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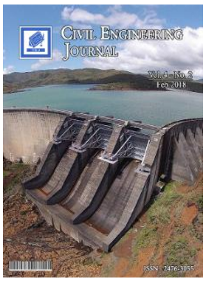

An Analytical Model for Estimating the Vibration Frequency of Structures Located on the Pile Group in the Case of Floating Piles and End-bearing Pile

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Abstract

Exact estimation of vibration fundamental period of structures plays a vital role in their designing procedure. The proposition of a relatively exact expression which considers the effects of a pile group on the fundamental period of the structures was of less interest to previous researchers. This study aims to propose an analytical model and expression so as to estimate the free vibration period of the structures located on a pile group. To reach the objectives of this study, several numerical analyses has been carried out using the method of equivalent spring which takes into account the effects of soil-pile-structure interaction on the fundamental period of the structures. In the next step of the study the effects of a pile group on the fundamental period of the structures have been analyzed analytically. In this analytical study two cases have been considered for the piles which are end-bearing and floating piles. In the case of floating piles a five degrees-of-freedom analytical model and its corresponding expression have been proposed considering the soil-pile-structure system. The numerical modelling has been performed using the direct method due to the neglect of the soil in analytical expression and the results have been compared with those of the proposed analytical expression. The soil mass participation coefficient (λ) has been obtained using the discrepancy between the results of the two different methods to modify the analytical expression. In the case of end-bearing piles an analytical model with three degrees-of-freedom and its corresponding expressions has been proposed. Then the soil has been neglected and a new analytical expression has been proposed using the mass participation coefficients adopted from other researches to calculate the fundamental period of the structures. The comparison between the results of the proposed expression and those of case and numerical studies confirms that the proposed expressions benefit from a relative accuracy and can be used as an initial criterion in designing procedure.

Keywords: Soil-Pile-Structure Interaction; Frequency of Free Vibration; Analytical Formula; Steel Frame; Numerical Study.

1. Introduction

The seismic assessment of steel frame buildings is typically based on the assumption that they are mounted on a rigid medium and that the effects of the Soil-Structure Interaction (SSI) can be ignored. In contrast, the SSI phenomenon can affect the response of structures tremendously. The fixed-base assumption is inappropriate for many structures, and structural systems that incorporate stiff vertical elements for lateral resistance (e.g., shear walls and braced frames) could be very sensitive to the small translational and rotational movements that are disregarded in the fixed-base assumption.

Lessons learned from recent earthquakes show that fixed-base assumption could be misleading, and neglecting the influence of SPSI could lead to unsafe design, particularly for structures founded on soft soils. The seismic design of

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buildings has been undergoing a critical reappraisal in recent years, with change of emphasis from ‘‘strength’’ to ‘‘performance’’. The development of capacity design principles in the 1970s was an expression of the realization that the distribution of strength through a building was more important than the absolute value of the design pillar shear which can be identified as the key point in the performance-based seismic design, where the overall performance of the building is controlled during the seismic design process. For determining the seismic response of structures, it is a common practice to assume that the structure is fixed at the pillar. In fact, if the ground is stiff enough (e.g.. structure founded on solid rock) it is reasonable to assume that the input motion of the structure due to a design earthquake is essentially identical to the motion of the free field, which is defined as the motion experienced at the same point before the structure is built. However, for structures constructed on soft soils, two modifications need to be considered for determining the seismic response. First, the imposed motion to the structure differs from the free field motion due to the presence of the structure. Secondly, additional dynamic deformations are induced within the structure due to the underneath soft soil. The process, in which, the response of the soil influences the motion of the structure and the response of the structure influences the motion of the soil is referred to as soil–structure interaction.

Mahshavari et al. (2004) studied the effects of SPSI in a coupled system numerically and the analyses were evaluated for two types of harmonic and transient excitation and the results were compared in both linear and nonlinear cases [13]. Kumar and Prakash (2004) carried out an analytical study investigating the fundamental period of buildings that are placed on pile [12]. Yingcai (2008) analyzed the dynamic response of the structures which are under the vibration of the reciprocating compressor in three different cases: considering the soil-pile-structure interaction, flexible building on a rigid foundation and flexible foundation-rigid structure, in numerical studies [15]. Yingcai (2008) conducted an analytical study investigating the effect of the pile group [15]. In this research, the stiffness matrix of soil-pile is formed by finite element method, then the general stiffness matrix of a single pile is assembled for different vibration modes and then the effect of the pile group is calculated from the interaction factors in DYNAN program. Rovithis et al. (2009) evaluated the seismic parametric response of a single-degree-freedom structure which is based on a pile that is on a soft clay [14]. For this purpose, the finite element method (3-D) has been used in the frequency domain along with harmonic excitation input in the profile of the soil. Chau et al. (2009) studied the effects of cavities created between the pile and the soil, on the maximum acceleration of the pile cap, using the results of the seismic table test [2]. Hosseinzadeh et al (2009) investigated the effects of the soil-structure dynamic interaction on the seismic-response of the individual and adjacent buildings using the seismic table tests on scaled models [3]. Lihua Zou et al. (2012) evaluated the effects of important soil and structure parameters on the vibrations control of the system [18]. Khary et al. (2013) studied the effects of seismic excitations, on the response of flexible piles, considering kinematic and inertial interactions [4]. Later, Chang et al. analyzed a single dimensional wave equation for piles under the shaking horizontal seismic ground [5]. Other studies on the seismic response of buildings with shallow and pile foundations were conducted by Hokmabadi et al, Ghanbari and Shirgir et al [6-8]. Fattah et al. [9] studied the excess pore water pressure generated around a single pile and pile group experimentally. The piles were excited by two opposite rotary machines embedded in saturated sandy soil. A small-scale physical model was built to perform the experiment. The effects of pile embedment depth ratio, piles spacing and operating frequency of the rotary machines. The results revealed that the generation of excess pore water pressure was affected mainly by pile slenderness ratio, machine operating frequency and the soil permeability. It was also found that using the pile foundation reduced the vibration amplitude. Li et al, [10] investigated the changes of undrained shear strength and pore pressure due to the installation of prestress concrete pile (PCP) in soft clay. The changes of undrained shear strength due to PCP installation are measured by vane shear test (VST). The piezocone penetration testing (CPTU) is conducted for detecting the change of pore pressure. The relation between the normalized excess pore pressure, and the ratio of radial distance to pile radius was proposed based on CPTU data. The same year Sharafi et al. [11] analyzed various experimental and numerical studies on slopes. Small-scale physical modeling of slopes under surcharge loads was performed on loose sand slopes. Safety factors and location of critical failure surfaces of various reinforced and unreinforced slopes are obtained and compared. They also analyzed the effects of soft bound interlayer, soil properties, pile spacing, pile position and surcharge.

1.1. Technical Approach

The analytical methods that have been introduced, include five major methods. These five methods are the direct method, the equivalent spring and damper methods, the substructure method, the discrete element method and the cone model method. Each one of these methods has some disadvantages and advantages that limit their use in different cases.

There has not been a comprehensive research on the dynamic characteristics of these buildings yet, which is available to correct the current code formula for steel frame buildings (on pile foundation). The soil–pile–structure interaction has not been examined comprehensively; and even the most crucial problem parameters have yet to be identified. An objective of the current paper is to present a simple formula for the estimation of the fundamental period of steel frame buildings by considering soil-pile-structure interaction.

There is no proper expression that takes into account the effects of the pile on the fundamental period of the buildings based on a pile group. Therefore, providing an analytical expression to estimate the free vibration period of the structures can be useful in designing structures.

As it was stated, the purpose of this research is to provide an analytical model and an expression for calculating the free vibration frequency of structures on the pile group. In this study, the effect of the presence of the pile group on the period of free vibration of the structures was studied analytically and numerically. In this way, two cases of floating piles and end-bearing piles were studied using the analytical method. In numerical method, the metal frame structures of the proposed model were investigated numerically by the equivalent spring method and direct method using finite element software and after verifying the software performance using the nomenclature equations and other researchers' results, it was used for verification of the results obtained by the analytical method. In the analytical method, for the case of floating piles, a new mass participation coefficient (λ) for the soil around the piles with shear modulus (G) and different Poisson ratios (υ) were introduced and a new analytical expression was proposed.

2. Equations of Motion and the Calculation of the Natural Frequency of the System

In general, a single-degree-of-freedom system can be represented as Figure 1. In this figure, the vibration of the system has been controlled by a spring and a damper. The equation of motion of the system can be obtained by balancing the spring and damper forces with inertia.

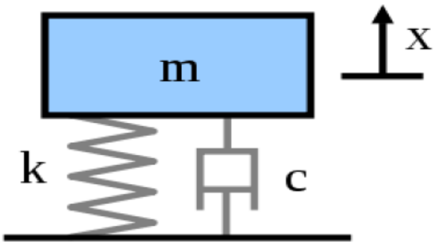

Figure 1. Single- degree-freedom system [7]

$$[K]\{U\}+[C]\{\dot{U}\}+[M]\{\ddot{U}\}=\{F\} \tag{1}$$

In which [K] is the equivalent stiffness matrix, [C] is the equivalent damping matrix, [M] is the equivalent mass matrix, {F} is the matrix of the input forces acting on freedom degrees of the system, {U} is the displacement matrix and, $\{\dot{U}\}$ and $\{\ddot{U}\}$are the first and the second derivatives of the displacement matrix with respect to time, respectively.

Usually, the above equation is written without regard to damping, because not only this quantity has a small effect on natural frequencies, but also the damping matrix is practically non-computable. (Raufi, 2008) Next, in order to solve this equation and to determine the response of each mode, it is clear that the motion is simple in a natural oscillatory mode where all the points of the structure vibrate with the frequency ω_i. Therefore, the form of the motion will be a sinusoidal function of time. For example, in a structure with two degrees of freedom, in the i^{th} mode, the displacements of the masses of m_1 and m_2 , have been represented with $u_{1(i)}$ and $u_{2(i)}$ respectively, the amplitude of these displacements represented with $A_{1(i)}$ and $A_{2(i)}$, the spatial response can be written in each mode as follows:

$$U_{(i)}=\begin{Bmatrix} u_{1(i)} \\ u_{2(i)} \end{Bmatrix}=\begin{Bmatrix} A_{1(i)} \\ A_{2(i)} \end{Bmatrix}\sin\omega_i t \tag{2}$$

In which the bracket index introduces the mode number. The acceleration equation is gained from the second derivative of the recent equation.

$$\ddot{U}_{(i)}=\begin{Bmatrix} \ddot{u}_{1(i)} \\ \ddot{u}_{2(i)} \end{Bmatrix}=-\omega_{(i)}^2\begin{Bmatrix} A_{1(i)} \\ A_{2(i)} \end{Bmatrix}\sin\omega_{(i)} t \tag{3}$$

It is achieved by substitution of $U_{(i)}$ and $\{\ddot{U}_i\}$ into equation 2 and omitting $\sin\omega_{(i)}t$

$$\left[K-\omega_{(i)}^2 M\right]\begin{Bmatrix} A_{1(i)} \\ A_{2(i)} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}$$

From the above equation, it is understood that the condition of $A_{1(i)}=A_{2(i)}=0$ is in fact the condition of equilibrium of the structure and consequently there is no vibration. Other solution can befound when the determinant of the coefficients of thissystem of equation equals zero.

$$\left[K - \omega_{(i)}^{2} M\right] = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5}$$

The magnitudes of ω_1 and ω_2 can be evaluated by solving the last equation. Similarly, the above method can be used to find the natural frequencies of an n-degree of freedom system. To find the natural frequencies of the system, wesubstitude the matrix ofstiffness and mass of the system into 5 and put the determinant of the obtained matrix equal to zero. By solving the above equation for ω, the values of the natural frequencies will be obtained.

3. Research Methodology

Modeling of the system's dynamic behavior is accompanied by many complexities that makes it difficult to validate the results. To obtain reliable results, it is necessary to choose a simple, yet accurate and realistic model. As it is shown in Figure 2, two analytical models have been represented with the consideration of simplifications for the floating and end-bearing piles. As it can be seen, in the modeling of the various components represented in this study, the structure has been modeled on a three degrees-of-freedom system, the piles on continuous mass and the soil on equivalent spring and damper.

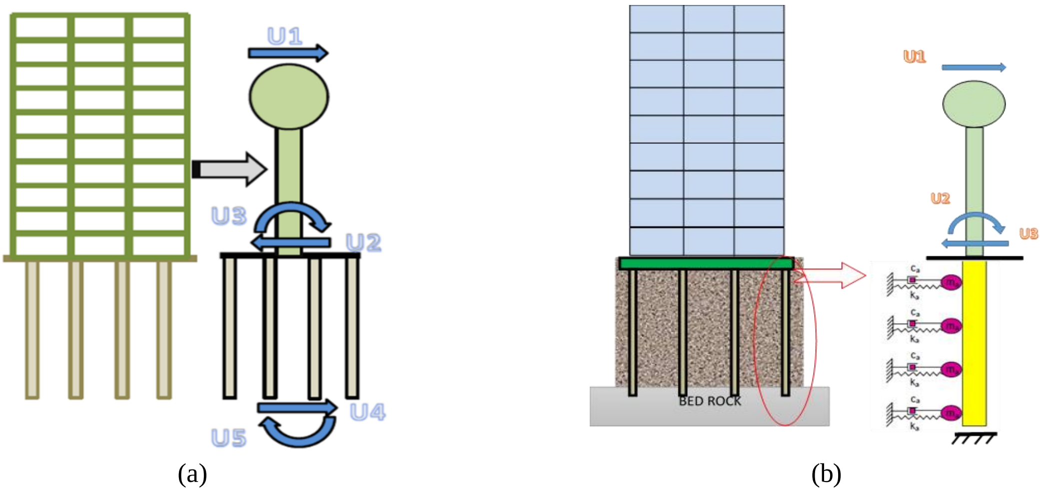

Figure 2.(a) The proposed analytical model for structures on the floating pile group; (b) for structures on a group of end-bearing piles [16, 17]

3.1. Modeling of Piles

A continuous mass beam element has been used for the modeling of the piles structure. In order to obtain stiffness, mass and damping matrices, the shape function matrix must be defined for the first and second degree-of-freedom. Equation 6 is the mode shape of the clamped-free beam element (pile) and equations 7 and 8 are the first and second derivatives of the mode shape respectively.

$$N : \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = \begin{bmatrix} 1-\cos(\frac{\pi x}{2L}) \\ x(1-\frac{x}{L})^2 \end{bmatrix} \tag{6}$$

$$N' : \begin{bmatrix} \frac{\pi}{2L}\cos(\frac{\pi x}{2L}) \\ 1-\frac{2x}{L}+\frac{3x^2}{L^3} \end{bmatrix} \tag{7}$$

$$N'' : \begin{bmatrix} \frac{\pi^2}{4L^2}\cos(\frac{\pi x}{2L}) \\ -\frac{2}{L}+\frac{6x}{L^3} \end{bmatrix} \tag{8}$$

Equation 9 is the stiffness matrix of the clamped-free beam element (pile). In equations 10 and 11 the procedure of obtaining this matrix is presented.

$$[K_b]=\int_0^L EI[N''(x)]^T[N''(x)]dx \tag{9}$$

$$N''.\,N''^T=\begin{bmatrix}\frac{\pi^2}{4L^2}\cos(\frac{\pi x}{2L})\\ -\frac{2}{L}+\frac{6x}{L^3}\end{bmatrix}\cdot\begin{bmatrix}\frac{\pi^2}{4L^2}\cos(\frac{\pi x}{2L}) & -\frac{2}{L}+\frac{6x}{L^3}\end{bmatrix}=\begin{bmatrix}\frac{\pi^4}{16L^4}\cos^2(\frac{\pi x}{2L}) & (-\frac{2}{L}+\frac{6x}{L^3})\pi^2\cos(\frac{\pi x}{2L})\\ (-\frac{2}{L}+\frac{6x}{L^3})\pi^2\cos(\frac{\pi x}{2L}) & (-\frac{2}{L}+\frac{6x}{L^3})^2\end{bmatrix} \tag{10}$$

$$K_b=\begin{bmatrix}\frac{EI\pi^4}{32L^3} & \frac{EI(\pi-6)}{L^2}\\ \frac{EI(\pi-6)}{L^2} & \frac{4EI}{L}\end{bmatrix} \tag{11}$$

Equations have also been written for the matrix of mass element and the Equation 12 is the mass matrix of the clamped-free beam element (pile) with its obtaining procedure presented in Equations 13 to 16. It should be noted that the m in equation 16 is the mass of the unit length of the pile.

$$[M_b]=\int_0^L m[N(x)]^T[N(x)]dx \tag{12}$$

$$[N(x)]^T[N(x)]=\begin{bmatrix}1-\cos(\frac{\pi x}{2L})\\ x(1-\frac{x}{L})^2\end{bmatrix}\begin{bmatrix}1-\cos(\frac{\pi x}{2L}) & x(1-\frac{x}{L})^2\end{bmatrix}=\begin{bmatrix}(1-\cos(\frac{\pi x}{2L}))^2 & (1-\cos(\frac{\pi x}{2L})).(x(1-\frac{x}{L})^2)\\ (1-\cos(\frac{\pi x}{2L})).(x(1-\frac{x}{L})^2) & x^2(1-\frac{x}{L})^4\end{bmatrix} \tag{13}$$

$$*\rightarrow\ \frac{L^2}{12}+\frac{4(\pi^2+4\pi-24)}{\pi^4}\cong\frac{L^2}{12} \tag{14}$$

$$[M_b]=\int_0^L m[N(x)]^T[N(x)]dx=\overline{m}\begin{bmatrix}\frac{L(3\pi-8)}{2\pi} & \frac{L^2}{12}\\ \frac{L^2}{12} & \frac{L^3}{105}\end{bmatrix} \tag{15}$$

$$\overline{m}=n^2\rho_{pile}A_{pile} \tag{16}$$

3.2. Soil Modeling

The foundation that has been considered for this system is a pile group. The foundation employs piles with length of L and elastic modulus of E. Furthermore, the piles have a circular cross section and their radius is assumed to be r. The pilecap is assumed to be in the term of square. If an independent spring (soil substitute) is used for each pile, one of the springs will always be under tension in each cycle, which causes the modeling error.

To eliminate this error, a spring has been used for a row of piles in the direction of the vibration (which is supposed to be transverse in this study), so that the spring tension can be considered as the pressure of the soil in the opposite direction. This spring is attached to all of the piles located in a row. According to the fact that the stiffness of a pile group is obtained by summing all piles' stiffnesses and that the pile group, used here in this study is assumed to have a square shape, so it can be written:

$(n\times n\times EI)$: The total equivalent stiffness of all piles (17)

Moreover, due to the fact that the stiffnessof the springs (soil) in each (transverse) row is summed together, it is possible at the end to put their total sum in the matrix. Equation (18) should be included in the general matrix to consider the stiffness of the soil in the system.

$$[K_a]=\int_0^L K_a[N(x)]^T[N(x)]dx \tag{18}$$

$$[K_a]=\begin{bmatrix}\frac{k_a\,n\,L(3\pi-8)}{2\pi} & \frac{k_a\,n\,L^2}{12}\\ \frac{k_a\,n\,L^2}{12} & \frac{k_a\,n\,L^3}{105}\end{bmatrix} \tag{19}$$

3.3. Structure Modeling

If the rotational and translational degrees of freedom at the bottom of the structure are assumed to be identical to the two degrees of freedom above the pile group, by using the slope-deflection method, it will be:

$$[K_s]=\begin{bmatrix} K_s & -K_s & -K_s h \\ -K_s & K_s h & K_s h \\ -K_s h & K_s h & K_s h^2 \end{bmatrix} \tag{20}$$

$$[M_s]=\begin{bmatrix} m_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

$$K=\begin{bmatrix} K_s & -K_s & -K_s h \\ -K_s & \dfrac{n^2EI\pi^4}{32L^3}+\dfrac{nK_aL(3\pi-8)}{2\pi}+K_s h & \dfrac{nK_aL^2}{12}+\dfrac{n^2EI(\pi-6)}{L^2}+K_s h \\ -K_s h & \dfrac{nK_aL^2}{12}+\dfrac{n^2EI(\pi-6)}{L^2}+K_s h & \dfrac{4n^2EI}{L}+\dfrac{nK_aL^3}{105}+K_s h^2 \end{bmatrix} \tag{22}$$

$$M=\begin{bmatrix} m_s & 0 & 0 \\ 0 & \dfrac{\overline{m}L(3\pi-8)}{2\pi} & \dfrac{\overline{m}L^2}{12} \\ 0 & \dfrac{\overline{m}L^2}{12} & \dfrac{\overline{m}L^3}{105} \end{bmatrix} \tag{23}$$

$$\overline{m}=n^2\rho_{pile}A_{pile}$$

The system equivalent frequency is a frequency in which the system response be infinity for a zero damping. Wolf (1989) showed that for a structure with three degrees of freedom, this case occurs when the following equation is established. The equivalent frequency of the system is calculated by substituting the first to third natural frequencies in equation (24)

$$\frac{1}{\tilde{\omega}^2}=\frac{1}{\omega_S^2}+\frac{1}{\omega_h^2}+\frac{1}{\omega_r^2} \tag{24}$$

4. Research Methodology

In order to ensure the accuracy of the presented analytical model for calculating the system equivalent period of the pile-soil-structure, it is necessary to compare the results with the analytical studies of other researchers, as well as the obtained results of analyzing a finite element software. In this study, ABAQUS finite element software has been selected for analysis and comparison. The numerical models have been made first in order to ensure the accuracy of the obtained results from Abaqus software and the obtained results were compared with the results of the analytical equations.

4.1. Verification by Alluvium Modeling

Calculating the free vibration frequency of an alluvium with an approximate dimensions of 30×120 m was considered for modeling verification in finite element software. Figure 3 shows the modeling of this alluvium in the software as a two-dimensional one and its results have been compared with the results of Equation 25. The specifications of alluvium have also been given in Table 1. Equation 25 presents the fundamental period of a homogeneous alluvium: (4th edition of Iran's 2800 letter).

$$\mathrm{T}=\frac{4\,H}{Vs} \tag{25}$$

Where in the proposed formula (H) is the depth of alluvium in meters and (Vs.) is the shear wave velocity in the alluvium materials in meters per second.

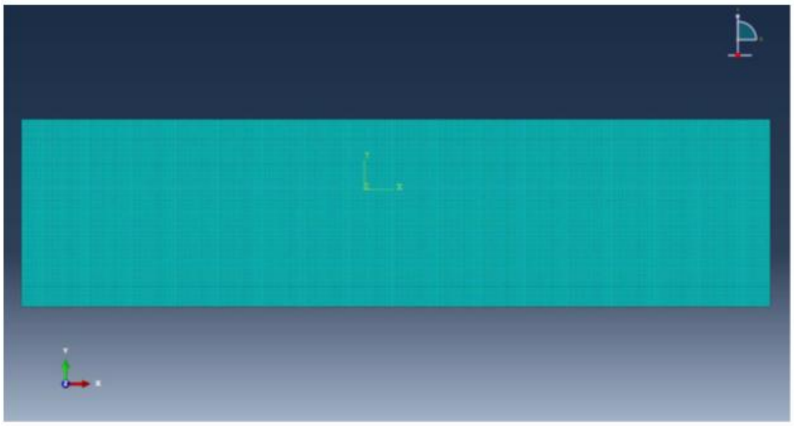

Figure 3. An Overview of Alluvium Modeling in Finite Element Software

Table 1.Specifications of Alluvium Materials

E (N/m^2)	ρ (kg/m^3)	Vs (m/s)	Gs (N/m^2)	ν	h (m)
2.4E+07	1900	70	9.3E+06	0.3	30
4.9E+07	1900	100	1.9E+07	0.3	30
2.0E+08	1900	200	7.6E+07	0.3	30
4.4E+08	1900	300	1.7E+08	0.3	30
7.9E+08	1900	400	3.0E+08	0.3	30
1.2E+09	1900	500	4.8E+08	0.3	30
1.8E+09	1900	600	6.8E+08	0.3	30
2.4E+09	1900	700	9.3E+08	0.3	30
3.2E+09	1900	800	1.2E+09	0.3	30
4.0E+09	1900	900	1.5E+09	0.3	30
4.9E+09	1900	1000	1.9E+09	0.3	30

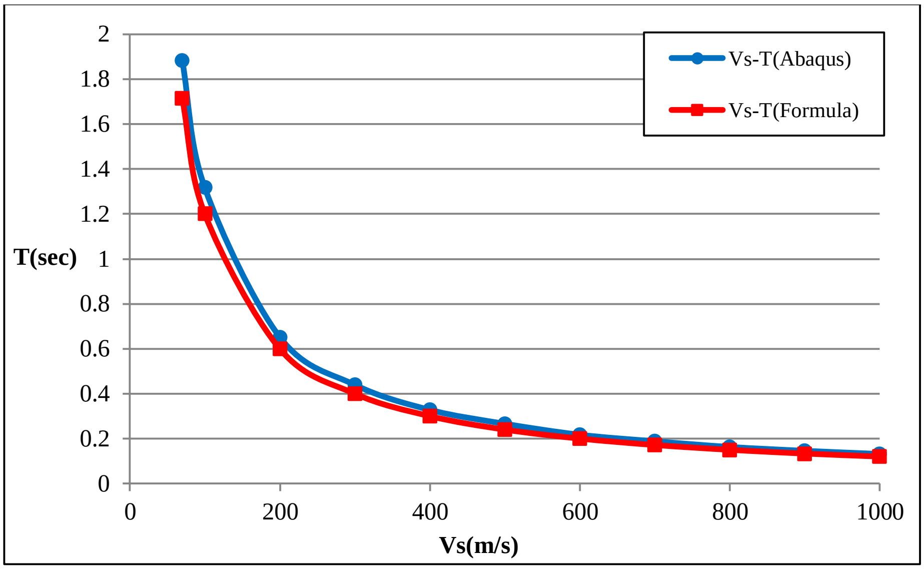

Figure 4. Comparison of the obtained results of the formula and numerical modeling

Figure 4 shows that there is a good agreement between the results of the numerical modeling in finite element software and that of the proposed analytical expression and can be used as a suitable validation for the software modeling.

4.2. Numerical Modeling for Steel Frames

In the numerical method, the soil has been removed and the equivalent spring and damper have been used. Considering the necessity of modeling of structures with logistic dimensions, the required information has been extracted from research studies of Minadiis et al. (2014) for modeling of structures and numerical analysis. In the study by Minadiis et al. (2014), just the structures with no piles which were based on soil have been investigated and their proposed expression has only examined the interaction between the soil and the structure, T_{SSI}. In the present study, numerical modeling results has been compared with UBC-97 and EC-8 regulations, and Minastiis et al. (2014). Table 2 provides the results for this section. According to Table 2 and comparison of the results in Fixed Base, there is a logical match between the outputs of the results.

Table 2. Comparison of the results of the research with the results of Minastiis et al. research (2014) and UBC-97 and EC8 in Fixed-Base

Frame	$T_{\text{Fixed Base Numerical}}$ Current Study	TEC8	TUBC-97	$T_{\text{Fixed Base Numerical}}$ Minasidis et al. (2014)
1	0.631791761	0.441672956	0.52510975	0.647
2	0.614175163	0.441672956	0.52510975	0.68
3	0.587889477	0.441672956	0.52510975	0.722
7	1.2426837	0.742802411	0.914269177	1.128
8	1.111111111	0.742802411	0.914269177	1.161
9	1.178272652	0.742802411	0.914269177	1.194
13	1.539408867	1.00679662	1.264582261	1.45
14	1.461988304	1.00679662	1.264582261	1.483
15	1.460984411	1.00679662	1.264582261	1.5
19	2.022940141	1.249239769	1.591835093	1.706
20	1.893939394	1.249239769	1.591835093	1.765
21	1.807370457	1.249239769	1.591835093	1.798
25	2.384756636	1.476822536	1.902944338	2.046
26	2.317228595	1.476822536	1.902944338	2.112
27	2.19389658	1.476822536	1.902944338	2.195

In the next step, the results of Minadiis et al. research (2014) and the present numerical study have been compared for the soil-structure interaction in the case of having pile. (Modeling shown in Figure 5) As it is shown in Fig. 6, there is a good agreement between the results of the two present researches.

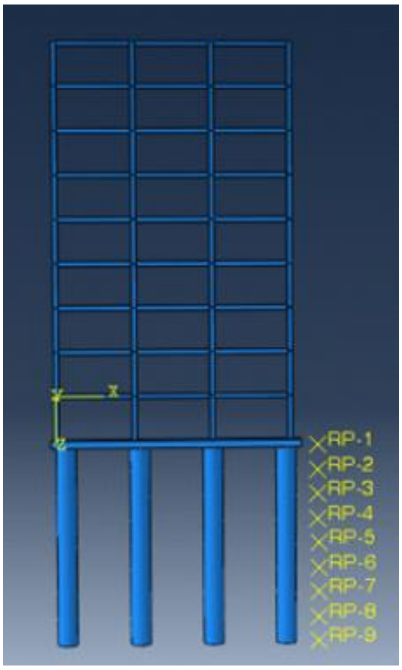

Figure 5. Structure modeling on the pile group in finite element software

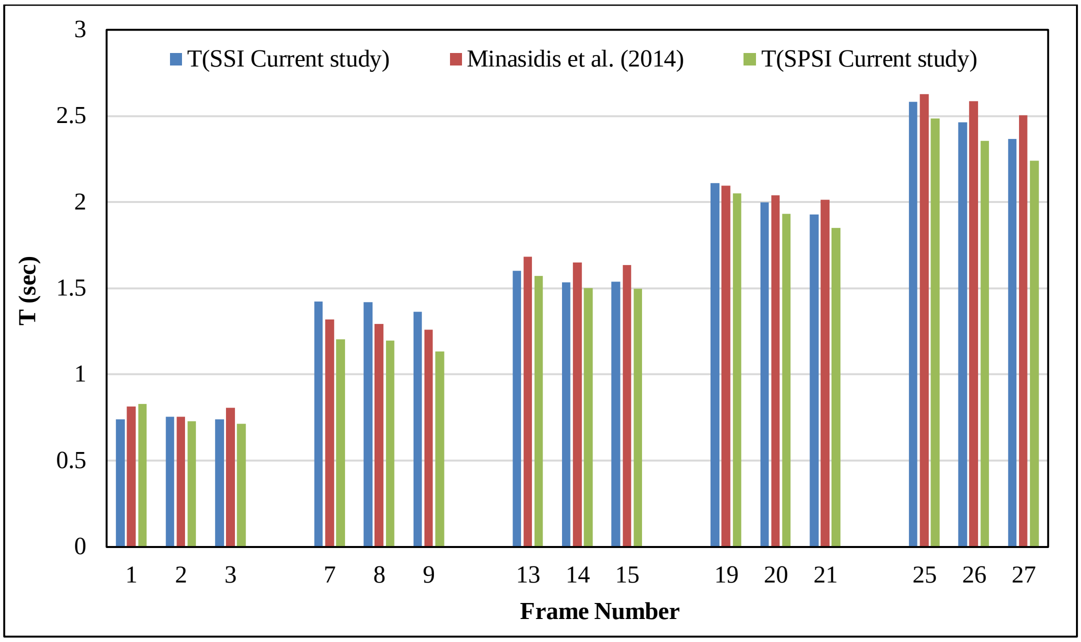

Figure 6. Comparison of T_{SSI} and T_{SPSI} cases of the present study with $T_{minasidis}$ research

According to the numerical studies carried out in this section, as it has been shown in Figure 6, the pile group decreases the free vibration period of the structure and the obtained period is greater than Fixed Base case and less than SSI case. Generally, the free vibration period of the structure decreases in the case of pile existence. In this research, one step further is taken than Minastiis research and an expression (T_{SPSI}) is proposed for metal frame structures, while adding a pile group and examining its effect. In the performed modeling in the presence of a pile group, the ratio of the length of the pile to the height of the structure has been equaled to 0.5 and the ratio of the diameter of the pile to the length of the pile has been equaled to 0.05, and the pile group has been considered as square and 4 × 4 for all frames. Therefore, equation 26 can be presented as in UBC-97 and UC8 regulations.

$$T_{SPSI} = 0.27 \times N^{0.8} \tag{26}$$

N: number of steel structures story located on the pile group in the case of soil-pile and structure interaction.

5. A Proposed Analytical Model for Frames with Floating Pile-Soil-Structure System

As it was stated in the proposed analytical method, a mass-spring-damper system have been used. In this model, the soil around the pile has been removed and its equivalent spring, damper and mass have been substituted. One of the fundamental differences between this model and the models presented by previous researchers is that the mass-participation coefficient of the soil has been added to this model. It should be noted that the coefficient is not dependent only on the Poisson ratio of soil, but also changes with the shear modulus of the soil and is more accurate than the coefficients used by the previous investigators. Pachko et al. (2008) presented the frequency independent coefficients for soil stiffness and damping using the statistical operations on Novak's research result (1974). Figure 7 shows a view of a one degree-of-freedom structure system model considering A) Floating piles, B) End-bearing piles.

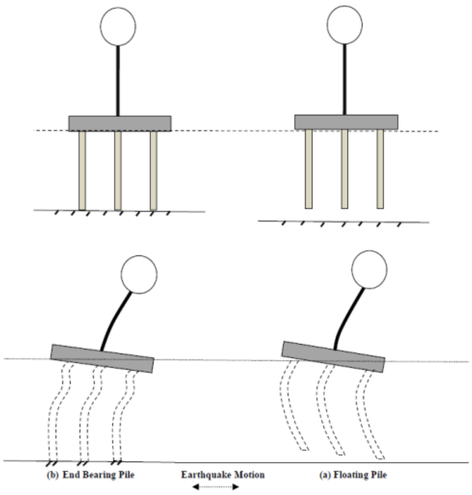

Figure 7. A view of a one degree-of-freedom structure system considering a) Floating piles, b) End-bearing piles

$$Ku = G\pi f(a_0,\nu,D) = G\pi\{\mathrm{Re}\,al[f(a_0,\nu,D)] + i\,\mathrm{Im}\,ag[f(a_0,\nu,D)]\} \tag{27}$$

$$\mathrm{Re}\,al[f(a_0,\nu,D)] \approx \alpha_K - \alpha_m a_0^2 \tag{28}$$

$$\mathrm{Im}\,ag[f(a_0,\nu,D)] \approx \alpha_C a_0 \tag{29}$$

$$Ku \approx G\pi(\alpha_K - \alpha_m a_0^2 + i\alpha_C a_0) \tag{30}$$

By comparing these equations and the results of Novak (1974), the stiffness, damping and mass of the soil values will be as follows:

$$Ku \approx K_a - m_a\omega^2 + ic_a\omega \tag{31}$$

$$K_a = G\pi\alpha_K \tag{32}$$

$$m_a = \pi r^2\rho\alpha_m \tag{33}$$

$$C_a = \pi r_0 V_S \rho\alpha_C \tag{34}$$

The structure has been modeled on a discrete mass with one horizontal degree-of-freedom, and the pillar and piles have four degrees of freedom. The matrix of stiffness, mass, and damping for the piles can be simplified as follows, where Ka is obtained by Equation 32.

$$[K_{Pile}] = \int_0^L EI[N''(x)]^T[N''(x)]dx + \int_0^L K_a[N(x)]^T[N(x)]dx \tag{35}$$

$$[M_{Pile}] = \int_0^L \rho A[N(x)]^T[N(x)]dx \tag{36}$$

$$[C_{Pile}] = \int_0^L c[N(x)]^T[N(x)]dx \tag{37}$$

If for a system with five degrees-of-freedom, Equation 38 exists between the frequency of the system and the frequency of each single degree of freedom, like what existed for the three degrees-of-freedom system then, we can calculate the system equivalent frequency by Equation 38 (Shirgir et al., 2015) [6, 8]:

$$\frac{1}{\omega_e^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} + \frac{1}{\omega_4^2} + \frac{1}{\omega_5^2} \tag{38}$$

$$\begin{aligned}
&\frac{1}{\omega_e^2} = \frac{m_s}{k_s} + \lambda\frac{G'^2L^8(\alpha) + G'L^4E'(\beta) + E'^2(\gamma)}{L^3G'(G'L^4+240E')(G'L^4+2800G')}\\
&(\alpha) = \frac{4}{3}\overline{m}'L^3 + \frac{16}{3}L^2 - 80m_sLh + 400m_sh^2\\
&(\beta) = 3040\overline{m}'L^3 + 9280m_sL^2 - 81600m_sLh + 260800m_sh^2\\
&(\gamma) = 10^3(448\overline{m}'L^3 + 896m_sL^2 + 2688m_sh^2 - 2688m_sLh\\
&G' = n\times G_{soil}\times\pi\times\alpha_k
\end{aligned} \tag{39}$$

Where n is the number of piles in a row of piles in a square pile group, ρ represents density, I is moment of inertia around the pile (circular), E denotes modulus of elasticity of the pile and G is the shear modulus of the soil [20].

5.1. Correction of the Analytical Equation Using Direct Numerical Method for Steel Frames

According to the fact that the mass participation coefficient which is introduced in Pacheco et al. (2008) is not used in the equation 39 and that the coefficient α_m was set to zero, in order to obtain the new mass participation coefficient (λ), some of the frames expressed in the finite element software have been modeled using the direct method. (Fig. 8) the coefficient λ is calculated using the equation 40.

$$\lambda = \frac{T_{DirectMethod} - T_{Fixed\,Base}}{T_{Analytical} - T_{Fixed\,Base}} \tag{40}$$

The coefficient λ can be obtained from Figures 8 and 9

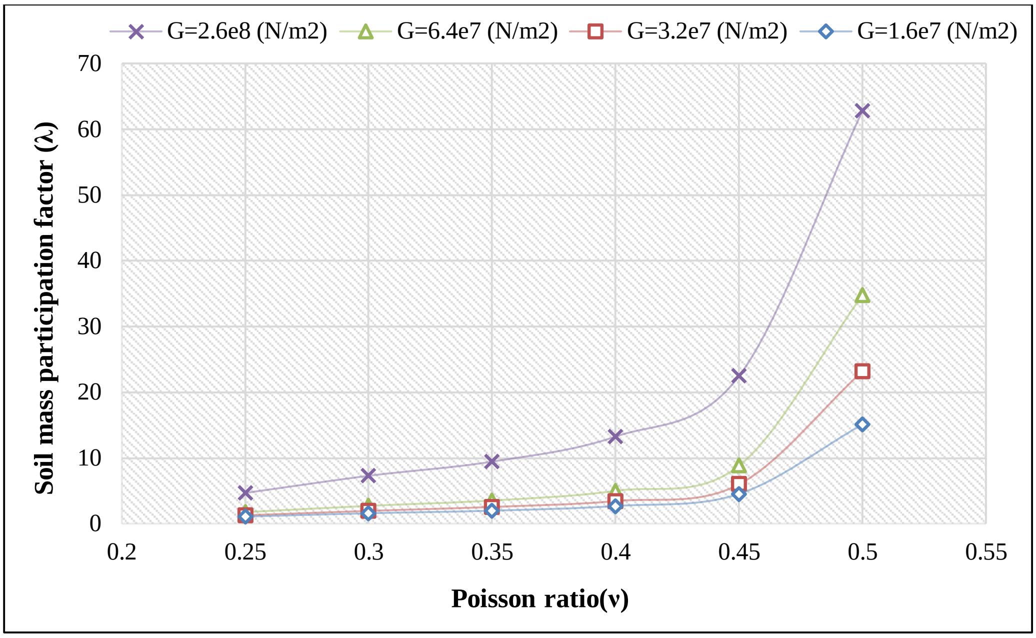

Figure 8. Soil mass participation coefficient of λ versus soil Poisson ratio

Now it is possible to modify the proposed analytical expression (Equation 39) and obtain the amount of λ from figure 8.

In Figure 9, changes in the ratio of free vibration period have been shown against the shear modulus. The higher the shear modulus (G) of the soil, the changes in the ratio of the structure period decrease and this number approaches one. According to figure 10, there is an inverse relation between the relative changes of the structure period with the Poisson ratio of the soil around the piles. As Poisson ratio decreases, the period ratio of the structure increases.In fact, it can be said that when the shear modulus is constant and the Poisson ratio increases, since in the linear behavior of the soil the equation E = 2G (1 + ν) is established, the modulus of elasticity increases accordingly. Increasing the modulus of elasticity means stiffening of the earth and reducing the effects of the soil and structure interaction.

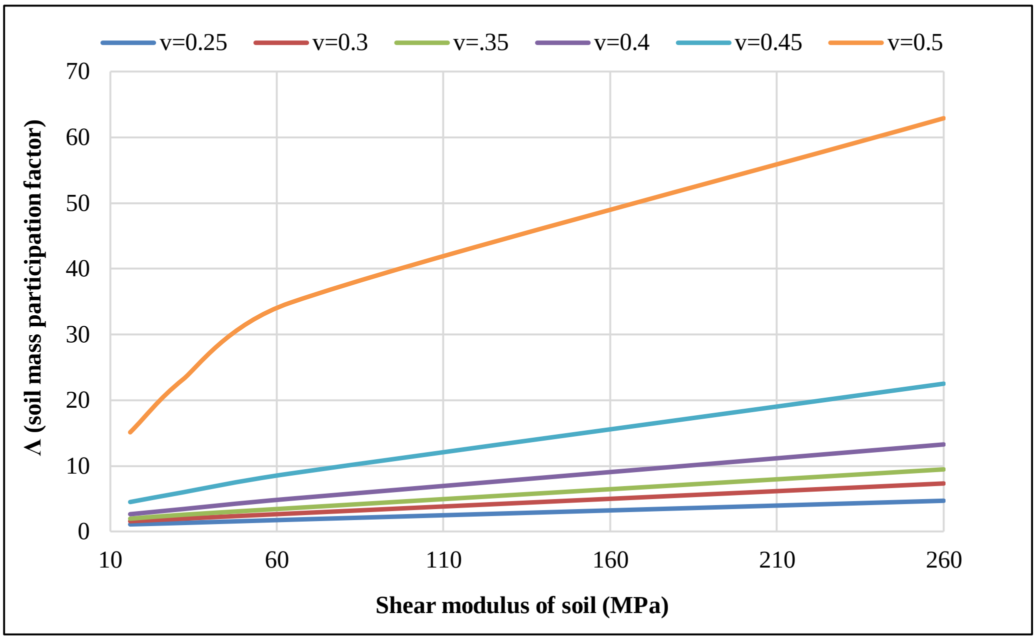

Figure 9. Mass participation coefficient of the soil around the piles versus soil shear modulus

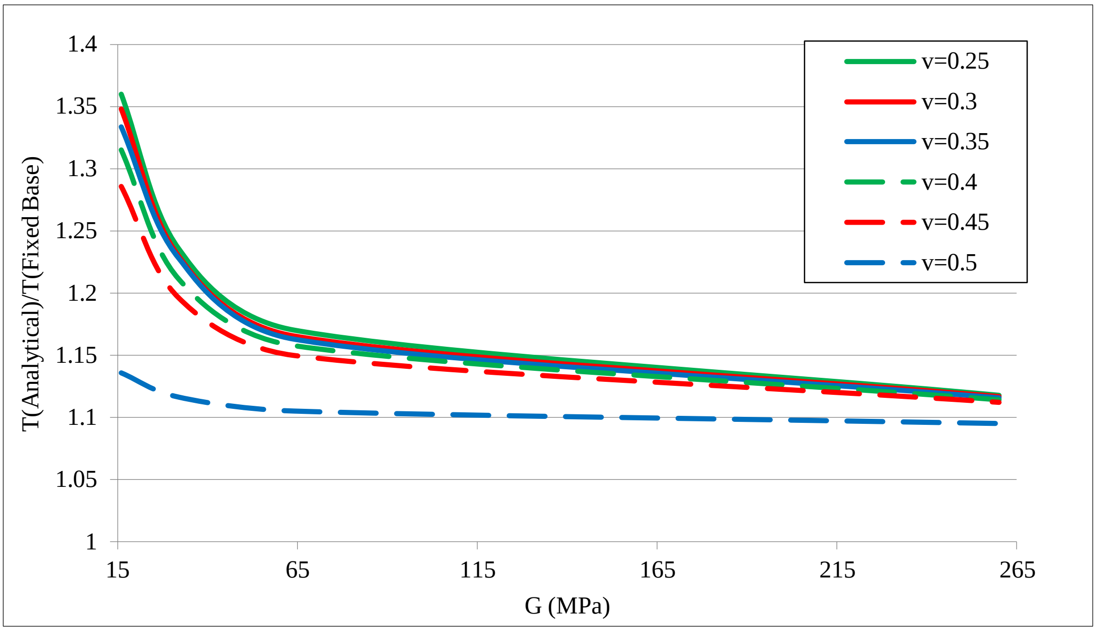

Figure 10. System period versus the shear modulus and different Poissons ratios of the soil

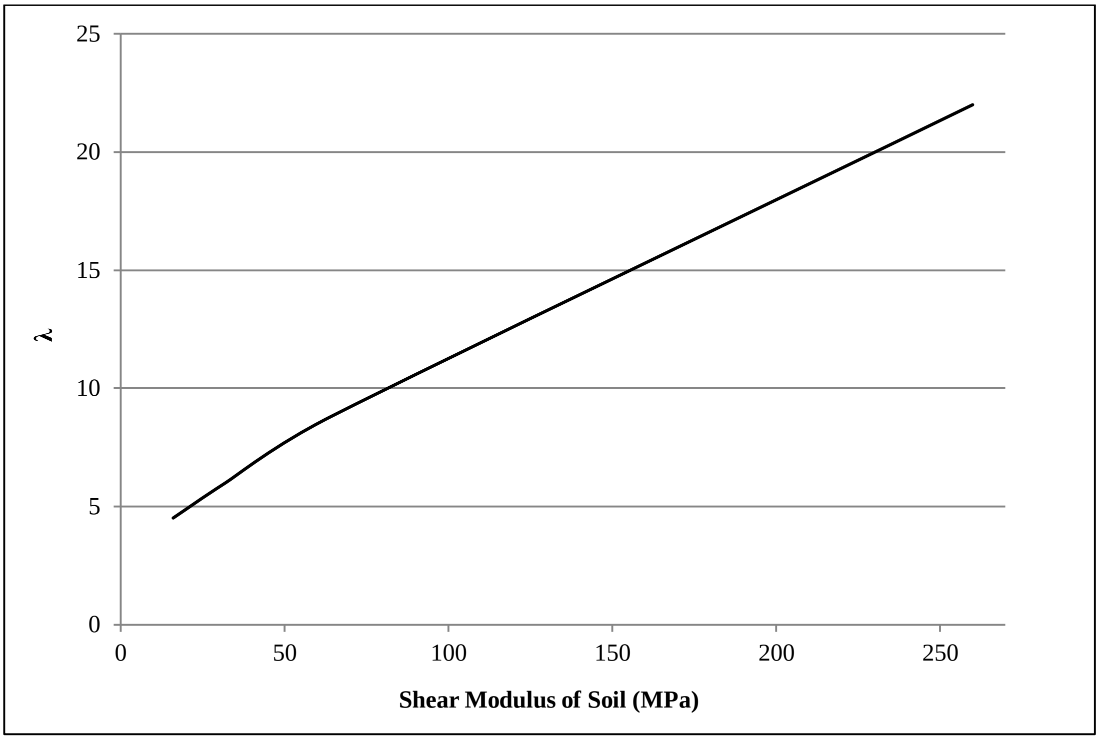

Figure 11. Coefficient of mass participation of the soil around the piles versus soil shear modulus

As shown in figure 11, as the shear module of the soil (G) around the piles increases, the value of the mass participation coefficient of the soil around the piles (λ) increases.

6. Proposed Analytical Model for structure-Soil-End-bearing Pile System

In view of the different behaviors of the piles in end-bearing and floating cases, this section presents a new analytical model for calculating the free vibration frequency of the structures on the end-bearing pile group. In this proposed analytical method, a model of discrete mass, spring, and damper has been used. In this model, the soil around the piles has been removed and its equivalent spring, damper and mass have been replaced. (Figure 2 (b)) In the presented model, the piles have been considered end-bearing. Therefore, the proper shape functions have been presented for end-bearing piles.

$$\begin{cases} N:\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix}=\left[1-cos(\frac{\pi x}{2L}) \quad x(1-\frac{x}{L})^2\right] \\ N':\left[\frac{\pi}{2L}cos(\frac{\pi x}{2L}) \quad 1-\frac{2x}{L}+\frac{3x^2}{L^3}\right] \\ N'':\left[\frac{\pi^2}{4L^2}cos(\frac{\pi x}{2L}) \quad -\frac{2}{L}+\frac{6x}{L^3}\right] \end{cases} \tag{41}$$

As it is shown in figure 12, Pacheco et al. (2008) have presented the frequency-independent coefficients for soil stiffness and damping using statistical operations on Novak's research results (1974).

$$Ku = G\pi f(a_0,\nu,D) = G\pi\{Real[f(a_0,\nu,D)] + i\,Imag[f(a_0,\nu,D)]\} \tag{42}$$

$$\begin{aligned} &Real[f(a_0,\nu,D)] \approx \alpha_K - \alpha_m a_0^2 \\ &Imag[f(a_0,\nu,D)] \approx \alpha_C a_0 \\ &Ku \approx G\pi(\alpha_K - \alpha_m a_0^2 + i\alpha_C a_0) \end{aligned} \tag{43}$$

By comparing these equations and the results of Novak (1974), the stiffness, damping and mass of the soil will be as follows:

$$\begin{aligned} &Ku \approx K_a - m_a\omega^2 + ic_a\omega \\ &K_a = G\pi\alpha_K \\ &m_a = \pi r^2\rho\alpha_m \\ &C_a = \pi r_0 V_S \rho\alpha_C \end{aligned} \tag{44}$$

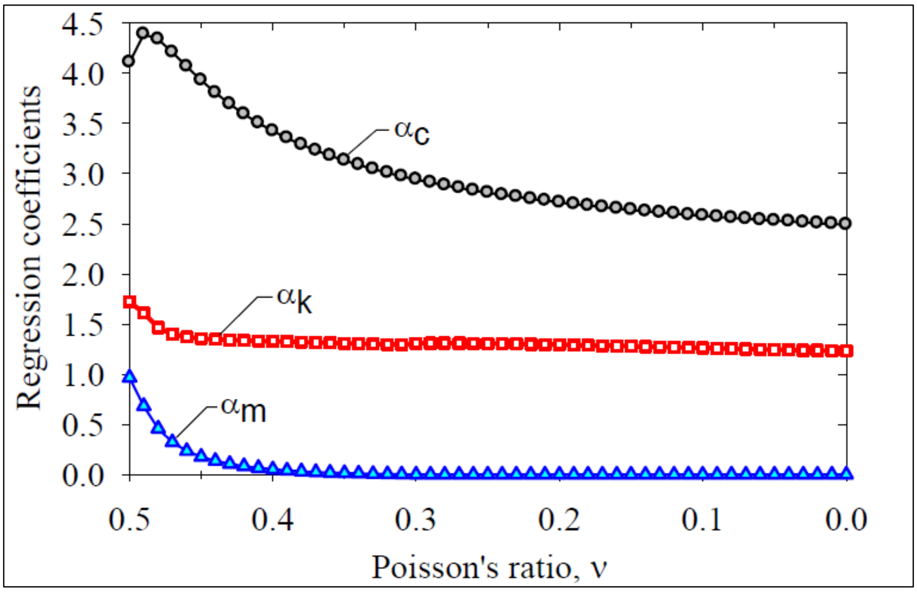

Figure 12.The required coefficients for the mass participation and soil stiffness. Pacheco et al (2008)

In a schematic model equivalent to SPSI system, the structure has been modeled as a discrete mass with a horizontal degree-of-freedom, and the pillar and piles have two degrees of horizontal and rotational freedom. It should be noted that in this case the piles have been modeled as end-bearing. The matrix of stiffness, mass and damping for piles can be simplified in this way, where k_a is obtainedfrom Equation 44.

In Figure 2b, the schematic model equivalent to SPSI system and its degrees-of-freedom is shown. The structure has been modeled as a discrete mass with a horizontal degree-of-freedom, and the pillar and piles have two degrees of freedom.

$$[K_{Pile}] = \int_0^L EI[N''(x)]^T[N''(x)]dx + \int_0^L K_a[N(x)]^T[N(x)]dx \quad (31)$$

$$[M_{Pile}] = \int_0^L \rho A[N(x)]^T[N(x)]dx \quad (32)$$

$$[C_{Pile}] = \int_0^L c[N(x)]^T[N(x)]dx \quad (33)$$

$(n \times n \times EI)$: Total stiffness of all piles (34)

In which c is the soil damping per unit length of the pile. By calculating the integrals in the above equations, the matrix of stiffness, mass, and damping are obtained for the pile, which includes the second and thirddegrees-of-freedom. Moreover, due to the fact that the stiffness of the springs (soil) in each (transverse) row is added to each other, it is possible to put their summed value in the matrix at the end. Now, to obtain the matrix of stiffness and mass of the structure, structures with different degrees-of-freedom are equalized with a structure with one degree-of-freedom, and matrices of stiffness and mass are obtained for it. If the rotational and translational degree-of-freedom on the pile cap are equal with the degree-of-freedom above the pile group, then we have.

$$[Ks] = \begin{bmatrix} K_s & -K_s & -K_s h \\ -K_s & K_s h & K_s h \\ -K_s h & K_s h & K_s h^2 \end{bmatrix} \quad (49)$$

$$[M_s] = \begin{bmatrix} m_s & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (50)$$

By integrating the matrices of the pile-soil and structure, the matrices of the whole system is obtained as follows:

$$K = \begin{bmatrix} K_s & -K_s & -K_s h \\ -K_s & \frac{n^2 EI\pi^4}{32L^3} + \frac{nK_a L(3\pi-8)}{2\pi} + K_s h & \frac{nK_a L^2}{12} + \frac{n^2 EI(\pi-6)}{L^2} + K_s h \\ -K_s h & \frac{nK_a L^2}{12} + \frac{n^2 EI(\pi-6)}{L^2} + K_s h & \frac{4n^2 EI}{L} + \frac{nK_a L^3}{105} + K_s h^2 \end{bmatrix} \quad (51)$$

$$M = \begin{bmatrix} m_s & 0 & 0 \\ 0 & \frac{\overline{m}L(3\pi-8)}{2\pi} & \frac{\overline{m}L^2}{12} \\ 0 & \frac{\overline{m}L^2}{12} & \frac{\overline{m}L^3}{105} \end{bmatrix} \quad (52)$$

$$= n^2 \rho_{pile} A_{pile}$$

$$\overline{m} = n^2 \rho_{pile} \times A_{pile} + n^2 A_{pile} \times \rho_{soil} \times \alpha_m$$

$$\overline{m} = n^2 A_{pile}(\rho_{pile} + \alpha_m \rho_{soil}) \quad (53)$$

If for a system with three degrees of freedom, the following relation exists between the frequency of the system and the frequency of each single degree of freedom, we can calculate the system's equivalent frequency by the following equation. Wolf, 1985:

$$\frac{1}{\omega_e^2} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} + \frac{1}{\omega_3^2} \quad (54)$$

After accepting the equation 54 using MATLAB software, and based on the presented method in equation 5, the corresponding eigenvalues of total stiffness and mass matrices of the system can be calculated. Next, with the aid of suitable simplifications, solving and substituting the mass and stiffness matrices into equation 5, equation (55) will be obtained as follows:

$$\frac{1}{\omega_e^2}=\frac{m_s}{k_s}\left(\frac{S+O+hkQ}{O-\frac{1392E'hk}{L}-3.3kyL^3}\right)$$
$$S=(\overline{m}\omega_s^2)*(590E'-3L^4y)$$
$$O=\frac{1393E'^2}{L^4}+\frac{1392E'hk}{L}-1.6E'L^4y^2+3.3hkyL^3+491E'y \qquad (55)$$
$$Q=\frac{1990E'}{L^2}+\frac{1059E'h}{L^3}-58L^2y+79Lhy+384h^2k-384hk$$
$$E'=E\times I\times n\times n$$
$$y=n\times G\times\pi\times\alpha_k$$

The n is the number of piles in a row of piles in a square pile group, ρ is density, I is first moment of inertia around the pile (circular), E is modulus of elasticity of the pile and G denotes shear modulus of the soil[19].

In Figure 13 the relative changes of the free vibration period of the SPSI case to fixed based case against the shear modulus is shown. When the shear modulus of the soil (G) increases, the changes in the period ratio of the structure $T_{SPSI}/T_{FixedBase}$ decrease and this number approaches to one. Note that in this figure, other soil specifications is considered constant and v is Poisson ratio of the soil around the piles. As it is shown in figure 13, there is an inverse relation between the relative changes of the structure period and the Poisson ratio of the soil around the piles and, by decreasing Poisson ratio of the soil, the ratio of the period of the structure , $T_{SPSI}/T_{FixedBase}$, increases ;in other words, the effects of the soil - pile (end-bearing) - Structures interaction increase.

Figure 14 is represented in order to investigate the changes in the effects of the soil-structure interaction, by changing the length of the pile to the height of the structure ratio, L_{pile}/h_S. According to Figure 14, it can be seen that for L_{pile}/h_S ratio, more than 0.8%, the soil-structure interaction stays constant and will not change considerably. Next investigation on the response of the system was continued by examining the changes in the effect of the soil-structure interaction as the soil stiffness to the pile bending stiffness ratio changes. With respect to Figure 15, which is studied for the same structure as in Figure 14, changes can be observed clearly. As it is shown in Figure 15, there is a certain ratio of G_{Soil}/EI_{Pile} in which the effects of the soil-structure interaction are minimal. For this particular structure, it is approximately 0.8.

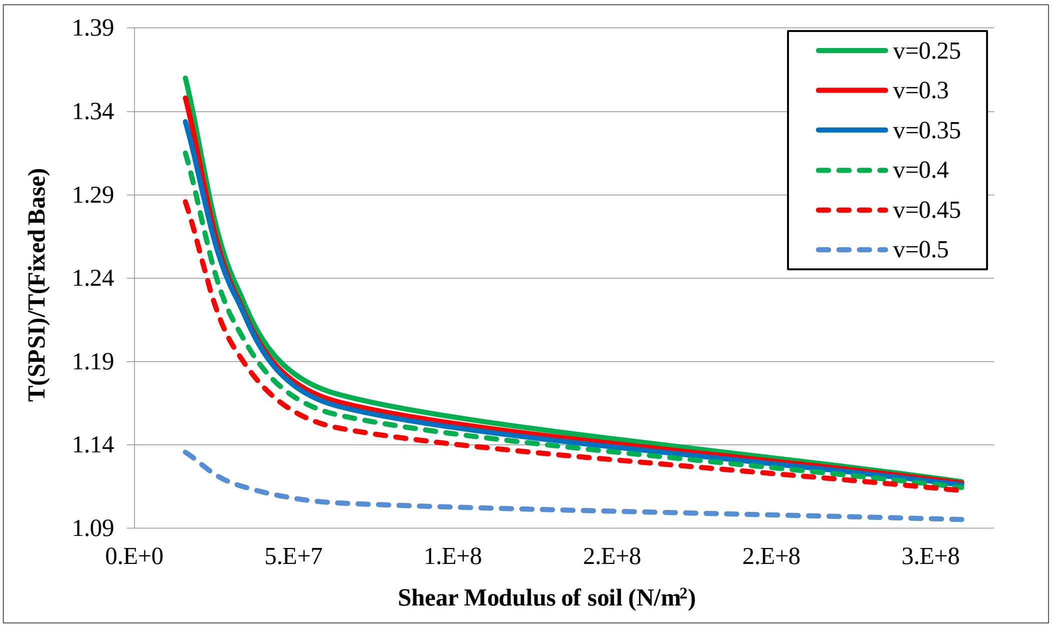

Figure 13. System period versus the shear modulus of the soil and different soil Poissons ratio

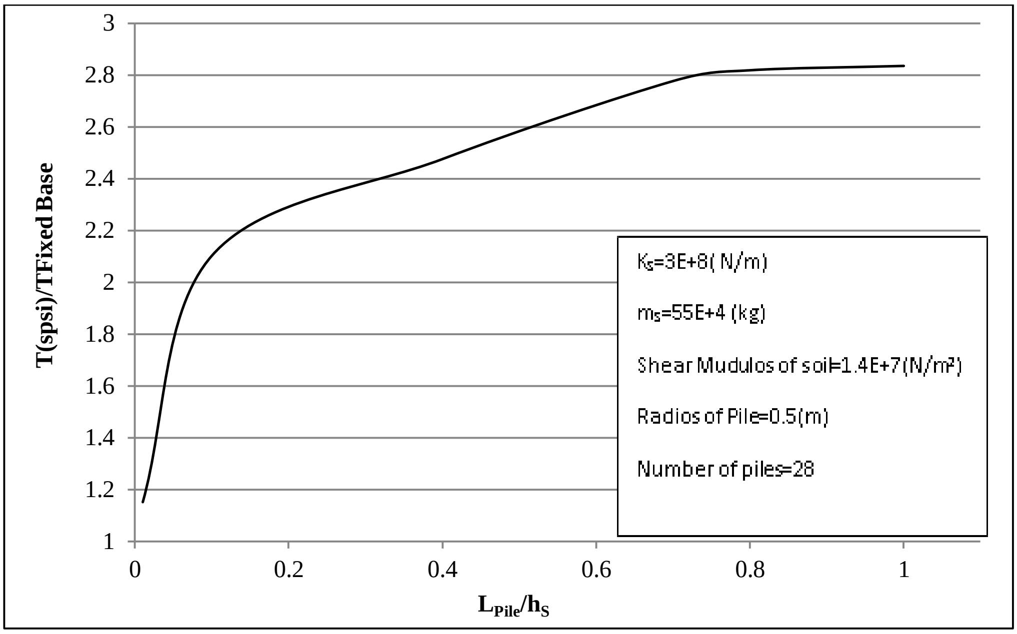

Figure 14. System period versus the length of the pile to the height of the structure ratio for a particular structure

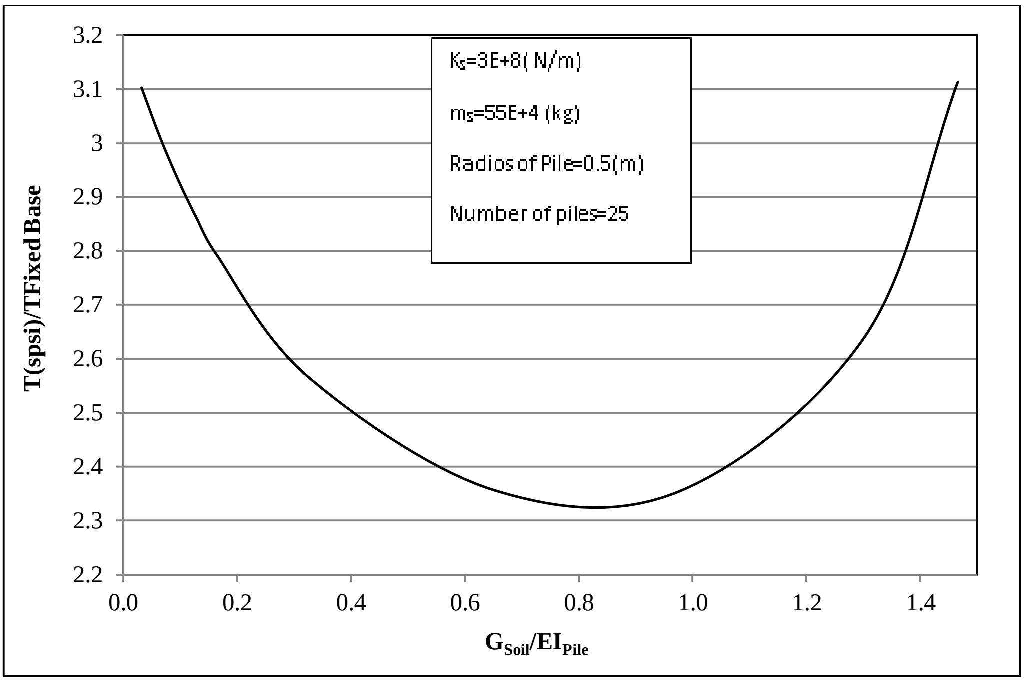

Figure 15. System period versus the shear modulus of the soil to the pile bending stiffness ratio for a particular structure

7. Discussion

To investigate the validity of the proposed analytical expression, the results of numerical studies in Fixed Base and floating piles were compared with the analytical results of end-bearing piles as it is shown in Table 3. As it was expected, when the piles areend-bearing, the equivalent period of the system is less than that of a floating piles and is closer to fixed based piles. It should be noted that in the numerical model, the piles are modeled as floating piles.

Table 3. Comparison of Analytical expression Results with those of the numerical studies for Fixed Piles

Frame	T-Fixed Base Numerical (Current Study)	T_{-SPSI}(Floating pile model) (Numerica-current study)	T_{-SPSI} (Propsed Analytical Method)
1	0.63	0.82	0.67
2	0.61	0.72	0.66
3	0.58	0.71	0.63
7	1.24	1.20	1.27
8	1.11	1.19	1.14
9	1.17	1.13	1.20
13	1.53	1.57	1.56
14	1.46	1.50	1.48
15	1.46	1.49	1.48
19	2.02	2.05	2.04
20	1.89	1.93	1.91
21	1.80	1.84	1.82
25	2.38	2.48	2.39
26	2.31	2.35	2.33
27	2.19	2.23	2.21

Based on the observed results, the question arises that does the difference between the vibration period of the structure in two different cases of end-bearing and floating piles decrese as the height of the structure increases? Perhaps the answer could be stated that the interaction of the soil structure tends to decrease the displacement of the structure, but the overall displacement increases as the foundation is able to translate and rotate. But in the case that the piles decrease the ability of the foundation's rotation, the displacements of the structure floors are more likely to result from the relative displacement of the floors, and the displacement due to the foundation rotation will have a small contribution to the total displacement of the system. For tall structures, this causes the amount of period of the structure to be close to its fixed foundation. In order to verify the validity of the proposed analytical expression, the obtained results should be compared, as shown in Table 4, with the results of the case studies. As it was expected, in the case where the proposed analytical model relates to the end-bearing piles, the system period presents less values due to the fixation on the base of the piles. According to this, it can be seen in Table 4 that, in almost all cases, if the piles areend-bearing, the system period has less values than floating piles. According to the above, if the percentage difference in response values of the two cases of floating and end-bearing piles is investigated, it can be seen that in the case of end-bearing piles, in average, the response of soil-pile-structure system would have 24 percent difference with that of the case with floating pile. For the case studies represented here in this study, the difference has been expressed as a relative of the results of the floating piles (Table 5)

Table 4. Comparison of the results of the proposed analytical expression in equation 49 (T-Floating Pile (s)) and (T-Fixed End Pile (s)) with those of the case studies

Bridge*	m_s (kg)	S (N/m)	h_s (m)	G_{Soil} (Pa)	L_{pile}(L)	N_{pile}	T_e^+ (s)	T(Fixed End Pile) (s)	T(Floating Pile) (s)
Dumbarton Bridge in California Fenves et al. (1992)	550,000	3.0E+08	16	1.4E+07	13	28	1.6	1.30	1.63
Northwest Connector in California Fenves and Desroches (1994)	280,000	2.0E+08	17	2.1E+08	15	78	0.55	0.24	0.31
Painter Street Bridge in California Makris et al. (1994)	250,000	1.2E+08	6	1.0E+08	7.62	20	0.27	0.31	0.46
Meloland Road Overpass in California Werner et al. (1987)	364,000	1.0E+08	7.95	1.0E+08	15.2	25	0.4	0.38	0.41
Ohba-Ohashi Road Bridge in Japan Ohira et al. (1984)	550,000	1.7E+07	10	1.1E+07	22	64	1.3	1.14	1.19
Landing Road Bridge in New Zealand Berrill et al. (2001)	210,000	2.5E+08	6	8.2E+07	9	8	0.4	0.18	0.38
Yachiyo Bridge in Japan Hamada (1992)	200,000	6.0E+07	9	5.5E+07	11	33	1.1	0.37	0.49
Hanshin Expressway in Kobe – Japan Gazetas and Mylonakis (1998)	1,100,000	2.0E+09	11.25	7.6E+07	15	17	0.7	0.493	0.63

* The data for the case studies are after Minasidis et al. (2014); + Equivalent period of vibration of the structure and soil.

Table 5. Comparison between end-bearing and floating pile responses

	End Bearing Pile Current Study	Floating Pile Shirgir et al (2015)	Difference (The percentage of Floating Piles results)
Dumbarton Bridge in California	1.3	1.635	0.204
Northwest Connector in California	0.24	0.3111	0.228
Painter Street Bridge in California	0.31	0.4653	0.333
Meloland Road Overpass in California	0.38	0.4191	0.093
Ohba-Ohashi Road Bridge in Japan	1.14	1.192	0.043
Landing Road Bridge in New Zealand	0.18	0.3892	0.537
Yachiyo Bridge in Japan	0.37	0.4921	0.248
Hanshin Expressway in Kobe – Japan	0.493	0.639	0.228
Average	0.239		

8. Conclusion

- Given that UBC97 and EC8 regulations have not provided any equations for the initial calculation of the free vibration period of metal structures considering the interaction of soil-pile-structure, numerous numerical studies have been carried out in this study for metal frames located on the pile group and a new expression was proposed.
- Due to the importance of the soil mass participation around the piles during free vibration, a more precise analytical expression than those proposed by previous researchers, was developed here in this study. In this regard, the parameter λ has been introduced as a soil mass participation. Comparing the results of this expression with those of the case and numerical studies showed that it has a good accuracy and can be useful as a primary expression to designers in the field of dynamic analysis of these types of structures.
- Based on the results of this study, by reducing Poisson ratio of the soil, the effects of soil-pile and structures interaction increases. Also, if the shear modulus of the soil around the piles increases, the free vibration period of the system decreases.
- Investigating the effects of soil-pile-structure interaction phenomenon shows that the presence of piles in the system decreases the free vibration period of the system compared to the case without considering the pile and this should be considered in further studies and designs.
- Based on the analytical model that has been presented for structures based on end-bearing piles, an expression has been proposed to estimate the vibration period of these structures, which can be used in early designing.
- For short frames modeled in this research (frames up to three floors), the end-bearing pile assumption has no accuracy for calculating the free vibration period.
- Comparison of the proposed model results with the measured results from the base of the bridges positioned on piles shows that the assumption of the fixed-end pile is often more precise than that of the floating pile.
- Comparison of the results of the proposed analytical model with those of the case studies showed that the structures that have been located on end-bearing piles have a shorter period than structures on floating piles. The period difference for the models studied here in this paper is about 24 percent. The presence of end-bearing piles in the system decreases the free vibration period of the system compared to a case without considering the end-bearing piles and this should be encountered in further studies and designing.
- Based on the obtained results of this study, if Poisson ratio of the soil decreases, the effects of soil-pile-structure interaction increases. Also, the higher the shear modulus of the soil around the piles, the lower the free vibration period of the system.
- Investigating the effects of soil-end bearing pile-structure interaction phenomenon shows that the change in the ratio of the length of the pile to the height of the structure L_{pile}/h_S will have effects on the system response that for the models of this study, for the ratio of more than 0.8, the interaction of the soil and the structure is constant and will not change much. Further research on the response of the system was continued by investigating the changes in the effect of soil-structure interaction due to the change in the soil stiffness to pile bending stiffness ratio. According to the studies, there is a certain ratio of G_{Soli}/EI_{pile} in which the effects of soil and structure interaction are minimal. For the structures in this research, it is approximately 0.8.

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