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Empirical Model of Unconsolidated Tephra Erosion: Verification and Application on Micro Catchment

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Abstract

Erosion is an important process that shapes the earth's surface. Given the complexity of the process, efforts to understand it are essential. Over the last 50 years, numerous models of soil particle erosion by surface runoff emerged, some of which share similar forms and parameters. The differences lie in the coefficient values of the parameters, attributed to the characteristics of the soil material such as texture, structure, and organic matter content. However, these erosion models tend to underpredict in the case of new volcanic deposit erosion. The erosion model for unconsolidated tephra, proposed by Yunita, was developed through laboratory experiments using volcanic material from Merapi Volcano, Indonesia. Nevertheless, the model has not been implemented for other cases. Therefore, this study aims to verify the erosion model for volcanic material in other cases, explore the possibility of broader implementation, identify the factors that influence its accuracy, and determine the model's limitations. To verify the model's potential for broader application, we applied it to micro-scale catchments in St. Hellens (USA), Sakurajima (Japan), and a laboratory scale plot in Merapi (Indonesia). The verification yielded satisfactory results for all three cases, especially for new tephra deposits. In the case of St. Helens, the extrapolation of model coefficients was proven to still be applicable even for thicker tephra layers. However, the erosion prediction was overestimated for tephra layer deposits older than 1 year, as the erosion rate decreases over time due to the compaction and stabilization of the tephra layer. In the Sakurajima, the model was also suitable for predicting long-term erosion amounts (daily and monthly). Meanwhile, in Merapi, the model provided accurate predictions for slopes of 20º and 25º but was less accurate for 30º slopes, where the measured erosion was due to both erosion and slope failure. These verification results demonstrate the potential of applying the empirical erosion model to micro catchments with relatively homogenous slopes and tephra properties. The sensitivity test revealed that slope, runoff, rainfall intensity, and volcanic ash thickness are strongly influence the erosion rate. This study also simplified the volcanic ash erosion model as a function of slope (S_0), runoff (q), and rainfall (i) by assuming the value of ($1-\tau_c/\tau_0$) is equal to 1. Further study using GIS tools is required for its application on several catchments with heterogeneous characteristics.

Keywords: Unconsolidated Tephra; Volcanic Material; Empirical Erosion Model; Verification.

1. Introduction

Erosion is an important process that shapes the Earth's surface. It is a complex process involving many variables, including the detachment, entrainment, transport, and sedimentation of soil particles [1, 2]. Naturally, erosion can be generated by wind, water, or mass movement [3]. Water, in the form of rainfall and overland flow, is one of the major agents of soil erosion in wet climate regions. Initially, the raindrops break the bond between soil particles so that when overland flow occurs, the soil particles are easily transported. Soil erosion occurs due to the impact of raindrops, defined as a function of the amount and the size of the raindrops that fall on the soil surface. The process intensifies as the water depth increases, making the applied shear stress exceed the critical shear stress of soil particles. At this point, the erosion process is dominated by overland flow rather than by rainfall.

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The quantity of soil erosion by overland flow in a catchment is heterogeneous, varying spatially and temporally depending on land cover, climate, and landscape characteristics, i.e., vegetation, soil type, topography, and drainage conditions [4-6]. Over the last 50 years, numerous models of soil particle erosion by surface runoff emerged, with most models being empirical formulas based on experiments. These models generally assumed that the erosion rate is a function of slope geometry, hydraulic characteristics, rainfall amount, and soil properties. Some models follow a similar equation form:

$$q_s = aS_0{}^b q^c i^d d_{50}{}^e \tag{1}$$

where q_s is erosion rate, S_0 is slope gradient, q is runoff discharge, i is rainfall intensity, and d_{50} is soil particle diameter. However, the differences among the formulas lie in the coefficient values of the parameters. An experiment, conducted on the Foothills Campus, Colorado University, proposed an erosion formula consisting of slope and runoff as its parameters, whose coefficients were respectively 1.66 and 2.035 [7]. The coefficients of slope, runoff, and soil diameter from an erosion experiment using Yongding riverbed material in China are 1.637, 1.269, and -0.345 [8]. Another empirical equation from a laboratory experiment in Wageningen, the Netherlands, showed that erosion was more sensitive to slope than to discharge and soil diameter, where the derived exponents were 2.89, 1.46, and -0.50 [9]. An experiment from Istanbul, Turkiye, included not only slope, runoff, and soil diameter but also rainfall in its erosion formula, and the coefficients of each component were 1.146, 0.899, -0.194, and 0.073 [10]. Another experiment from China using purple soil in Wang Jia Qiao Watershed within the Three Georges Dams Catchment gave a simple composition of erosion formula with all exponents of slope, runoff, and rainfall equal to 1.00 [11]. Meanwhile, an erosion experiment in Selangor, Malaysia, defined an empirical formula for erosion as a function of discharge, rainfall, and Reynold number with the coefficients, respectively, 2.628, 1.562, and -0.085 [12]. From the previous experiments, the coefficient of slope (b) generally ranges from 0.980 to 2.890, the coefficient of runoff (c) ranges from 0.899 to 2.035, the coefficient of i (d) varies widely with a range of -1.652 to 1.000, and the coefficient of soil particle diameter (d_{50}) is negative ranging from -0.500 to -0.085. These different results showed that various combinations experimental conditions can lead to variety erosion formulas. The primary factor influencing the difference in the coefficient values is the characteristics of the soil material used in the experiment, such as texture, structure, and organic matter content.

The erosion formulas have strict limitations due to their boundary conditions. The previous erosion formulas tend to underpredict in the case of new volcanic material erosion [13]. The boundary that needs to be considered is that most of the erosion models mentioned were developed from the experiments using soil material originating from the weathering process in the continental region, which has characteristics distinct from volcanic material. Those soil particles, having undergone weathering and transport far from their sources, typically have a spherical and smooth shape [14, 15]. This particular shape was gained from the process of colliding, grinding, and smoothing as the results of particles' interaction with other particles and water. So, the farther a soil particle traveled, the more spherical and smoother its shape. Meanwhile, volcanic material particles, produced through explosive processes such as those from the Merapi volcano in Indonesia, are more fragmented and have sharp edges as a consequence of their formation process [16]. The shape of volcanic particles depends on the eruption type [17]. An explosive volcanic eruption ejects a large mass of magma fragments from meter-sized blocks to micron-sized ash particles, which form due to bubble magma disruption, gas expansion, and fragment collisions [18]. The shape particle difference gives fundamental distinction so that the previous erosion formulas are not suitable to be applied to the case of new volcanic material erosion.

Additionally, unlike common soil, a new volcanic material contains no organic material. The new material deposit from volcanic eruptions, pristine tephra, contained no organic carbon but may contain some inorganic carbon, which originated from volatile elements emitted during a volcanic blast in the form of carbon dioxide, carbon monoxide, methane, and carbonyl sulfide [19]. Organic matter content will influence the physical and mechanical properties of soil. Organic matter absorbs more water because it provides soil pores and lower bulk density, which is great for retaining water [20]. The absence of organic material in fresh tephra decreases the soil's capability to infiltrate rainfall. Therefore, in the period of post-eruption, the runoff will increase significantly on the slope covered by new volcanic material. This particular condition increases the sensitivity level of runoff as an erosion parameter, which has not been accommodated yet in the previous erosion model. It is also found that the previous erosion models give lower predictions in volcanic material cases. For instance, the Zhang et al. model gave a prediction of about 0.74 of the measured erosion, the Kılınç and Richardson model gave a prediction of approximately 0.19 of the measured data, while the Ali et al. and Aksoy et al. models resulted in less than 0.05 of the measured erosion [13]. Therefore, it is important to develop a specific erosion model for volcanic areas, especially to predict the amount of volcanic material that will be transported downstream by rainfall through an erosion mechanism.

An erosion model for unconsolidated tephra was created through laboratory experiments using volcanic material from the Merapi volcano and showed promising results. The model defined erosion as a function of slope, runoff, rainfall, and the ratio of applied shear stress to critical shear stress of soil particles, in which the interaction of each parameter is determined by the volcanic material layer thickness [13]. However, this model has not yet been applied to other cases, nor has its applicability for larger-scale erosion in the field been evaluated. Therefore, this study aims to verify the erosion model for volcanic material based on experimental erosion data from other cases, explore the possibility of broader implementation of the model, identify the factors that influence its accuracy, and determine the model's limitations. In this study, we collected data from previous experiments related to new tephra erosion, selected datasets used for verification, applied selected datasets to the proposed erosion model, conducted a performance test based on the prediction results, adjusted the coefficient model as an improvement, and finally tested the sensitivity of the parameter to simplify the erosion formula. Through this study, we hope that the erosion model for unconsolidated tephra can be implemented for broader cases, especially for predicting the volcanic material that will be transported downstream by the erosion mechanism as supporting information for the mitigation effort.

2. Material and Method

2.1. Methodology

Validation of an empirical erosion formula is necessary to ensure the model can be applied in other cases. Verification in this study aimed to assess opportunities for its implementation, examine factors affecting deviations, and adjust the parameter coefficient for the model's suitability. There are several methods to verify a model. Nevertheless, the most common way is by applying other measurement data from different locations to the tested model. However, the tephra erosion observation and measurement on the field after an eruption is difficult to carry out because the volcano eruption is a rare event. Moreover, the location of tephra deposition is generally remote and also dangerous to reach. To overcome these difficulties, we use secondary data from three different erosion experiments. We extracted the data set needed for verification, including slope, runoff, rainfall, mechanical properties, tephra thickness, and measured erosion amount from those experiments. Then, the data set was applied to the proposed volcanic material erosion model, and the prediction results were tested using the Nash-Sutcliffe model Efficiency (*NSE*), Index of Agreement (IOA), and Root Mean Square Error (RMSE) to demonstrate its performance. We also conducted iterations to adjust the formula of model coefficient parameters, testing the parameter sensitivity, and simplifying the model formula. The overall methodology used in the present study is schematically represented in Figure 1.

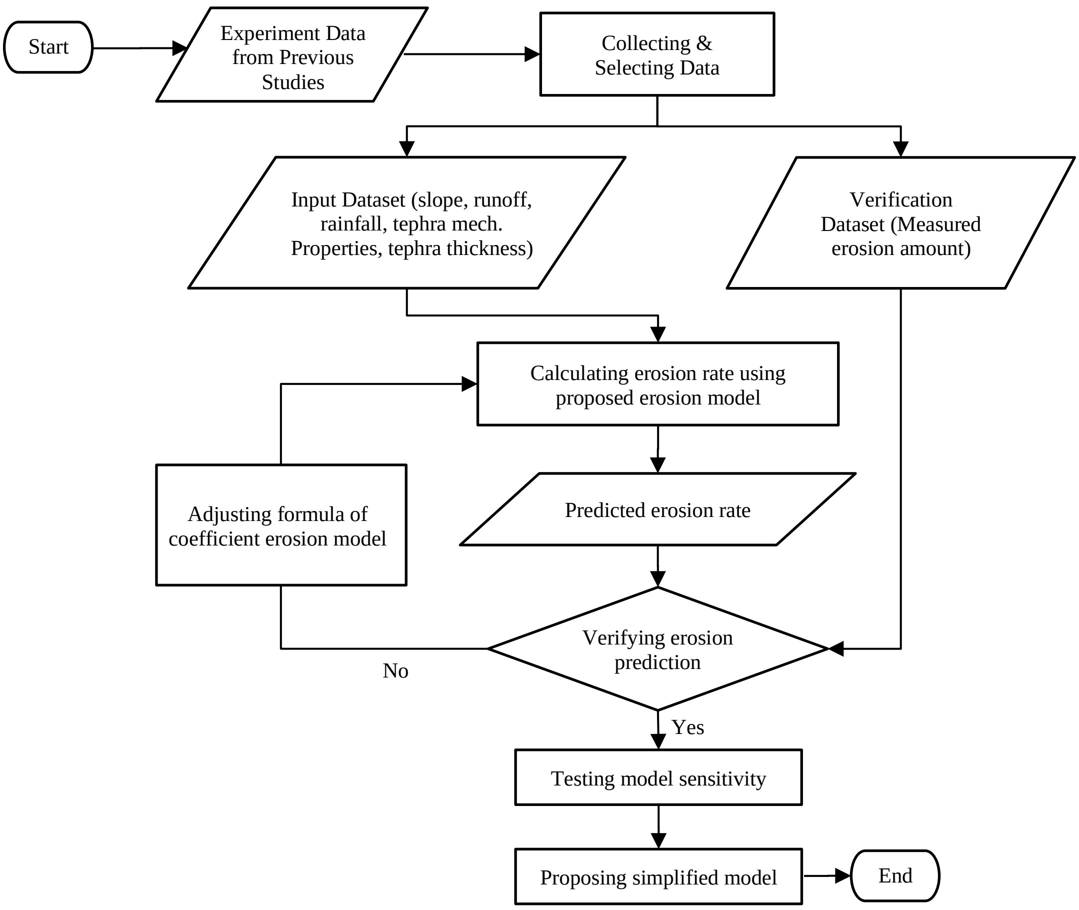

Figure 1. The scheme of research methodology

2.2. Data Collecting

Research related to the erosion of volcanic ash has not been widely conducted due to the limited availability of volcanic ash material and the difficulty of simulating volcanic ash conditions similar to those in the field. To examine the application of the volcanic ash erosion model proposed in different cases, we carried out a verification using data from previous studies. For this verification, we selected two erosion experiment data sets obtained from St. Hellens Volcano in Washington, USA [21] and Sakurajima Volcano in Japan [22]. Additionally, we also utilized data sets from an infiltration laboratory experiment of the volcanic ash layer from Merapi, Yogyakarta, Indonesia [23]. These three cases were chosen because they represent the diversity of catchment characteristics, volcanic materials, and ranges of parameters. The characteristics of each experiment case are presented in Table 1.

Table 1. Experiment characteristics

Description	Leavesley et al. (1989) [21]		Teramoto et al. (2006) [22]	Duhita et al. (2021) [23]
Catchment	Shultz Creek Plot 1	Shultsz Creek Plot 2	Hikinohira	Laboratory flume
Volcano	St. Helens	St. Helens	Sakurajima	Merapi
Area	143 m^2	157 m^2	11 m^2	1.125 m^2
Length	33.5 m	24.4 m	5 m	1.5 m
Grain size	Non-uniform d_{50} = 0.5 mm*	Non-uniform d_{50} = 0.5 mm*	Non-uniform d_{50} = 0.15 mm	Non-uniform d_{50} = 0.8 mm
Tephra thickness	20 cm	20 cm	5 cm	10 cm
Slope	11% (6.3°)	33% (18.3°)	38.4% (21°)	36.4% (20°); 46.6% (25°); 57.7% (30°)
Rainfall	Sprinkler, 13.1-34.4 mm/hour (1 hour)	Sprinkler, 13.1-34.4 mm/hour (1 hour)	Natural, 13.4-107 mm (total rainfall)	Sprinkler, 116.3 mm/hour (2 hours)
Overland Flow	0.5-2.3 $cm^3.s^{-1}.cm^{-1}$	0.5-2.3 $cm^3.s^{-1}.cm^{-1}$	145-971 $cm^3.m^{-1}$	0.01-0.19 $cm^3.s^{-1}.cm^{-1}$

* Data taken from May 25, 1980 eruption [24].

The experimental data from St. Hellens Volcano used to verify this model is 13 datasets. The data was obtained from a research by Leavesley et al. on the erosion rate model of volcanic ash deposits after the eruption of St. Helens on May 18, 1980. The research was conducted on two plots within the Shultz Creek catchment. To induce erosion on the slopes, a rainfall simulator in the form of a sprinkler was installed at a height of about 3.1 m from the surface of the slope plots. The rainfall intensity applied to the plot ranged from 13.1-34.4 $mm.h^{-1}$ with a duration of 1 hour. Measurements were taken twice, in September 1980 (4 months after the eruption) and in August 1981 (15 months after the eruption). The thickness of the fall tephra in the plot area ranges from 15-20 cm with the material's gradation in the deposition layer naturally sorted by gravity. This tephra layer consists of volcanic ash in the upper layer, sand to silt in the middle layer, and sandy gravel in the lower layer. Based on measurements in September 1980, the bulk density value in the volcanic ash layer was 1.02 $gr.cm^{-3}$, and in the gravel-sand layer was 0.80 $gr.cm^{-3}$, while the porosity value of the material was around 0.52-0.53. The bulk density value increased by about 20% in the measurement in August 1981, while the porosity value decreased to around 0.46-0.48. The diameter of the material in the experiment by Leavesley et al. was not mentioned, so it is assumed to be similar to the average diameter of the fall-tephra from May 25, 1980 eruption, which was 0.50 mm [24]. This assumption was based on the fact that the volcanic material that adjected from the same creater relatively similar even from different eruption event.

The experimental data from Sakurajima Volcano used in this study comprise 12 datasets. The data was obtained from Teramoto et al. who conducted a study on the effect of the volcanic ash layer on hydrological processes and sediment loads in a micro-watershed on the west slope of Sakurajima Volcano. Measurements of surface runoff and erosion were conducted on a plot (Hachitani line) of 11 m^2 in the Hikinohira watershed during the period from May 16th to August 17th, 2003, for slope conditions without volcanic ash and from August 17th, 2003 to March 14th, 2004, for slope conditions covered with volcanic ash 5 cm thick. To verify the volcanic ash erosion model, we used only measurement data of slopes covered with volcanic ash. The mean diameter of volcanic ash is about 0.15 mm, while the slope is 38.39% or about 21°. Teramoto et al. did not specify the mass density of volcanic ash used in their experiment. So it is assumed that the mass density of volcanic ash was approximately 1.607 $gr.m^{-3}$ as mentioned in another study that took volcanic material samples at ±2.2 km from the Mainamidake crater on the north slope of Sakurajima Volcano [25]. Although it was a different eruption event from Teramoto et al., generally the chemical and physical properties of volcano remains the same in a long period of time, unless there is a significant tectonic event that changes the mineral content in the magma chamber.

From an infiltration laboratory experiment of Merapi volcanic ash layer conducted by Duhita et al., we obtained 36 erosion data sets. The laboratory plot size was 1.5 m in length and 0.75 m in width. The volcanic material was non-uniform, with a mean grain size of 0.8 mm taken from Gendol Tributary at Merapi Volcano. The mass density of volcanic ash is 2.711 $gr.m^{-3}$. The volcanic ash layer thickness was 10 cm. The rainfall was set constant at approximately 116.5 $mm.h^{-1}$ over a 2-hour duration, generated by artificial rainfall apparatus with sprinklers installed 7 m above the flume base. The experiment plot was set at three variation slopes: 20 º, 25 º, and 30 º. The amount of volcanic ash erosion was collected at the lowest end of the experiment plot and measured every 10 minutes during the experiment.

2.3. Tephra Erosion Model

The erosion induced by rainfall starts as the raindrops fall and break the bond between soil particles which are held by cohesion and inter-particle friction, later when the soil surface layer gets saturated, a thin runoff forms, and as the critical water depth is reached, the soil particles begin to move. Based on the concept, the erosion model of unconsolidated tephra proposed by Yunita et al. is defined as a function of parameters related to slope, rainfall, overland flow, and physical properties of soil and water. It is expressed as the following function [13]:

$$q_s = f\left(i,\ \rho,\ \nu, q, S_0,\ X_r,\ \frac{\tau_c}{\tau_0},\ g\right) \tag{2}$$

where q_s is erosion rate of tephra (kg s^{-1} m^{-1}); q is flow discharge (m^3 s^{-1} m^{-1}); i is rainfall intensity (m s^{-1}); X_r is slope length (m); ρ is the mass density of water (kg m^{-3}); ν is the kinematic viscosity of water (m^2 s^{-1}); S_0 is slope gradient; τ_c is critical shear stress (Pa), τ_0 is applied shear stress (Pa), and g is gravity (m s^{-2}). These variables have been reduced to four dimensionless parameters, $\frac{q_s}{\rho\nu} = f\left(S_0,\ \frac{iX_r}{\nu}, \frac{q}{\nu}, \frac{\tau_c}{\tau_0}\right)$, allowing that the erosion model for volcanic ash on the slope to be expressed as [13]:

$$q_s = a\rho\nu {S_0}^b \left(\frac{q}{\nu}\right)^c \left(\frac{iX_r}{\nu}\right)^d \left(1 - \frac{\tau_c}{\tau_0}\right)^e \tag{3}$$

According to Yunita et al., the coefficients a and e are constant, spesifically 10.353 and 1.139, while the value of the exponent b, c, and d vary with volcanic ash thickness. Therefore, the general formulas of volcanic ash erosion rate for each tephra thickness are presented below [13]:

For 1 cm tephra thickness:

$$q_s = 10.353\rho\nu \cdot {S_0}^{1.280} \left(\frac{q}{\nu}\right)^{1.458} \left(\frac{iX_r}{\nu}\right)^{-0.560} \left(1 - \frac{\tau_c}{\tau_0}\right)^{1.139} \tag{4}$$

For 2.5 cm tephra thickness:

$$q_s = 10.353\rho\nu \cdot {S_0}^{1.591} \left(\frac{q}{\nu}\right)^{1.361} \left(\frac{iX_r}{\nu}\right)^{-0.474} \left(1 - \frac{\tau_c}{\tau_0}\right)^{1.139} \tag{5}$$

For 5 cm tephra thickness:

$$q_s = 10.353\rho\nu \cdot {S_0}^{1.769} \left(\frac{q}{\nu}\right)^{1.191} \left(\frac{iX_r}{\nu}\right)^{-0.306} \left(1 - \frac{\tau_c}{\tau_0}\right)^{1.139} \tag{6}$$

As shown in Equation 4 to 6, for 1 cm, 2.5 cm, and 5 cm volcanic ash thickness, the values of b are 1.280, 1.591, and 1.769, the values of c are 1.458, 1.361, and 1.191, and the values of d are -0.560, -0.474, and 0.306 respectively. To simplify the application of the formula across different volcanic ash thickness, the coefficient values b, c, and d are plotted on a graph as a function of volcanic ash thickness (T_{va}) as shown in Figure 2. Figure 2 demonstrates that the variation in the values in the three coefficients strongly correlates with the thickness of volcanic ash, following a power function as described by the Equations [13]:

$$\mathrm{b} = 1.288{\mathrm{T}_{\mathrm{va}}}^{0.202} \tag{7}$$

$$c = 1.482{T_{va}}^{-0.123} \tag{8}$$

$$-d = 0.589{T_{va}}^{-0.366} \tag{9}$$

where T_{va} is volcanic ash thickness (cm). With Equations 7 to 9, the value of each coefficient for any given volcanic ash thickness can be calculated.

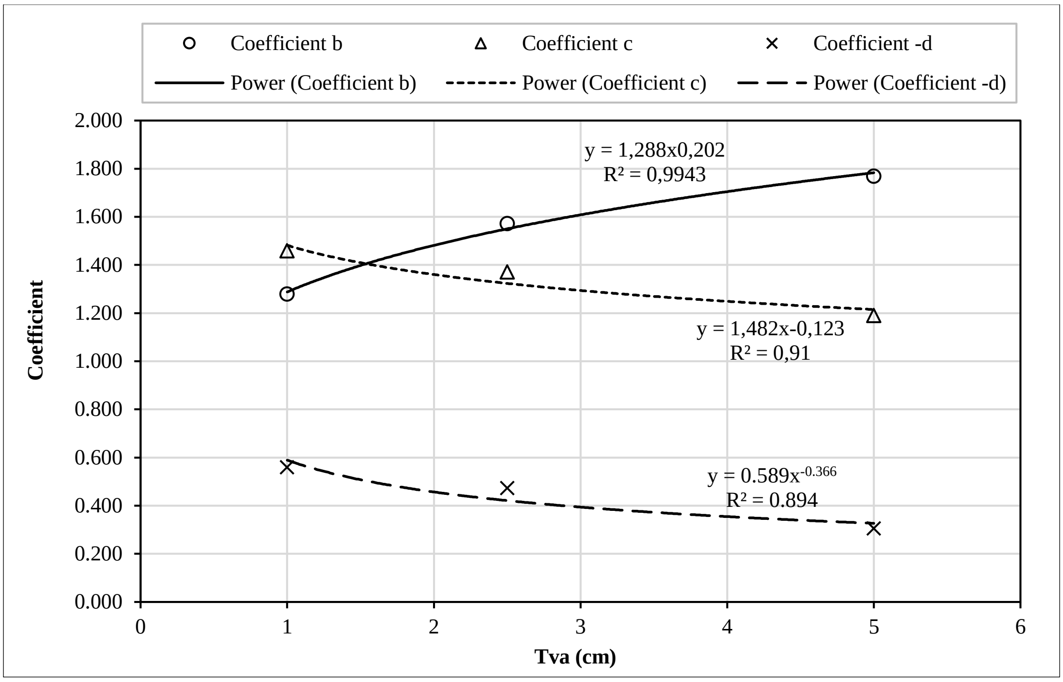

Figure 2. The correlation of b, c, and d coefficient towards volcanic ash thickness (T_{va})

2.4. Verification Model Performance

In this study the application of the tephra erosion model proposed by Yunita et al. was verified by using the Nash-Sutcliffe model Efficiency (*NSE*), Index of Agreement (IOA), and Root Mean Square Error (RMSE) to test the model's performance in various cases. NSE is commonly used in hydrology-related modeling, where model performance is determined based on the relative magnitude of the residual variance as compared to the variance of the measured data. The ideal NSE value is 1, the NSE value greater than 0.50 is considered good, and a value greater than 0.75 is regarded as very good. The NSE value is calculated using the following equation [26]:

$$\mathrm{NSE} = 1 - \frac{\sum_{i=1}^{n}(O_i - P_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O})^2} \tag{10}$$

IOA is a statistical measure of the correlation between predicted and observed values, which ideally has a value of 1, if the value is greater than 0.50, it indicates that the model's performance is good. Meanwhile, a value greater than 0.75 is considered indicative of very good model performance. The IOA value is calculated using the following formula [27]:

$$\mathrm{IOA} = 1 - \frac{\sum_{i=1}^{n}(O_i - P_i)^2}{\sum_{i=1}^{n}(|P_i - \bar{O}| + |O_i - \bar{O}|)^2} \tag{11}$$

Meanwhile, RMSE quantifies the difference between the predicted and the observed values to assess model precision. An ideal RMSE value is close to 0, and the formula is expressed as follows:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(O_i - P_i)^2}{N}} \tag{12}$$

For the three performance test formulas, O_i is the observed value at time i, P_i is the predicted value at time i, $\bar{O}$ is the average of observed values, and N is the amount of data points.

3. Result and Discussion

3.1. Erosion Model Cerification

3.1.1. Erosion Prediction based on Leavesley et al. (1989) Experiment Data

The volcanic ash layer in Leavesley et al. experiment was thicker than the variation in Yunita et al. experiment. Therefore, to derive the erosion formula for this case, we extrapolate the coefficient formula, particularly for the exponent b, c, and d, for a 20 cm volcanic ash thickness using Equations 7 to 9. Thus, the volcanic ash erosion model for T_{va} = 20 cm is obtained as follows:

$$q_s = 10.353\rho v {S_0}^{2.360} \left(\frac{q}{v}\right)^{1.025} \left(\frac{iX_r}{v}\right)^{-0.197} \left(1 - \frac{\tau_c}{\tau_0}\right)^{1.139} \tag{13}$$

As the values of ρ and v depend on water temperature, we assumed that the water temperature in Washington from August to September was approximately 17.6^{0}C. Figure 3 presents the value of predicted q_s values compared to those measured q_s in Leavesley et al. experiment.

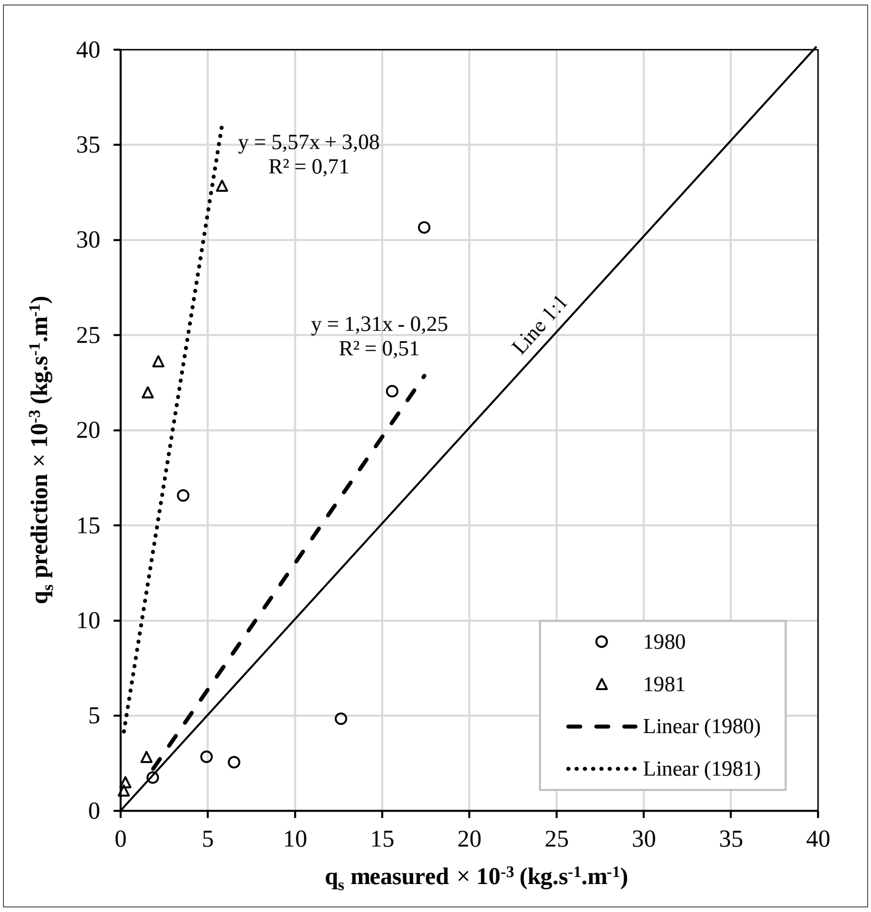

Figure 3. The predicted q_s compared to the measured q_s from Leavesley et al. Data

Figure 3 reveals that the predicted value of q_s is higher than the measured q_s value. The predicted q_s is approximately 1.31 times higher than the q_s measured in September 1980 and about 5.57 times greater than the q_s measured in August 1981. The reasons for higher erosion estimation (overestimation) are possibly because of the different characteristics of volcanic material, a relatively long-time lag between the time of measurement and the eruption, and the thickness of the tephra layer.

The distinct characteristics of the volcanic material produced by the St. Helens are indicated from the lower density of its tephra, while the average diameter of tephra is 5 times larger than that of volcanic material from Merapi Volcano. These parameter values suggest that St. Helens' tephra is more porous than Merapi's, facilitating easier water penetration, which in turn reduces surface runoff and the potential for erosion of the tephra layer. The time lag between the measurement and the eruption time also contributed to the increase of tephra density due to consolidation. As mentioned by Leavesley et al. that there was a 20% increase in density and a decrease in the porosity of the tephra layer from the first measurement to the second measurement, which means that in the period of time between September 1980 to August 1981, the tephra layer naturally compacted and became denser.

Besides by internal factors in the erosion process, discrepancies in predictions can also arise from deviations in determining the coefficient of the erosion model formula. The volcanic ash erosion model was developed based on experiments with volcanic ash thicknesses between 1-5 cm, whereas the ash layer thickness in the Leavesley et al. experiment was much larger, specifically 20 cm. The formula coefficients were determined based on extrapolation which led to the deviations. Based on this assumption, a re-evaluation of the erosion model formula, particularly for the coefficients b, c, and d, was conducted. This time, the calculation was performed in reverse, where optimization focused on the values of the coefficients b, c, and d to derive a predicted q_s value that closely matches the measured q_s value in September 1980, under the assumption that there was no significant consolidation process of the ash layer within 4 months after the eruption. From the optimization results, thus the coefficients of Equation 13 were thus corrected:

$$q_s = 10.353\left(\rho v {S_0}^{2.360}\left(\frac{q}{v}\right)^{0.944}\left(\frac{iX_r}{v}\right)^{-0.181}\left(1-\frac{\tau_c}{\tau_0}\right)^{1.139}\right) \tag{14}$$

Figure 4 presents the value of the predicted q_s after correction compared to the measured q_s.

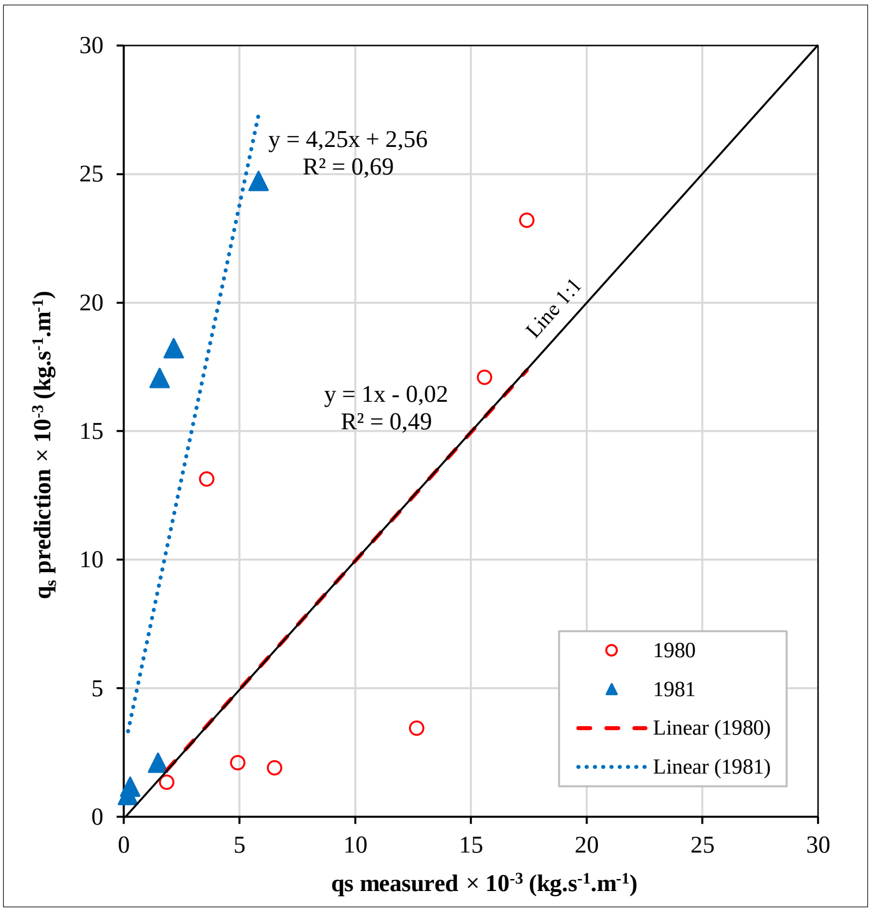

Figure 4. The predicted q_s compared to the measured q_s from Leavesley et al. Data after correction

Figure 4 shows that the predicted q_s for the September 1980 measurement almost align with the ideal 1:1 line, with the predicted q_s value equaling the measured q_s value. However, the predicted q_s value for the August 1981 measurement is still significantly larger, approximately 4.25 times the q_s measurement. The reason is that, more than a year after the eruption, the volcanic ash layer had consolidated and become more stable. Collins and Dunne stated that gully and sheet erosion continued to decrease over a three-year period following the 1980 eruption of St. Helens, attributed to more stable gully networks, more permeable soil layers, and the recovery of vegetation on the slopes. This also indicates that the volcanic ash erosion model developed by Yunita et al. has limitations with respect to time, the longer the time lag between the measurement and the eruption time, the greater the deviation in erosion predictions. Therefore, further studies are necessary to understand the pattern of decreasing erosion rates of volcanic ash on slopes over several years post-eruption to refine and improve the existing erosion models.

3.1.2. Erosion Prediction based on Teramoto et al. (2006) Experiment Data

The Teramoto et al. experimental data used in the verification include cumulative rainfall (r), runoff depth (h), and thickness of erosion (h_s) over 3 to 44 days, where the units of these three parameters are volume per plot area. To address the differences in units in the experiments of Teramoto et al., a modification of the erosion model formula was made by assuming that $q \approx hX_r$ and $i \approx rX_r$, in order to obtain the following formula:

$$q_s{}' = 10.353\left(\rho v {S_0}^{1.783}\left(\frac{hX_r}{v}\right)^{1.215}\left(\frac{rX_r}{v}\right)^{-0.327}\left(1-\frac{\tau_c}{\tau_0}\right)^{1.139}\right) \tag{15}$$

where $q_s{}'$ is total amount of erosion (kg m^{-1}), h is water depth (m) and r is total rainfall (m). The coefficient values $b = 1.783$, $c = 1.215$, and $d = -0.327$ are obtained based on Equations 7 to 9 for $T_{va} = 5\ cm$. While the values ρ and v are calculated using the monthly average seawater temperature in Kagoshima based on data https://www.seatemperature.org/asia/japan/kagoshima-shi.htm, as Sakurajima Volcano located in the middle of Kagoshima Bay. The comparison of the predicted $q_s{}'$ value of the volcanic ash erosion model against the measured q_s' value from the Teramoto et al. study is presented in Figure 5.

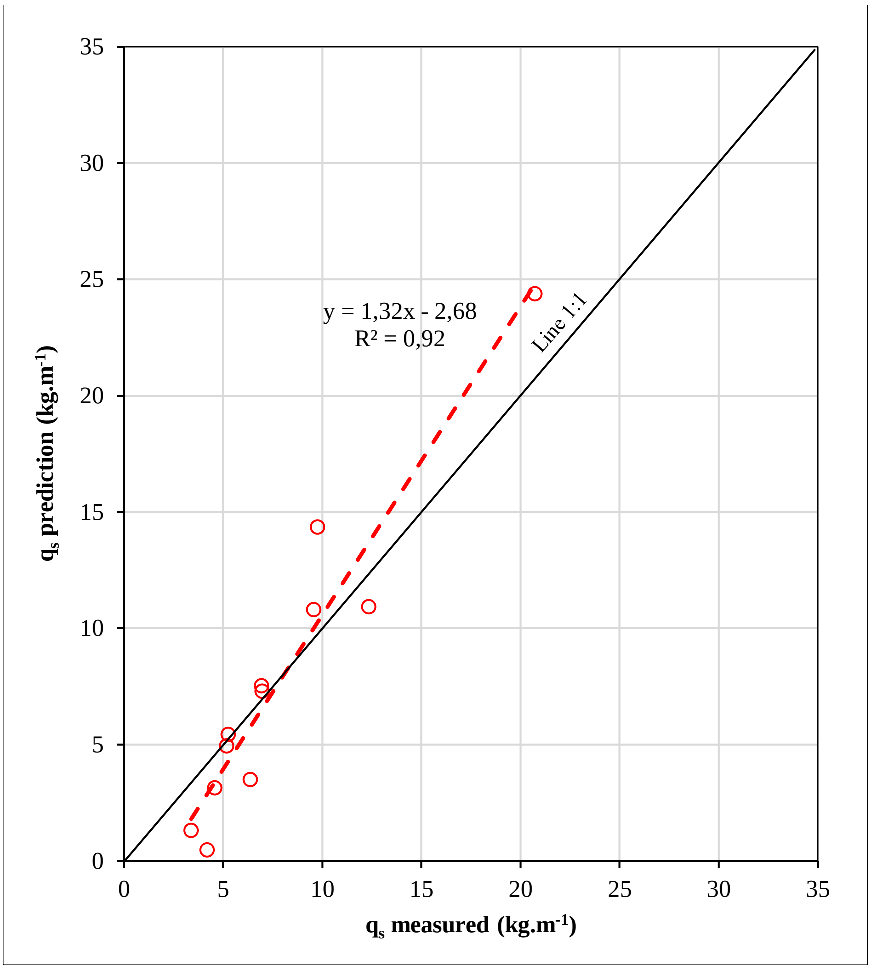

Figure 5. The predicted $q_s{}'$ compared to the measured $q_s{}'$ from Teramoto et al. Data

Figure 5 demonstrates that the prediction results for volcanic ash erosion are sufficiently close to the ideal 1:1 line, with a predicted $q_s{}'$ value of around 1.33 from the measured $q_s{}'$ and a correlation number (R^2) of about 0.92. A slight deviation of the predicted value is expected due to measurements being carried out over a long period (7 months) and the use of cumulative rainfall data. The erosion rate of volcanic ash will decrease over time as the layers of volcanic ash material consolidate and become more stable, alongside the recovery of vegetation on the slopes. This decrease means that prediction results will deviate more with increasing time (overestimated), as also observed in the verification using data from Leavesley et al. for the August 1981 erosion measurement, which was 15 months after the eruption of St. Helens. The use of cumulative rainfall data in predictions presents several issues, as it does not accurately reflect the intensity of rainfall. High cumulative rainfall values can result from low-intensity rainfall over a long duration, which does not trigger runoff and erosion, whereas low cumulative rainfall can cause erosion if it occurs in a short period, meaning the rain intensity is sufficient to form surface runoff, which can transport volcanic ash material.

However, given that the deviation was not significant enough, the potential for prediction error could also stem from inaccurate coefficients in the formula. Despite the correlation coefficient being quite high (more than 0.85), there is still a potential for error, especially for the coefficients c and d, where the correlation coefficient value is less than 0.95. Based on this, a re-evaluation of the erosion model formula was conducted, particularly for the coefficients c and d. The optimization of these coefficients aimed to achieve a predicted $q_s{}'$ value close to the measured $q_s{}'$ value in the Teramoto et al. experiments. Thus, a new equation with corrected coefficients was obtained.

$$q_s{}' = 10.353\left(\rho v {S_0}^{1.783}\left(\frac{hX_r}{\nu}\right)^{1.174}\left(\frac{rX_r}{\nu}\right)^{-0.304}\left(1-\frac{\tau_c}{\tau_0}\right)^{1.139}\right) \tag{16}$$

The graph of the predicted $q_s{}'$ compared to the measured $q_s{}'$ after correction is given in Figure 6.

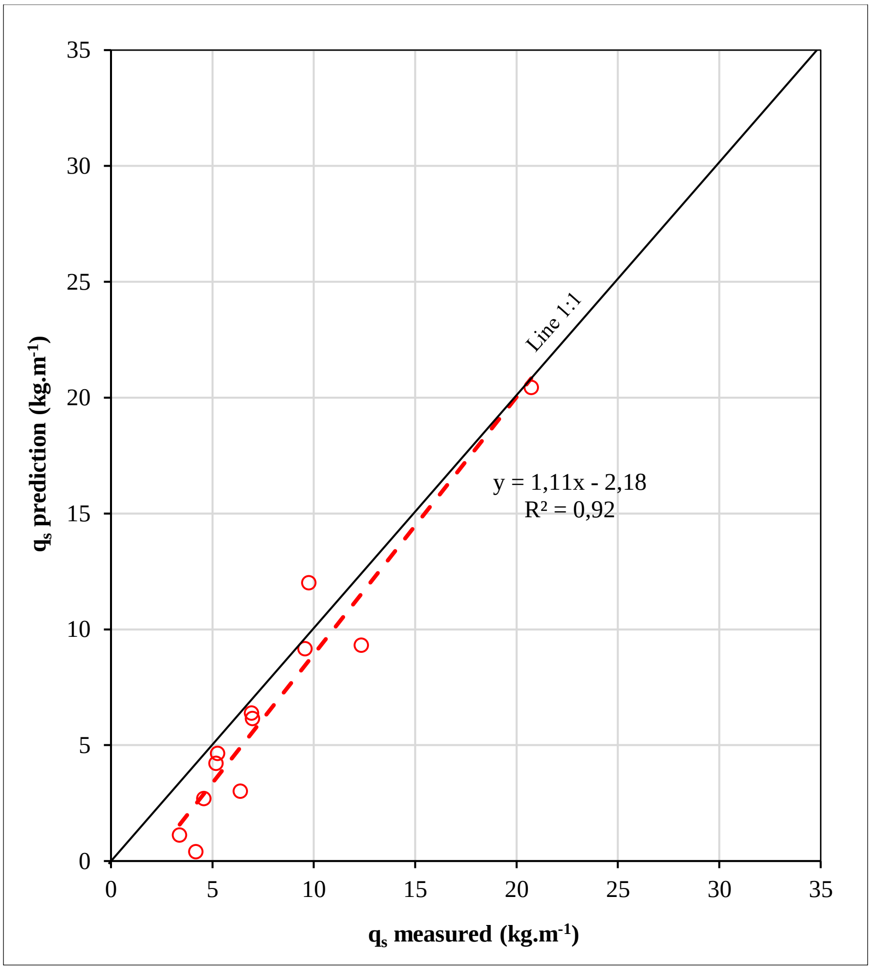

Figure 6. The predicted q_s' compared to the measured q_s' from Teramoto et al. Data after correction

In conclusion, the results of the Teramoto et al. data verification indicate that the volcanic ash erosion model is sufficiently consistent to be applied up to 7 months after the ash layer has been applied to the slopes. In the case of Teramoto et al., the consolidation process occurring during this period was not significantly impactful, which did not majorly affect the prediction results, unlike in the Leavesley et al. case. We suspect that the thickness of the volcanic ash layer contributes to the significance of the consolidation process, as in Leavesley et al. the volcanic ash thickness was 20 cm. The thicker the tephra layer, the more significant the consolidation occurs because the self-weight of the thicker layer is greater. This highlights that the consolidation or compaction rate of the volcanic ash layer varies across different scenarios, which is crucial to acknowledge as it influences the accuracy of the volcanic ash erosion model.

3.1.3. Erosion Prediction based on Duhita et al. (2021) Experiment Data

The thickness of volcanic ash in the experiment by Duhita et al. is greater than the range of volcanic ash erosion research experiments. Therefore, in order to derive the erosion rate equation in this case, extrapolation was conducted for the coefficients b, c, and d, as verified in the experimental case of Leavesley et al. The volcanic ash erosion formula for Duhita et al.'s experiment is as follows:

$$q_s = 10.353\left(\rho v {S_0}^{2.051}\left(\frac{q}{v}\right)^{1.116}\left(\frac{iX_r}{v}\right)^{-0.254}\left(1-\frac{\tau_c}{\tau_0}\right)^{1.139}\right) \quad (17)$$

The coefficient values b, c, and d for volcanic ash thickness, T_{va}=10 cm was calculated using Equations 7 to 9. The values of ρ and v were calculated by assuming the average temperature of the water during the experiment is the same as the volcanic ash erosion model study, which is 27°C due to the same experimental location. The comparison of the predicted q_s values and the measured q_s from Duhita et al. presented in Figure 7, where the value of q_s is plotted based on the slope.

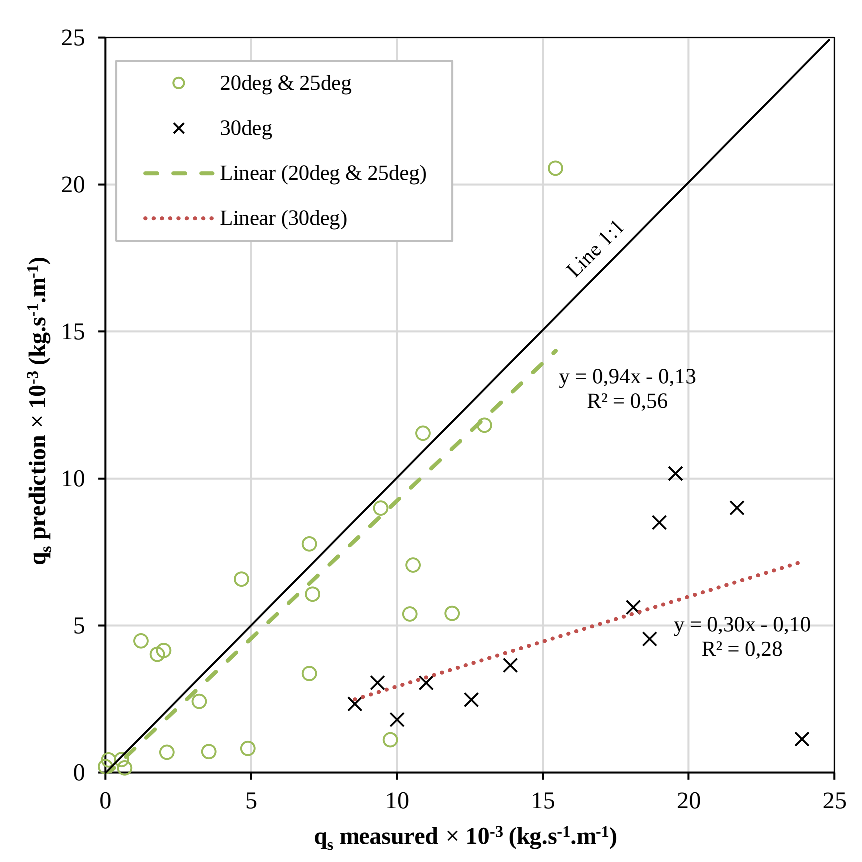

Figure 7. The predicted q_s compared to the measured q_s from Duhita et al. Data

From Figure 7, it is observed that the predicted q_s from the volcanic ash erosion model provides values that are very close to the measured q_s from Duhita et al.'s experiments, especially on the 20° and 25° slope scenarios. This is attributed to the similarity in the physical parameters of the volcanic ash material, used in the experiment by Duhita et al. to those in the volcanic ash erosion model experiment in Yunita et al.'s study. The differences related to a thicker ash layer (10 cm) and a larger average diameter of volcanic ash (0.8 mm), found not resulting a significant difference as compared to the experiment by Leavesley et al. In laboratory experiment the measurement was conducted soon after the tephra layer was setup, so there is no time lag between the tephra deposition to measurement time as it is in Leavesley et al. experiment. Meanwhile, in the 30° slope, the predicted q_s is lower (under-estimated), which is only around 0.31 of the measured q_s. This deviation is attributed to the steepness of slope, if the slope is greater than 25^0, the soil movement will be dominated by mass movement mechanism, such as slope failure, rather than the erosion mechanism.

Slope stability of non-cohesive materials such as sand, which is not compacted, is determined by the angle of repose (φ). The angle of response for dry sand is 34°, but when the pores within the sand are filled with water, this value can be lower, namely between 15° to 30° [28]. In the experiment conducted by Duhita et al., under conditions of wet volcanic ash layers due to infiltration, the angle of response was lower than 30°. Thus, in this case, the movement of volcanic ash downslope resulted not only from erosion but also from slope failure. Moreover, the high rainfall intensity (116.3 mm/hour) over a long duration (2 hours) in the Duhita et al.'s experiment further weakened the soil layer as it became saturated with rainfall. These two factors caused the volume of material carried downstream in the Duhita et al. experiment was greater.

The verification using experimental data from Duhita et al. revealed that the volcanic ash erosion model cannot be applied to steep slopes greater than 25^0. Deviations in the prediction results of erosion that occur due to the movement of material on the slopes, were dominated by the slope failure mechanism, not the erosion process. Additionally, the parameters of rainfall intensity and duration significantly influence the initiation of slope failure. Therefore, before applying this volcanic ash erosion model, it is necessary to identify the angle of repose of volcanic material, as well as the relationship between the intensity and duration of rainfall that triggering slope failure.

3.2. Correction for Erosion Model Coefficient

Over the last 50 years, nmeruous models of soil particle erosion due to surface runoff have been developed, most of which are empirical formulas based on experiments. Some of them have the same formula with the volcanic ash erosion model presented in this study, particulary in relation to parameters such as slope (S_0), surface runoff (q), and rain intensity (i). The differences between these formulas lie in the coefficient values of the parameters. The coefficient of the slope parameter generally ranges from 0.98 to 1.66. However, for the erosion model by Ali et al. (2012), which assigns a significantly higher value of 2.89 to this coefficient, the range for the parameter q or Q spans from 0.899 to 2.035, while the coefficient of parameter i or R varies widely, ranging from -1.652 to 1.000. As mentioned before, the primary factor influencing the variance in coefficient values is typically the characteristics of the soil material used in the experiment, such as texture, structure, and organic matter content. However, from the verification we also revealed that in the case of volcanic material those characteristics can be changes over time because of the consolidation and vegetation recovery process. Figure 8 depicts the variability of those coefficients from other models compared to the coefficients of the parameters in the volcanic ash erosion model towards the average flow discharge (q) of the experiment.

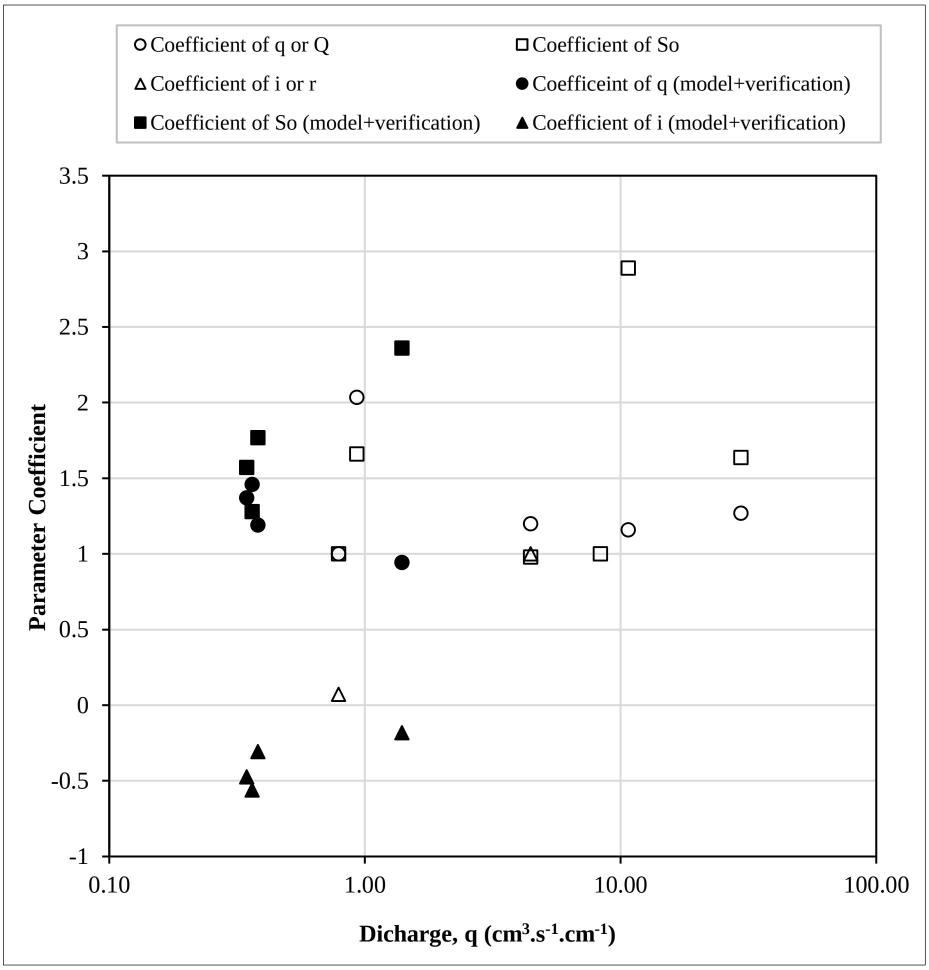

Figure 8. The coefficients of erosion from other models compared to the volcanic ash erosion model

From the verification results of the volcanic ash erosion model using experimental data from Leavesley et al., Teramoto et al., and Duhita et al., the coefficient values of b, c, and d were determined for variations in volcanic ash thickness. Based on the experimental coefficient values and model verification, a regression analysis was performed once again to correct Equations 7 to 9, as shown in Figure 9.

Figure 9 illustrated that the value of the correlation coefficient R^2 for the three graphs increases with the addition of data from the verification, especially for the coefficients c and d. The correlation coefficient for the graph representing the function coefficient c increased from 0.93 to 0.96, while that for the function coefficient d rose from 0.89 to 0.97. Given the absence of significant change, there is no formula correction for coefficient b as the Equation 7 still suitable in all cases. The equations for the coefficients c and d after verification are as follows:

$$c = 1.503T_{va}^{-0.145} \quad (18)$$

$$-d = 0.594T_{va}^{-0.388} \quad (19)$$

However, the coefficients of the volcanic ash erosion model still require further verification and refinement, particularly for various unverified variations of volcanic ash thickness.

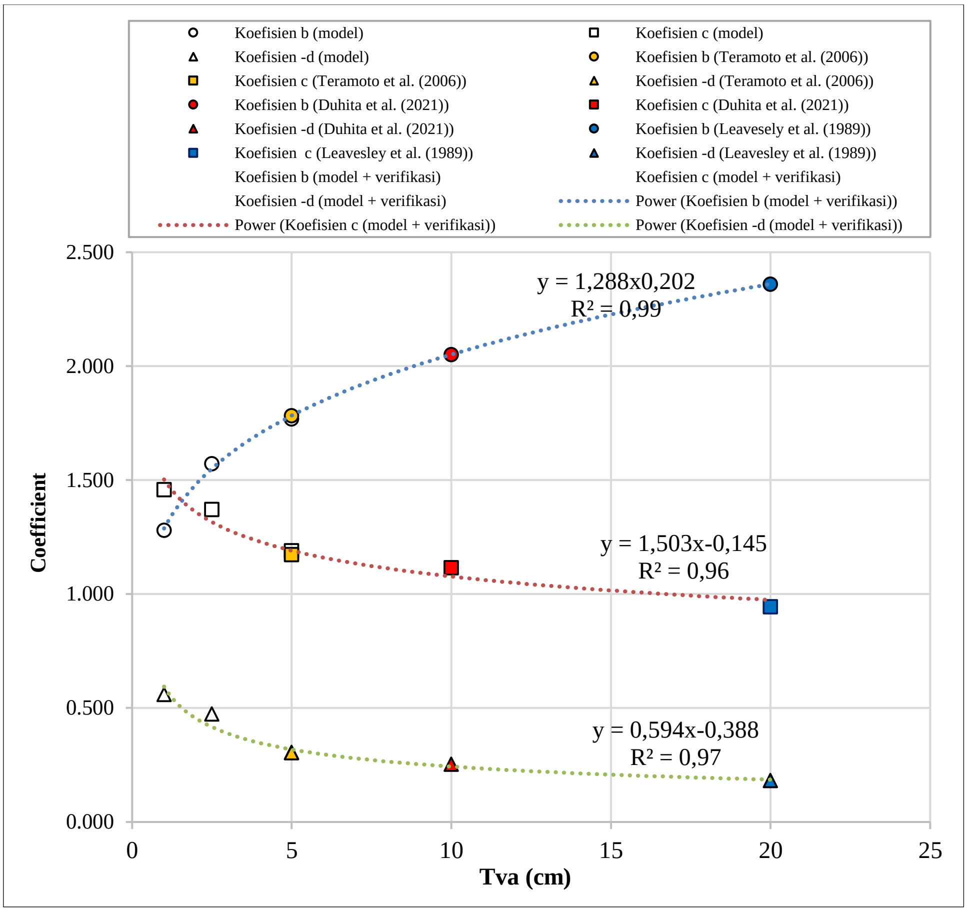

Figure 9. The coefficients of the volcanic ash erosion model after verification

3.3. Model Performance Test

To test the performance of the volcanic ash erosion rate model with corrected coefficients, the volcanic ash erosion rate was recalculated using both experimental and verification data. The prediction results are then plotted against the measured values, as presented in Figure 10, to demonstrate the model's performance after correction. Additionally, the entire erosion rate dataset was analyzed to determine the correlation coefficient (R^2), as well as the NSE, IOA, and RMSE values. Based on the overall data calculation, the (R^2) value of the model is 0.97. Meanwhile, the NSE, IOA, and RMSE values are 0.94, 0.98, and 0.71, respectively. The results of this performance test indicate that the volcanic ash erosion rate model provides excellent predictive value and can be applied to cases of volcanic ash from different volcanoes. However, the equation for the coefficients of the volcanic ash erosion rate model still requires ongoing verification and refinement, especially for various unverified variations in volcanic ash thickness. Additionally, to support Lahar flood mitigation efforts effectively, this erosion rate model still needs to be tested on a watershed scale.

3.4. Sensitivity Test and Simplification Model

The proposed volcanic ash erosion rate model formula is an alternative method for estimating the volume of new volcanic material that potentially carried downstream by runoff. However, an applicable model is not only determined by its accuracy but also by its simplicity. As the erosion itself is a complex process which involves many factors, the erosion formulas developed empirically are often complicated as well. Therefore, simplification on the erosion formula is necessary to make it easier to use. In this study, the erosion formula simplification was conducted by first testing the sensitivity of the parameters in the erosion model formula.

Sensitivity tests were carried out to determine which parameters give significant influence on the volcanic ash erosion. However, the surface flow parameter (q) is apparently a function of rain intensity (i) and infiltration which is particularly related to volcanic layer thickness (T_{va}), while the parameter ($\frac{\tau_c}{\tau_0}$) is determined by slope slope (S_0), rainfall intensity (i), and volcanic ash diameter (d_s). Thus, the parameters assessed in the sensitivity test toward the erosion rate of volcanic ash, including thickness of volcanic ash layer (T_{va}), slope (S_0), rainfall intensity (i), and diameter of volcanic ash (d_s). In addition, this research also tested the sensitivity of these parameters to the value of the critical shear stress parameter. Table 2 describes the simulations of several variation values for the parameter sensitivity test. The values of the parameters being tested are determined with variations of 50%, 75%, 100%, 125% and 150% of the base value. The result of sensitivity test was given in Figure 11.

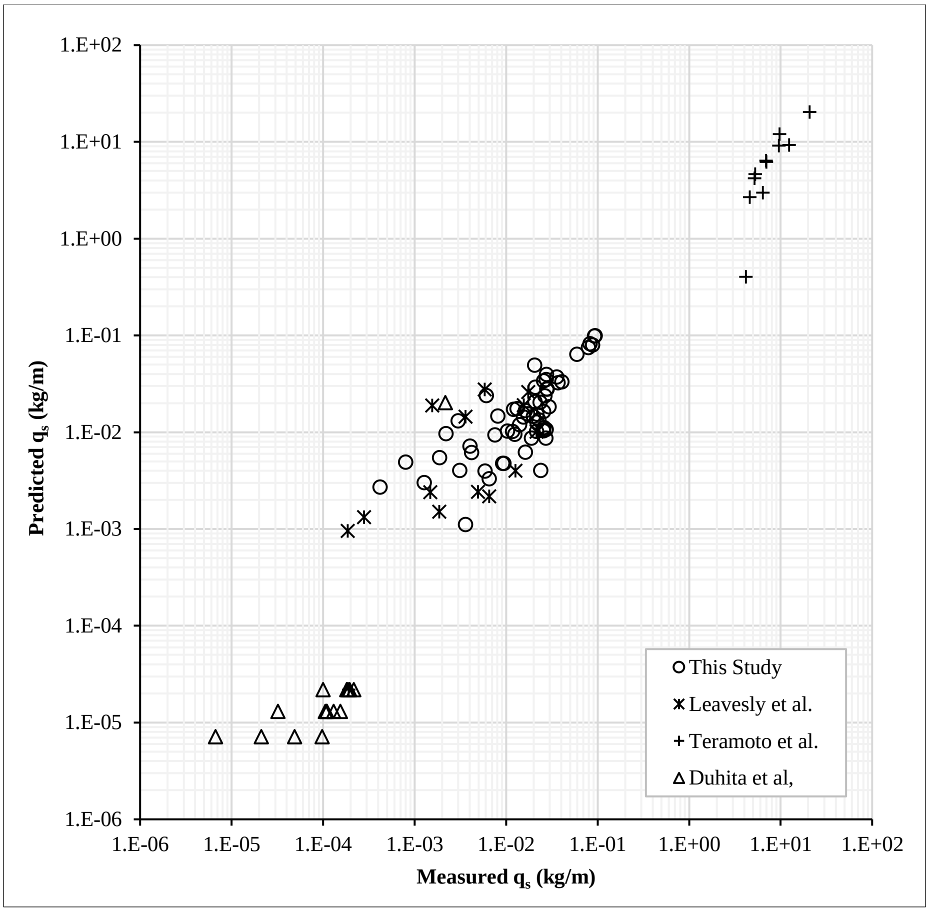

Figure 10. The comparison of model predicted q_s values with measured q_s from experimental data and verification data

Table 2. The variation values of parameter for the sensitivity test

Variation	T_{va}	S_0		i	d
	cm	deg. (º)	%	mm	mm
50%	0.75	7.50	13	60	0.250
75%	1.50	11.25	20	90	0.375
Base value	2.00	15.00	27	120	0.500
125%	2.50	18.75	34	150	0.625
150%	3.00	22.50	0.41	180	0.750

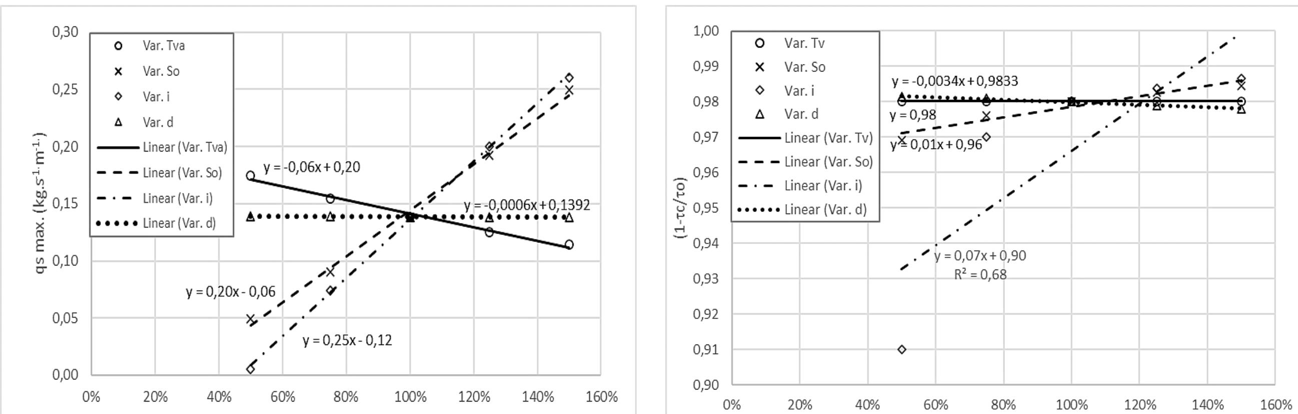

Figure 11. The result of parameter sensitivity test towards volcanic ash erosion rate (left) and the critical shear stress parameter (right)

Figure 11 (left) shows that rainfall intensity and slope are the parameters that significantly influence the erosion rate values. The graph shows that the erosion rate increases 0.25 kg/s/m per 100% increase of rainfall intensity, while the erosion rate increases 0.20 kg/s/m per 100% increase of slope. Meanwhile, increasing 100% of the thickness of volcanic ash and the diameter of volcanic as only reduces the erosion rate respectively about 0.06 kg/s/m and 0.0006 kg/s/m. Based on Figure 11 (right), it is revealed that variations in the values of all parameters do not have a significant effect on she value of $(1-\frac{\tau_c}{\tau_0})$. The influences of rainfall intensity, slope, and volcanic ash diameter are respectively 0.07, 0.01, and 0.003 per 100% increase in parameter values. Meanwhile, changes in the thickness of volcanic ash have no effect on the value of $(1-\frac{\tau_c}{\tau_0})$. Besides that, Figure 11 (right) also captures that the value of $(1-\frac{\tau_c}{\tau_0})$ tends to be close to 1 because the shear resistance of volcanic ash particles is much smaller than the stress acting due to flow (τ_c<<τ_0). In this case, when the surface flow thickness is much greater, we can assume the value of $(1-\frac{\tau_c}{\tau_0})$ is equal to 1, so that the erosion formula can be simplified as follows:

$$q_s = 10.353\rho v {S_0}^b \left(\frac{q}{v}\right)^c \left(\frac{iX_r}{v}\right)^d \tag{20}$$

From the sensitivity test, we found that the parameters that significantly influence the volcanic ash erosion are rainfall intensity, slope, and volcanic ash layer thickness. We also conclude that volcanic ash erosion is strongly sensitive to the changes of runoff, as the runoff itself is influenced by rainfall and volcanic ash thickness. Equation 20 is better applied for catchment scale prediction where the value of $(1-\frac{\tau_c}{\tau_0})$ can be neglected. However, for micro-catchment or laboratory experiment, the value of $(1-\frac{\tau_c}{\tau_0})$ is still important to be considered for the accuracy of prediction

4. Conclusion

The model verification results for the volcanic material from Leavesley et al. (1989) demonstrated that the volcanic ash erosion model was quite accurate, even for greater volcanic ash thickness that had been deposited for 4 months. However, for material conditions that have been deposited for 1 year, the prediction results showed a much larger value (over-estimated) due to the soil layer consolidation process. Model verification results based on experimental data from Teramoto et al. (2006) indicated that the prediction results of the volcanic ash erosion models are quite accurate, even when predicting cumulative volcanic ash erosion over long periods (daily/monthly) up to 7 months since the volcanic ash layer was applied. Model verification results based on experimental data from Duhita et al. (2021) revealed that applying this model to slopes greater than 25º yields inaccurate results because the movement of material on the slopes is dominated by slope failure.

This volcanic ash erosion model has been proven to be applicable to volcanic areas other than Merapi Volcano, accommodating different volcanic ash characteristics. The model is adequate for predicting the amount of erosion within one year after the eruption, a crucial period for quick response in the post-eruption phase. However, erosion is a complex process which involves many factors, the erosion formulas developed empirically are often complicated as well. To simplify its application, the volcanic ash erosion rate (q_s) was redefined as a function of slope (S_0), runoff (q), and rainfall intensity (i) by assuming the value of $\left(1-\frac{\tau_c}{\tau_0}\right)$ is equal to 1. It is hoped that with this model, the amount of volcanic ash potentially running downstream through the erosion process can be predicted, allowing for actions to be taken to mitigate the threat. However, further study is still needed to estimate the erosion rate on a watershed scale using GIS tools with heterogeneous characteristics and to understand the effects of erosion reduction due to vegetation recovery and the consolidation process of the volcanic ash layer over time.

5. Declarations

5.1. Author Contributions

Conceptualization, F.T.Y.; methodology, F.T.Y.; formal analysis, F.T.Y.; investigation, F.T.Y; data curation, F.T.Y.; writing—original draft preparation, F.T.Y., I.S., J.N., and U.B.S.; writing—review and editing, F.T.Y., I.S., J.N., and U.B.S.; supervision, I.S., J.N., and U.B.S. All authors have read and agreed to the published version of the manuscript.

5.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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5.5. Conflicts of Interest

The authors declare no conflict of interest.

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