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# Numerical Modelling of Flow in Morning Glory Spillways Using FLOW-3D

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#### Abstract

Suspended load amount with flow is one of the factors which are disregarded in designing morning glory spillways. It is due to the fact that physical modeling of sediment load with flood flow is very difficult and costly. Suspended sediments load with flow can change the density of passing water, leading to changing most of assumptions existing in spillways' design. With its unique potential to model dense flows and flows contain suspended loads, numerical model of FLOW-3D can provide valuable information in this regard. In the present study, flow was calibrated and validated using FLOW-3D through physical model. Then, by adding suspended load to flow, the values of discharge passing through the morning glory spillway were determined. In this regard, applying suspended load (3000, 6000, 9000, and 12000 ppm), flow discharge values were investigated for various heads over the spillway. The research findings revealed that increasing suspended flow load leads to decreasing values of flow passing through the morning glory spillways; such that, deceased values strongly depend on suspended load.

Keywords: Suspended Load; Flow Discharge; Morning Glory; Numerical Modeling; FLOW-3D.

# **1. Introduction**

Sediment materials entered into dam reservoirs are the result of soil and river bed erosion and are carried by flow. Erosion amount depends on various climatic conditions, geological characteristics, topography status, vegetation, and land use type. Therefore, soil erosion height may changes from 0.06 mm to 0.16 mm. On average, about 132 tones sediments are resulted per square kilometer per year [1]. Most of eroded materials are carried as wash load, suspended load and bed load in rivers. After arriving at natural or artificial lakes, just like dam reservoirs, due to hydraulic conditions changes or through dams' lower dischargers and spillways, sediments are transferred to downstream or sequestrated behind the dam. With respect to the fact that hydraulic performance of morning glory spillways is highly influenced by passing discharge values, suspended flow load issue should be highly considered in such spillways.

Ervine and Ahmed (1982) studied on flow aeration characteristics in a vertical shaft spillway [2]. In 2006, Zhao conducted physical model studies on hydraulic parameters of flow in morning glory spillways under different flow conditions [3]. Nohani and Emamgheis (2014) investigated impact of number and thickness of vortex breaker plate on strength of spiral vortices and efficiency of the spillway discharge system by making an experimental model of morning glory spillway and conducting experiments. The results showed that discharge coefficient of morning glory spillway enhanced up by 20% via increasing number of plates and 9% via increasing both number and thickness of plates [4]. Shemshi and Samani (2017) explored into effect of number, thickness and angle of vortex breaker plates on discharge coefficient of morning glory spillway by making a physical model and conducting experiments [5]. Coleman et al. (2004)

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study was aimed to determine discharge coefficient of morning glory spillways under geometrical conditions of spillway crest using an experimental model. The study was carried out on morning glory spillways at different diameters with/without vortex breakers in spillway crest. It should be noted that type of crest edge was also assessed in this study. So, two kinds of crest edges, including sharp edged and flat edged crests, were examined in this study [6].

Xianqi (2015) examined flow characteristics in morning glory spillway of large dams. Five different geometric and hydraulic plans were applied using a physical model and equation of discharge coefficient was determined for each morning glory spillway [7]. In Petaccia and Fenocchi (2015) studies, an experimental investigation of pressure fluctuations and flow profiles in morning glory spillways has been studied [8]. In the case of morning glory spillways related to vertical and semi-vertical shafts in studies [9-11] there are several discussions. Nohani et al. (2015) numerically evaluated impact of vortex breaker plates on inflow pattern in morning glory spillways via Flow3D software. The results indicated that in a vortex breaker structure with dimensions of  $5 \times 8 \times 10$ , presence of vortex breaker significantly increases water flow in fixed water heights compared to control situation and discharge rate enhances by increasing number of vortex breakers [12].

#### 2. Research Methodology

#### 2.1. FLOW-3D and Governing Equations

FLOW 3D is an appropriate model for complex fluids problems. This numerical model is widely used, particularly for unsteady 3-dimensional flows with free level and complex geometry. In this model, finite volume method is used in regular rectangular grid generation. Due to using finite volume method in a regular grid, the form of the employed discrete equations is similar to discrete equations in finite difference method. Accordingly, FLOW-3D contain first and second-order reliability methods which are explained in the following. Also, this software uses five turbulence models such as k- $\epsilon$  and RNG. In FLOW-3D, two methods have been simultaneously used for geometrical simulation. The first method is volume of fluid (VOF) which is used to show the behavior of fluid at free level. The second method is fractional area-volume obstacle representation (FAVOR) which is used to simulate solid levels and volumes such as geometrical boundaries.

Equations governing fluid dynamics are obtained from the law of conservation of mass and the law of conservation of momentum. These equations are in the form of partial differential equations. In general, to obtain flow equations, three steps should be considered: selecting accurate base laws, applying laws by an appropriate model and adopting mathematical equations showing the above physical laws. The main equations to simulate 3-dimensional flow are three differential equations including continuity relations and movement size in x, y and z directions.

A flow continuity equation is obtained from the law of conservation of mass and writing balance equation for a fluid element. General continuity equation is presented as Equation 1.

$$V_f \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u A_x) + \frac{\partial}{\partial y} (\rho v A_y) + \frac{\partial}{\partial z} (\rho w A_z) = 0$$
(1)

Where  $V_f$  indicates the fraction of open volume to flow;  $\rho$  indicates fluid density; (u, v, w) indicate velocity components in the directions of (x, y, z);  $A_x$  indicates the fraction of open level in x direction;  $A_y$  and  $A_z$  indicate the fraction of open level in y and z directions.

Fluid movement equations with velocity components of (u, v, w) in three different directions, i.e. Navier-Stokes equations are presented as following:

$$\frac{\partial u}{\partial t} + \frac{1}{V_F} \left( uA_x \frac{\partial u}{\partial x} + vA_y \frac{\partial u}{\partial y} + wA_z \frac{\partial u}{\partial z} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + G_x + f_x$$

$$\frac{\partial v}{\partial t} + \frac{1}{V_F} \left( uA_x \frac{\partial v}{\partial x} + vA_y \frac{\partial v}{\partial y} + wA_z \frac{\partial v}{\partial z} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial y} + G_y + f_y$$

$$\frac{\partial w}{\partial t} + \frac{1}{V_F} \left( uA_x \frac{\partial w}{\partial x} + vA_y \frac{\partial w}{\partial y} + wA_z \frac{\partial w}{\partial z} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial z} + G_z + f_z$$
(2)

In these equations,  $(G_x, G_y, G_z)$  indicates mass acceleration and  $(f_x, f_y, f_z)$  indicate viscosity accelerations [13].

In FLOW-3D, to simulate transfer, erosion, deposition, and change of sediments establishment status is due to fluid flow. Sediment model of this numerical model uses two concentration fields including suspended sediments and bed sediments.

Displacement of suspended sediment loads with fluid is due to local pressure's gradient changes. These suspended sediments may be created due to input flow containing suspended particles or due to bed erosion. Since bed erosions have been limited by neighbouring particles and are not easily displaced, they can move only in case of changing into

suspended load in shared level of bed and fluid. Suspended load can be changed into load when the velocity of depositing is higher than the velocity of bed erosion [14]. A part of control volume occupied by solid particles of sediment is  $(f_S)$  and the rest is defined from accumulated fluid of  $(f_L)$  such that:

$$f_S + f_L = 1 \tag{3}$$

Suspended load causes to the increase of real fluid viscosity. This increase continues until solid particles' volumetric element reaches to volumetric cohesion element. After that, increasing suspended load does not lead to the increase of viscosity but it causes that particles start to behave in solid manner. In this state, average viscosity of fluid is computed from the following relation:

$$\mu^* = \mu_f \left( 1 - \frac{Min(f_S, f_{sco})}{f_{sCR}} \right)$$
(4)

Where  $\mu_f$  indicates fluid's viscosity and  $\mu^*$  indicates average viscosity of sediment particles' critical element. Apparent density of  $\bar{\rho}$  is assumed as linear function of sediments volume where  $\rho_s$  and  $\rho_L$  indicate apparent density of sediment and fluid, respectively.

Drift refers to the deposition of sediment particles under the impact of floating forces affecting sediment particle. In the model of sediment washing in FLOW-3D, sediment particles are assumed in spherical shape such that it is influenced by fluid's viscosity effect. Therefore, deposition coefficient is automatically computed according to the following relation:

$$D_{f} = \frac{d_{50}^{2} \times (\rho_{s} - \rho_{L})}{18\mu}$$
(5)

Therefore, deposition velocity is computed through the following relation:

$$u_{drift} = D_f \times f_L \frac{\nabla P}{\overline{\rho}} = \frac{f_L \times d_{50}^2}{18\mu} \frac{\nabla P}{\overline{\rho}} (\rho_s - \rho_L)$$
(6)

Where in the above equation,  $(\nabla P/\overline{\rho})$  indicates mechanical potential of gradient or acceleration and is limited to 10 times more than particle's weight and causes to omit numerical fluctuations in pressure value. Near to fluid's free level, the value of,  $(\nabla P/\overline{\rho})$  is replaced with acceleration (g). The coefficient of  $f_L$  has been employed in the above equation since sedimentation is possible only at the presence of solid particles (sediment). Therefore, if control volume is full of sediments,  $f_L = 0$  and then,  $u_{drift} = 0$ .

At the level of bed sediments, shearing stress is active and causes to erosion and displacement of sediment at bed sediment level. This erosion is a function of fluid's sear stress at surface, critical shearing stress and sediment and fluid's density. The parameter of critical shields indicates the minimum shear stress required to lift sediment particles from the shared surface of fluid and active bed.

$$\theta_{cirt} = \frac{\tau_{crit}}{g(\rho_L - \rho_s)d} \tag{7}$$

Where  $\theta_{cr}$  indicates critical shields;  $\Gamma_{cr}$  indicates the minimum shear stress necessary for bed length to lift sediment particles. The purpose of developing and explaining this model is to estimate and predict the magnitude of semimetal flow which has been worn out. To this end, the parameter of shear velocity is defined to measure removal power of flow. So, the velocity of removing sediments from bed can be presented through the following equation:

$$U_{lift} = \alpha n_s \sqrt{\frac{\tau - \tau_{crit}}{\rho}}$$
(8)

Where  $n_s$  indicates normal vector of bed surface;  $\alpha$  indicates dimensionless parameter indicating the probability of sediment particles removal from bed (usually 1). In static fluid, internal friction angle of sediment particles determine the minimum slope through which sediments' walls can be steady. Internal friction angle above sediments indicates steady wall slope in steep slopes such as clay. In low angels of walls, there is a strong intention to collapse and move forwards such as sand.

In downstream hole where sediments are compiled together and create a mass of sediments, sediments establishment status makes an angle with horizon surface which indicates internal friction angel. In the model, this angle is signified by  $\xi$ . Natural establishment angle of sediments in various temporal and spatial conditions is computed through the following relation:

$$\varphi = \frac{n_{\text{interface}} \cdot g}{|g|} \tag{9}$$

Where  $n_{interface}$  equals normal vector of surface and g indicates gravity acceleration. After scour or transferring sediments suspended in the surface, critical shear stress occurring in sloped surface for each surface is computed through the following relation:

$$\tau_{cr} = \tau_{cr}^0 \sqrt{1 - \frac{\sin^2 \varphi}{\sin^2 \xi}} \tag{10}$$

According to the above equations, when natural slope of sediments equals their internal friction ( $\varphi = \xi$ ), critical shear stress equals zero, indicating that bed surface undergoes erosion due to every kind of imposed shearing stress. Also, when ( $\varphi > \xi$ ), we have ( $\tau_{cr} < 0$ ), indicating that higher internal friction angel of sediment particles leads to higher wall slope ( $\varphi$ ) since the wall of scour or sediment washing hole undergoes erosion without shear stress ( $\tau_{cr} = 0$ ). The movement of suspended sediments in system is expressed through convection-diffusion equation.

$$U_{j}\frac{\partial c_{s}}{\partial x_{j}} - \omega_{s}\left(\frac{\partial c_{s}}{\partial z}\right) = \frac{\partial}{\partial x_{j}}\left(\Gamma\frac{\partial c_{s}}{\partial x_{j}}\right)$$
(11)

Where  $C_s$  indicates sediments' concentration,  $\Gamma$  indicates diffusion coefficient and  $\omega_s$  indicates particles' collapse velocity which equals:

$$\omega_s = u_{lift} - u_{drift} \tag{12}$$

Therefore, the above equation is entered into problem solving as following:

$$\frac{\partial c_s}{\partial t} + u \cdot \nabla c_s = \Gamma \nabla^2 c_s - u_{lift} \cdot \nabla c_s - u_{drift} \cdot \nabla c_s \tag{13}$$

The concentration of sediments suspended in shared surface of sediments' bed and water before starting sediment washing (t = 0) equals:

$$C_{s0} = f_s \times \rho \tag{14}$$

The above computational equations and algorithm in FLOW3D are used to discharge bed sediments.

#### 2.2. Three-Dimensional Modelling Method

Firstly, it is necessary to construct and design 3D geometry of the morning glory spillway with the region topography through Solidworks Software. Solidworks is a powerful software for designing, especially 3d designing. In this software, there are separated parts for modeling and operations of welding, casting, molding as well as analyzing tension and modeling the behavior and resistance of the piece under various loadings. The main advantage of this software, compared to similar software, is its' easy and simple environment. In spite of ease of use, it has a significant ability in designing. Therefore, Solidworks2011 has been used to geometrically simulate the morning glory spillway and its components. In the following figure, the geometry of the morning glory spillway with vortex breaker of the input span has been shown.

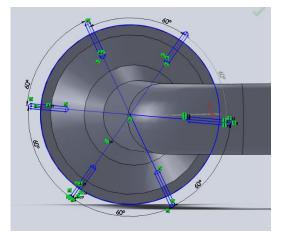


Figure 1. The inlet of vertical shaft of morning glory spillway of Haraz dam with vortex breaker and crown caps

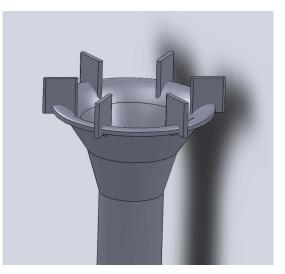


Figure 2. The morning glory spillway of Haraz dam with vortex breakers of the spillway

Applying boundary conditions existing in the numerical models should be introduced to the model based on the available laboratory conditions. Therefore, in boundary conditions, the grid bock created in FLOW-3D, boundary conditions in Ymax and Ymin directions, with respect to the fact that impermeable wall has been placed for hydraulic model in real models, boundary conditions of Wall (which is similar to real condition) has been selected in this numerical model. In minimum boundary conditions (Zmin) of the channel, just like the walls in real model, impermeable walls have been used such that, boundary conditions of Wall have been selected in the simulated numerical model as well. To apply boundary conditions for Zmax in the numerical model, given that flow has free surface and in direct contact with air in real model, boundary conditions should be used. Therefore, the gradient of air pressure changes is considered within and out of zero. Also, since in FLOW-3D, there is no need to define air phase as a new phase in hydraulic problems, the conditions of free surface and direct contact with air are met by relative air pressure [15]. To properly select and apply input and output boundary conditions of flow in Xmax and Xmin based on experimental studies, a stable flow with a certain height of the fluid over the morning glory spillway should be introduced to the model. Hence, using boundary conditions existing in FLOW-3D, the fluid height for Xmin is applied with the boundary conditions of Fluid Height. It is fully in accordance to the model through which the results validation and calibration is performed. On the other hand, after passing flow through the morning glory spillway, the flow should be directed to the output boundary out of the numerical model where output flow in the third block has been applied like the laboratory hydraulic model's conditions as outflow. All the boundary conditioned applied have been shown in the following figures. Notably, all the above mentioned boundary conditions for larger block should consider computational space and other blocks have symmetry boundary conditions to avoid creating any effect of boundary conditions on the model.

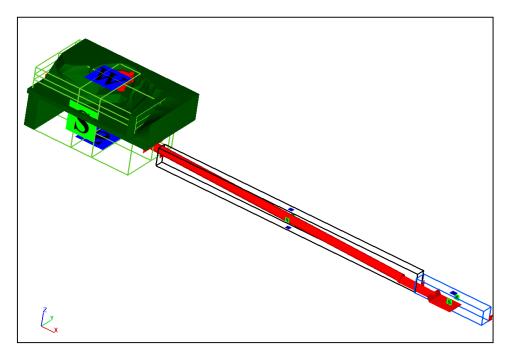


Figure 3. The boundary conditions applied on the flow simulation on the morning glory spillway

After gridding and applying boundary conditions to the numerical model, suspended flow load should be added to the numerical model. Accordingly, using the option of adding suspended sediments, sediments condensation level can be added at the onset of computations. The following figure shows the amount of sediments condensation.

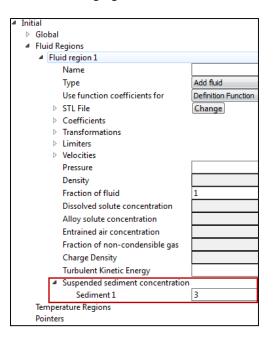


Figure 4. The boundary conditioned applied in simulating flow with suspended load, for example, suspended load equals 3000 ppm (3 kg/m<sup>3</sup> suspended sediments)

## 3. Discussion and Results

The first step in a numerical model is to calibrate model. That is, the effects of external factors are minimized and the model conditions makes closer to real conditions. The present numerical model has been performed based on experimental studies data. Therefore, the numerical model is calibrated and validated based on the experimental data. Calibrating the numerical model in terms of boundary conditions and simulation is important. To extract the proper and exact data of a numerical model, it is necessary to achieve stable status. In the studied numerical model, after investigating several models, appropriate time for extracting results from the model was considered 100 *sec*. The following figure shows the way of passing flow through the morning glory spillway in different times. After 100 sec, the flow reached to stability state over the spillway. In the figure, the flow passing through the morning glory spillway with vertical and horizontal shaft has been depicted.

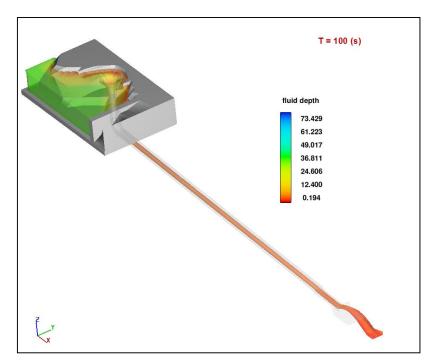


Figure 5. The flow in the morning glory spillways in different times of simulation

To investigate the effect of suspended load on outflow discharge of the spillway in four different states, the suspended load values of 3000, 6000, 9000 and 12000 ppm have been applied for various heights over the spillway. The following figure shows the diagrams pertained to each of suspended load values.

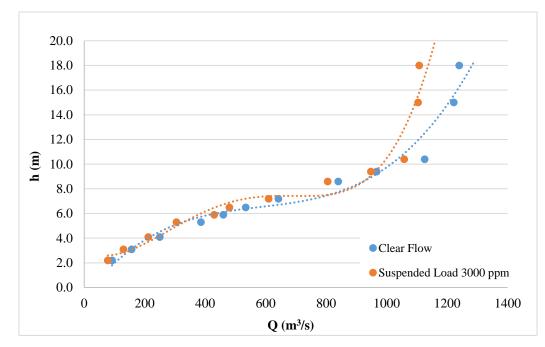


Figure 6. The diagram of outflow of the morning glory spillway based on free flow height (rating curve) for net flow and flow with suspended load of 3000 ppm

Investigating the results for all discharge values revealed that adding suspended flow load causes to decrease outflow passing through the spillway. The change of discharge values especially to control discharge is higher in tunnel. Comparing the average outflow values for all flow heights over the spillway in pure water mode equals  $600 m^3/sec$ . Adding suspended load about 3000 ppm causes to decrease the average outflow from the morning glory spillways up to  $605 m^3/sec$ . In other words, adding suspended load to outflow of the spillway about 3000 ppm leads to the decrease of the average outflow about 3.8%. In the following figures, the effect of adding suspended flow load to 6000 ppm on the outflow values has been explored.

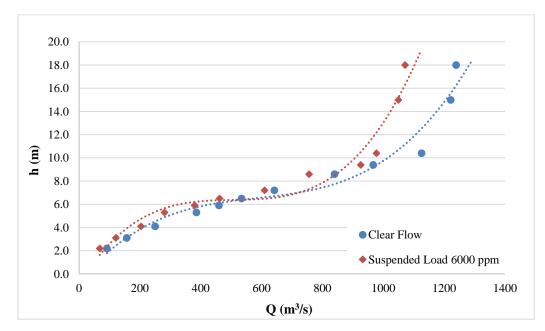


Figure 7. The diagram of outflow of the morning glory spillway based on free flow height (rating curve) for net flow and flow with suspended load of 6000 ppm

Investigating the results for all discharge values revealed that adding suspended flow load for the second time about 6000 ppm causes higher decrease in outflow passing through the spillway. Also, the change of discharge values especially in discharge control is higher in tunnel and reaches to flow control area in the spillway throat. Comparing the

average outflow values for all flow heights over the spillway in pure water mode equals  $600 m^2/sec$ . Adding suspended load about 6000 ppm causes to decrease the average outflow from the morning glory spillways up to  $575 m^2/sec$ . In other words, adding suspended load to outflow of the spillway about 6000 ppm leads to the decrease of the average outflow about 87.12%. In the following figure, the effect of adding suspended flow load to 9000 ppm on the outflow values has been explored.

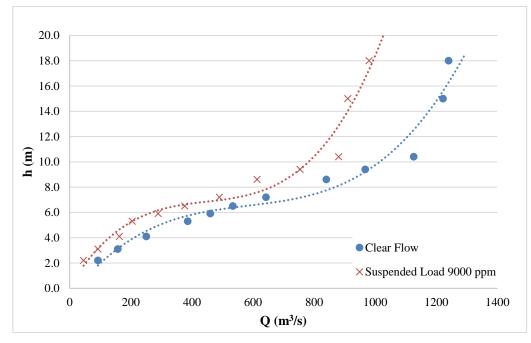


Figure 8. The diagram of outflow of the morning glory spillway based on free flow height (rating curve) for net flow and flow with suspended load of 9000 ppm

Investigating the results for all discharge values revealed that adding suspended flow load for the third time about 9000 ppm causes higher decrease in outflow passing through the spillway. Also, the change of discharge values especially in discharge control is obvious in the tunnel and reaches to flow control area in the spillway throat although the presented diagram of crown control area has well shown the decrease of suspended load amount. Comparing the average outflow values for all flow heights over the spillway in pure water mode equals  $660 m^3/sec$ . Adding suspended load about 9000 ppm causes to decrease the average outflow from the morning glory spillways up to  $575 m^3/sec$ . In other words, adding suspended load to outflow of the spillway about 9000 ppm leads to the decrease of the average outflow about 7.18%. In the following figure, the effect of adding suspended flow load to 12000 ppm on the outflow values has been explored.

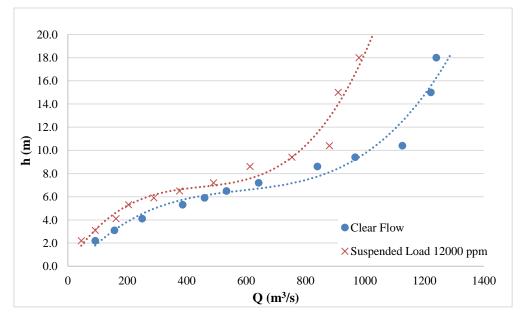


Figure 9. The diagram of outflow of the morning glory spillway based on free flow height (rating curve) for net flow and flow with suspended load of 12000 ppm

Investigating the results for all discharge values revealed that adding suspended flow load for the fourth time about 12000 ppm causes higher decrease in outflow passing through the spillway. Moreover, the change of discharge values especially in discharge control is obvious in the tunnel and reaches in flow control area in the spillway throat although the presented diagram of crown control area has well shown the decrease of suspended load amount. Comparing the average outflow values for all flow heights over the spillway in pure water mode equals  $660 m^2/sec$ . Adding suspended load about 12000 ppm causes to decrease the average outflow from the morning glory spillways up to  $483 m^2/sec$ . In other words, adding suspended load to outflow of the spillway about 12000 ppm leads to the decrease of the average outflow about 26%.

# 4. Conclusion

Investigating the results for all discharge values showed that adding suspended flow load, outflow values from the spillway are decreased. In all conditions, discharge values change, especially in discharge control conditions is higher in the tunnel. Adding suspended load about 3000 ppm to outflow of the morning glory spillways decreased the average outflow about 3.8%. Also, adding suspended load about 6000 ppm to outflow of the morning glory spillways decreased the average outflow about 3.12%. Furthermore, adding suspended load about 9000 ppm to outflow of the morning glory spillways decreased the average outflow about 7.18%. Comparing the average outflow for all flow heights over the spillway in pure water state is  $660 \ m^2/sec$ . this value is decreased to 483 m3/sec by adding suspended load about 12000 ppm. Finally, adding suspended load about 12000 ppm to outflow of the morning glory spillways causes the decrease of the average outflow about 26%.

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