways with 21 and 26 steps in comparison with flat stepped spillways with the same number of steps. While flat stepped spillways with 10 and 64 steps experienced more energy dissipation in comparison with pooled stepped spillways with the same number of steps. In some of the experimental studies, the effect of the porosity of the steps on energy dissipation was examined. Felder and Chanson [18, 19] used porous step pools in a spillway with 10 steps and compared its energy dissipation was greater at the end of the spillway when porosity was not used. Guenther et al [20, 21] examined flow aeration and energy dissipation on a stepped spillway with 10 steps and 4 types of configurations. They came to the conclusion that pooled stepped spillways with staggered configuration of flat and pooled stepped spillways and flat stepped spillways regarding the matter of energy dissipation. Wuthrich and Chanson [22] examined gabion stepped spillways consisting of 10 steps with a 26.6 degree slope. They compared their results with that of flat stepped spillways (with flat and impermeable steps) and showed that relative energy dissipation over flat stepped spillway was greater than of gabion stepped spillways in transition and skimming flow regimes.

In the past decade, researchers were concentrated on using numerical methods because of their high accuracy in hydraulic parameters simulation. A lot of numerical researches were conducted on stepped spillways. Tabara et al. [23] used k- ε model for numerical simulation of stepped spillway with different step configurations and examined the energy dissipation and water surface profile. Xiangju et al. [24] simulated a two-phase flow on a stepped spillway. They examined the hydraulic parameters including velocity and pressure and compared the VOF and Mixture methods. Zhi-Yong [25] simulated the flow over a stepped spillway numerically through using the VOF method. Their results included pressure and velocity profiles and the friction factor. Carvalho and Amador [26] utilized the VOF method and compared velocity and turbulence intensity between the numerical and experimental results. The results indicated that they were fairly consistent. Zhongdong et al [27] compared different turbulence models for simulating flow over stepped spillways. They examined pressure distribution in their study. They also came to the conclusion that the k-ɛ turbulence model had the most capability in simulating the flow over stepped spillways in comparison with the other turbulence models. Baylar et al. [28] examined energy dissipation and flow pattern on the stepped chutes. They concluded that flow aeration efficiency increased as the energy dissipation increased. Also, nappe flow aeration was more than that of skimming flow. Zhenwei et al. [29] used the VOF method for simulating the passing flow three dimensionally. Their pressure, velocity, and flow surface profile results was fairly consistent with the experimental results. Nikseresht et al. [30] used the VOF and Mixture method and compared stepped spillways with different slopes. They came to the conclusion that as the discharge increased, energy dissipation decreased for a given slope. Attarian et al. [31] examined the energy dissipation and velocity profiles in their numerical simulation. Regard to the above-mentioned numerical studies that numerical methods are highly capable of modelling the flow over stepped spillways.

Based on other research conclusion, the geometrical properties of stepped spillways have a great impact on energy dissipation and other hydraulic parameters. In this study, the flow over pooled stepped spillways with different number of steps and discharges was simulated using commercial software *FLOW-3D*. According to the authors of the present paper, this topic has so far. Also, the effect of the number of steps on energy dissipation, turbulent kinetic energy, and the flow pattern was examined.

2. Materials and Methods

Felder et al. [8] experimental data were used to verify the numerical model. They used a 52 cm width stepped spillway with 10 steps. The steps were 20 cm width and 10 cm height. All the steps had pools which were 3.1 cm height and 1.5 cm thick. Figure 1. shows the schematic design of this stepped spillway.

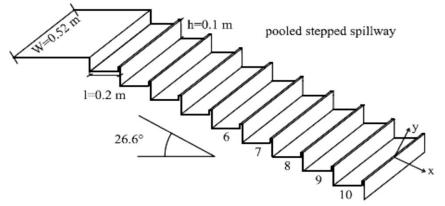


Figure 1. Schematic design of the pooled stepped spillway used in present study

3. Numerical Model: Meshing and Runs Features

In this research, commercial cod *FLOW-3D* was used .This software has high capability to simulate flow pattern in hydraulic structures especially in stepped ones. In this CFD program FAVOR technique has been utilized to simulate obstacles more precisely. Used pooled steps had so small dimensions in comparison with step and crest geometries. FAVOR method enjoys this ability to simulate the small dimension of obstacles. Furthermore full geometry of spillway could be incorporated by using AutoCAD or solid modeller existing in *FLOW-3D* which in this research spillway has been designed by CAD file.

The grid used in the computational domain is shown in Figure 2. Cells with varied sizes were used near the walls and pools of the steps to create dense grids. A uniform mesh was used however in the channel width. The grid continued a 1.7 meters away from the end of the spillway so that the downstream boundary did not affect the results of the numerical model around the spillway. According to the experimental condition all boundary conditions has been employed. In experimental research, predetermined discharge (Q) had been used at the beginning of spillway channel that specific discharge was used in the present paper subsequently. Furthermore the experimental results and flow condition had just been presented above steps and we couldn't choose other boundary condition including specific pressure or specific velocity for output. Hence outflow boundary condition was used at the output to prevent the downstream boundary effects on last steps results. The "Wall" boundary condition was also used for the walls and channel bed. The symmetry boundary condition was used for the upper boundary which was located on top of the air phase.

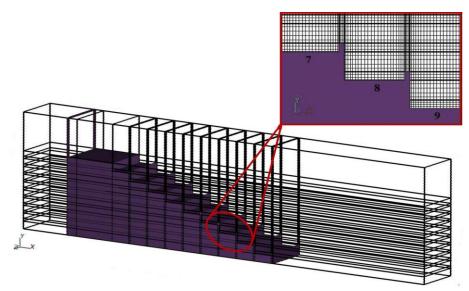


Figure 2. Meshing pattern of the pooled stepped spillway

Tables 1. illustrates the dimensional features of 4 meshes of 1.356502 (Run 17), 1.569420 (Run 25), 2080344 (Run 33) and 2.396521 (Run 34) million volumes, respectively. In all runs the minimum and maximum size of cells was considered in adjacent of steps and end of the downstream respectively. Based on that, results at the end of the downstream did not presented and this paper just deal with the data above steps, maximum size of cells was selected at the downstream. Moreover in all states just meshing size were changed. In this software maximum aspect ratios in X/Y, X/Z and Y/Z should be less than 4 to simulate flow pattern more precisely. This principle was considered in all runs subsequently. In all runs, same boundary conditions were considered. Furthermore, in all states results were presented in the condition of the flow that was reached to the final steady state.

Table 1. Meshing dimension properties for assessment of mesh convergence

Runs	Run 17	Run 25	Run 33	Run 34
Min. size of cells (mm). X direction	6.2	6.5	5	4.7
Max. size of cells (mm). X direction	23.2	21	17.5	15
Min. size of cells (mm). Y direction	5.51	5.3	4	3.8
Max. size of cells (mm). Y direction	13.75	12.8	11.9	11.2
Min. size of cells (mm). Z direction	5.31	4.9	4.2	4
Max. size of cells (mm). Z direction	15.1	13.97	12.84	12.3

4. Governing Equations

In the present study, the flow pattern over pooled stepped spillways with different steps and discharges was simulated using commercial software. In this model, The Finite-volume method was used to discretize the governing equations on a Cartesian coordinate system. The governing equations are the continuity and Navier-Stokes equations for incompressible fluid as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

Where $u_i(i=1,2,3)$ are the Reynolds-averaged velocity components in X, Z and Y directions, respectively

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P^*}{\partial x_i} + \frac{\partial}{\partial x_j} \left((v + v_t) (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) \right)$$
(2)

Where ρ is the density of fluid, ν and ν_t are the molecular and eddy viscosity, respectively, and t shows time.

 $P^* = P - \rho g h$ is the modified pressure including the gravity terms where P is total pressure and h is distance from the reference in Y direction.

The VOF method is used to simulate the water surface profile. Volume fraction (F) is calculated with regard to equation 3 at each time step:

$$\frac{\partial F}{\partial t} + \left(\frac{\partial}{\partial x}(Fu) + \frac{\partial}{\partial y}(Fv) + \frac{\partial}{\partial z}(Fw)\right) = 0.0$$
(3)

Where F represents the volume occupied by air in each computational cell. The entire cell volume is filled with air where F=1, and the entire volume of the cell is filled with water if F=0. When 0 < F < 1, a part of the cell is occupied with water and another part is occupied with air.

5. Results and Discussion

5.1. Comparison of Experimental and Numerical Results

Felder et al. [8] experimental results with the discharges presented in Table 2. were used in this section to verify the numerical model. d_c and h represent the critical depth and step height, respectively [8].

Configuration	$Q=(m^3/s)$	d _c /h	Reynolds number
	0.075	1.29	5.73×10 ⁵
Pooled stepped spillway	0.09	1.45	6.87×10 ⁵
	0.113	1.7	8.72×10 ⁵

In all figures of the present paper, velocity distribution are shown in terms of the dimensionless functions V/V_c and (y+w)/dc. w and y represent step pool height and flow depth from the step pool to the flow surface in the direction vertical to the spillway slope, respectively. V_c is the critical velocity which is obtained as below:

$$V_c = \left(g \times d_c\right)^{1/2} \tag{4}$$

Figures 3, 4 and 5 displays velocity profiles, in order to compare the numerical and experimental data. In the figure 4, results are presented for Sensitivity analysis of mesh size in discharge $0.09m^3/s$. As it could be seen in the figure 4 results for Run 33 and Run 44 are approximately similar. According to this, all of the figures were obtained with Run 33. The numerical results are consistent with the experimental data as it could be seen in the figures and also with regard to the relative error values which are presented in table 3. The maximum level of consistency between the numerical and experimental model is in the middle area of the depth up to the flow surface in all three figures.

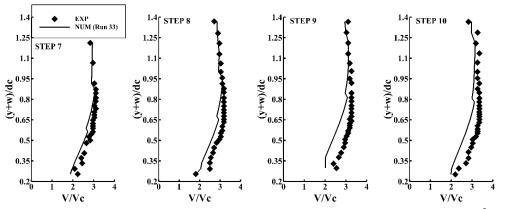


Figure 3. Velocity distribution on steps 7, 8, 9, and 10 of pooled stepped spillway ($Q = 0.075m^3 / s$)

Table 3. Mean relative error percentage for velocity distribution in the simulation of different steps

Spillway	$Q = (m^3/s)$	STEP 7	STEP 8	STEP 9	STEP 10
	0.075	6/7	7/16	12/87	11/4
Pooled stepped spillway	0.09	5/66	6/38	8/4	9/37
	0.113	-	8/18	9/16	9/37

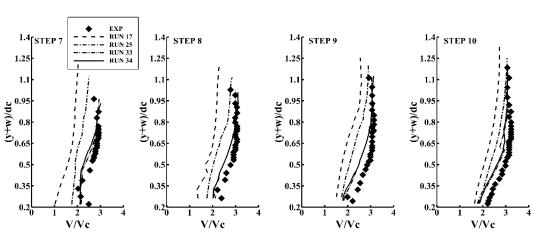


Figure 4. Velocity distribution on steps 7, 8, 9, and 10 of pooled stepped spillway ($Q = 0.09m^3 / s$)

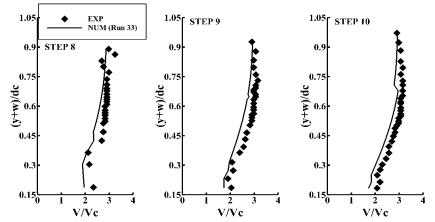


Figure 5. Velocity distribution on steps 8, 9, and 10 of pooled stepped spillway ($Q=0.113m^3/s$)

Figure 6. shows the comparison of the results of the residual head at the end of the pooled stepped spillway between numerical and experimental results. The relative error percentages obtained for the different discharges $0.075m^3/s$, $0.09m^3/s$, and $0.113m^3/s$ are equal to 3.55, 3.59, and 4.38 percent, respectively. The numerical and experimental models are fairly consistent with regard to the relative error percentage obtained for the residual head at the end of the spillway.

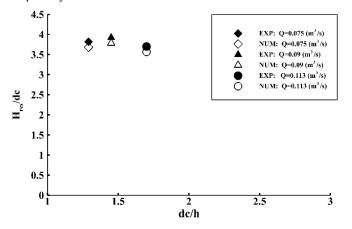


Figure 6. The results of the residual head at the end of the pooled stepped spillway

5.2. Velocity and Energy Dissipation

After making sure of the numerical model performance in the simulation of velocity and residual head at the end of the spillway, the flow pattern with different discharges over the pooled stepped spillway were studied with different number of steps. The schematic design of these spillways is presented in Figure 7. In order to do that, the pooled stepped spillways with the geometrical properties presented in Table 4. were used. In all cases, the step pool height is 3.1 cm and the spillway slope is constant.

Table 4. Geometrical properties of the simulated pooled stepped spillway				
Stepped spillway	Number of steps	Step width (cm)	Step height (cm)	
1	6	36	16/67	
2	8	25/714	12/5	
3	10	20	10	
4	12	16/636	8/333	
5	14	13/846	7/142	

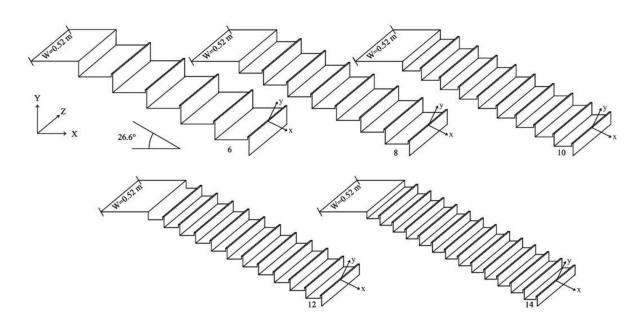


Figure 7. Schematic design of the pooled stepped spillway with different number of steps

The comparison between the velocity distributions of pooled stepped spillway with different number of steps for discharge $0.075m^3/s$, are shown in Figure 8. It could be seen in this figure that the flow velocity is minimum in a stepped spillway with 6 steps in comparison with other states. However, the velocity near the flow surface is minimum in the 8-step stepped spillway in comparison to other states. The velocity is almost equal near the step pool in stepped spillways with 6 and 8 steps. This velocity is almost minimum in comparison to other states. In the case of 12 steps, the flow velocity is maximum about in the whole depth. However, the maximum velocity near the flow surface is occurred in the 14-step stepped spillway.

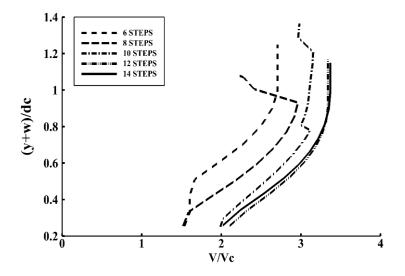


Figure 8. Comparing velocity distribution at the end of the pooled stepped spillways with different number of steps $(Q = 0.075m^3/s)$

The flow velocity distributions are compared between pooled stepped spillways with different number of steps for discharge $0.09m^3/s$ in Figure 9. It could be stated with regard to this figure that the minimum velocity is obtained in the stepped spillway with 6 steps in about the whole depth. However the flow velocity is minimum near the step pool in the pooled stepped spillway with 8 steps. But in contrast with the previous state, the minimum velocity near the water surface can be seen in pooled stepped spillways with 12 and 14 steps. In this discharge, the velocities near the step pool are almost equal in spillways with 10, 12, and 14 steps.

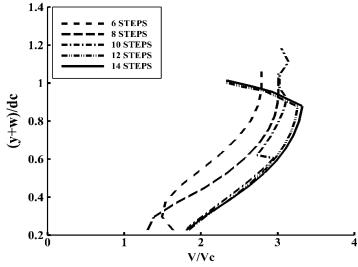


Figure 9. Comparing velocity distribution at the end of the pooled stepped spillways with different number of steps ($\mathbf{Q} = 0.09m^3/s$)

Figure 10. shows the comparison between velocity distributions of pooled stepped spillways with different number of steps in discharge $0.113m^3/s$. Like the previous state, the minimum velocity was obtained for the stepped spillway with 6 steps almost over the whole depth. In addition to that, the flow velocity decreases near the flow surface in a stepped spillway with 6 steps. The maximum velocity is obtained near the stepped spillway with 10 steps. The velocity is minimum and almost equal in the stepped spillways with 8 and 12 steps in the above-mentioned discharge.

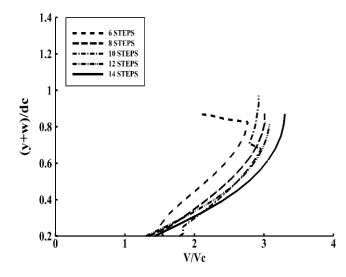


Figure 10. Comparing velocity distribution at the end of stepped spillways with different number of steps ($Q = 0.113m^3 / s$)

The residual head in the pool of the last step is shown in Figure 11. for different number of steps and different discharges. H_{res}, the residual head, in the end of the pooled stepped spillway in this figure is obtained through the following equation [5]

$$H_{res} = d \times \cos\theta + \frac{U_w^2}{2 \times g} + w \tag{5}$$

Where w, U_w , θ , d, and g represent the step pool height, the mean flow velocity along the central axis of the spillway, the spillway slope, the equivalent clear water flow depth, and the gravitational acceleration, respectively.

It could be seen in Figure 11. that the minimum residual head in the downstream is related to the stepped spillway with 6 steps for all three selected discharges. In the discharge $0.075m^3/s$ the increase in the number of the steps increases the residual head at the end of the spillway. In the discharge $0.09m^3/s$ increasing the number of steps up to 10 steps increases the residual head at the end of the spillway, but the residual head remains almost constant for more steps number. The maximum residual head in a discharge equal to $0.113m^3/s$ is obtained for the stepped spillway with 14 steps.

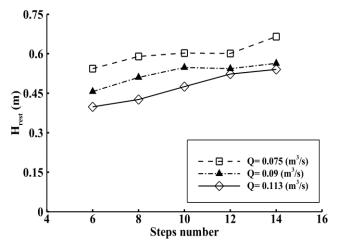


Figure 11. The residual head at the end of the pooled stepped spillway with different number of steps

The relative energy dissipation $\frac{\Delta H}{H_{\text{max}}}$ is shown in terms of dimensionless function, $\frac{\Delta Z_0}{d_c}$, in Figure 12. H_{max} is obtained through the following equation [6].

$$H_{\max} = H_{dam} + \frac{3}{2}d_c \tag{6}$$
$$\Delta H = H_{\max} - H_{res} \tag{7}$$

$$\Delta H = H_{\max} - H_{res}$$

Where H_{dam} is equal to the spillway height and ΔH is equal to the total head dissipation. In Figure 12. ΔZ_0 is defined as the vertical distance between the spillway crest and the pool of the last step. It could be stated with regard to Figure 12. that the maximum relative energy dissipation is obtained for the stepped spillway with 6 steps for all three discharges. The relative energy dissipation decreases as the number of the steps increase for all discharges. Only for discharges $0.09m^3/s$ and $0.113m^3/s$ the relative energy dissipation in the stepped spillway with 12 steps is slightly more than the stepped spillway with 10 steps.

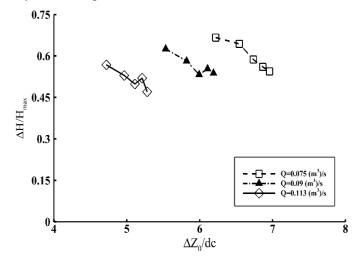


Figure 12. Relative energy dissipation in the pooled stepped spillway with different number of steps

With regard to the fact that the maximum relative energy dissipation is obtained for the stepped spillway with 6 steps, Table 5. quantitatively compares other states with the stepped spillway with 6 steps. It could be concluded with regard to this table that the minimum relative energy dissipation percentage in comparison with the stepped spillway with 6 steps is obtained for the stepped spillway with 8 steps 6 steps for the discharge 0.075 m³/s. According to the results of this table, it could be seen that the greatest difference is related to discharge 0.075 m³/s and is obtained for the stepped spillway with 14 steps.

Table 5- Comparing the relative energy dissipation between the pooled stepped spillway with 6 steps and the pooled stepped
spillways with different number of steps in percentage

\mathbf{Q} (m^3/s)	dc (m)	Number of steps	Decrees percentage of the relative energy dissipation in comparison with the stepped spillway with 6 steps
0.075 0.129		8	3.406
	0.120	10	11.915
	0.129	12	15.913
		14	18.350
0.09 0.145		8	7.012
	10	14.836	
	12	11.464	
	14	14.051	
0.113 0.17		8	6.727
	0.17	10	12.184
	0.17 12	8.463	
		14	17.166

5.3. Turbulent Kinetic Energy

Turbulent kinetic energy (TKE) is formed in the spillways due to high gradient of velocity [32-34]. Taking into consideration the fact that the change process of the TKE was the same for all three selected discharges, only the results related to discharge 0.113 m^3 /s are shown in Figure 13. It could be seen in this figure that the maximum TKE is created near the step pool on each step. The value of the TKE also increases along the spillway which was similar to the previous researches [32, 34-35]. On the other hand, an increase in the number of steps decreases the TKE.

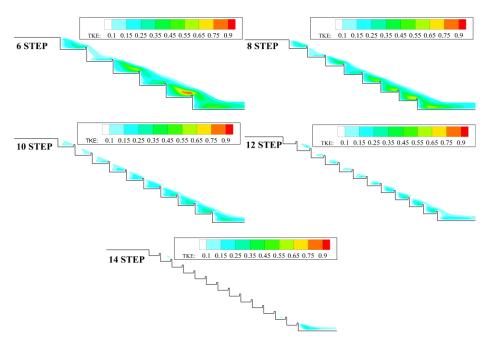


Figure 13. Turbulent kinetic energy (m^2/s^2) for different number of steps for discharge Q= 0.113 m³/s

Maximum TKE for different discharges are shown in Figure 14. for pooled stepped spillways with different number of steps. It could be seen in this figure that the maximum TKE is created for the stepped spillway with 6 steps for all three selected discharges. Moreover, as the number of the steps increases the value of TKE decreases. It could also be seen that as the discharge increases the value of TKE increases. Slope of the graph in Figure 14. is steeper in the distance between the stepped spillway with 8 and 10 steps in comparison with the other sections which indicates that the obtained value for TKE decreases the most in this distance in comparison with the rest of the graph.

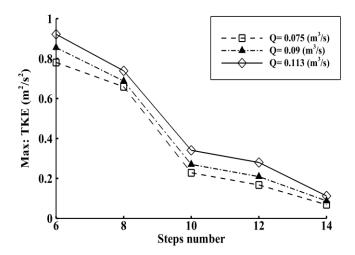


Figure 14- The maximum TKE (m²/s²) value in the pooled step spillway with different number of steps

5.4. Flow Pattern

The pattern of the flow passing over the stepped spillway depends on the discharge and the geometry of the steps [1, 7-8]. Chanson [1] presented the following equations for specifying the flow regime passing over stepped spillways. The upper limit of the nappe flow regime can be obtained using Equation. 8:

$$\frac{dc}{h} = 0.89 - 0.4 \times \frac{h}{l} \qquad \text{NA-TRA} \tag{8}$$

The lower limit of the skimming flow regime can be obtained through Equation. 9:

$$\frac{dc}{h} = 1.2 - 0.325 \times \frac{h}{l} \qquad \text{TRA-SK} \tag{9}$$

Where h and l are the height and length of the step, respectively. NA, TRA, and SK represent the nappe, transition,

and skimming flow regimes, respectively. The equations 10 and 11 are valid for range 0.05 < h/l < 1.7 [1].

Figure 15. shows the water surface profile for different discharges for the stepped spillways with different number of steps. It could be seen in this figure that as the discharge increases the flow depth increases. Also a change in the number of steps does not change the flow depth on the spillway crest for similar discharges. According to the equations 10 and 11 and also Baylar et al. [28] classification, it could be stated that the flow surface has a series of fluctuations in all three discharges in the stepped spillway with 6 steps, in other words the transition flow regime is present in this state. The transition flow regime is only created in the discharge 0.075 m³/s when 8 steps are used. The skimming flow regime is formed on other cases.

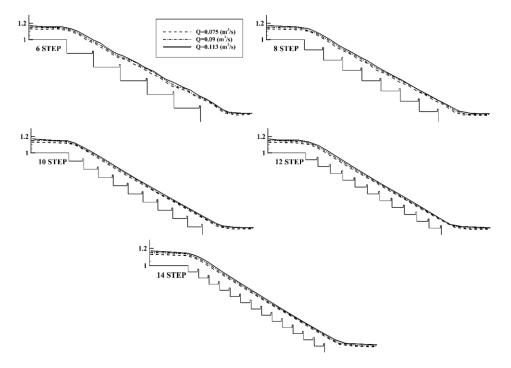


Figure 15. The water surface profiles for the pooled stepped spillway with different number of steps

The streamlines for discharge 0.113 m^3 /s are shown in Figure 16. It could be seen in this figure that vortex flows are formed in an incomplete manner in a number of the steps in the stepped spillway with 6 steps. Moreover, the locations of these flows occurrence are different from each other. In the stepped spillway with 8 steps vortex flow is formed in a number of steps adjacent the vertical dimension of the step. This flow occupies more than half of the step length on other steps. These flows are formed in smaller sizes in the middle of the steps on the stepped spillways with 10, 12, and 14 steps, vortex flows are formed on all steps and they cover more than half of the step length.

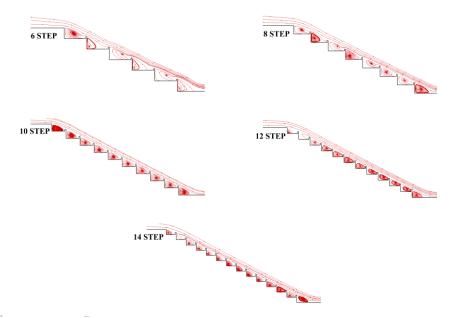


Figure 16. Flow pattern over pooled stepped spillway with different number of steps

6. Conclusion

In the present paper three dimensional simulation of pooled stepped spillway with different number of steps was studied using commercial software. The VOF method was used to simulate the water surface profile and k- ε (RNG) turbulence model was used for flow turbulence. Comparing the numerical results with the experimental data indicated that the model had an acceptable level of accuracy in the velocity over the pooled stepped spillways for different discharges. The results demonstrated that a decrease in the number of steps of pooled stepped spillways decreased the flow velocity at the end of the spillway. The common point obtained for all the discharges for the pooled stepped spillways with different number of steps was that a decrease in the number of steps increased the relative energy dissipation. While an increase in the number of steps in different discharges resulted in different change trend of the relative energy dissipation. The obtained results indicated that the value of turbulent kinetic energy increased along the spillway. The maximum turbulent kinetic energy was formed near the pool of each step. Furthermore, discharge increase increase the value of the turbulent kinetic energy. Also a decrease in the number of steps led to turbulent kinetic energy value increase. With regard to discharge, a decrease in the number of steps changed the flow pattern over the pooled stepped spillways. Vortex flows were formed on all steps for all cases. However, in the stepped spillways with 6 and 8, the vortex flows were incompletely formed on a number of steps.

7. References

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