



Evaluation of Progressive Collapse Performance in Double layer Diamatic Domes

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Abstract

Double-layer spatial domes are one of the most common spatial structures, the stability and progressive collapse of which are of great importance in design, construction and maintenance of such special structures. In this paper considering three loading cases and two types of support conditions, the collapse behaviour of double layer Diamatic dome has been investigated utilizing non-linear static analysis and alternate path method usage. In order to modelling compressive member behaviour, effective buckling modes have been obtained by eigenvalue buckling analysis for all of the members. Behaviour of compressive members has been obtained via definition of initial imperfection and non-linear static analysis. Riks arc-length method has been utilized for non-linear static analysis. The numerical results have indicated that reducing the number of the supports and focusing of load in a local area of the dome extremely impact on its vulnerability to failure, as in similar loading condition, decreasing the number of the supports reduces the capacity of damage resistance in spatial domes up to 50 percent. Investigating some models has shown that removing the critical members of the top layer has little effect on load-bearing capacity of the dome and it causes a slight failure in the structure. In this condition, structural redundancy can be considered equal to static indeterminacy. Load bearing capacity of the structure decreased up to 39 percent when compressive members of the web and bottom layers were removed. In this condition, the structure failure is considered moderate.

Keywords: Progressive Collapse; Alternate Path Method; Nonlinear Static Analysis; Double Layer Spatial Dome.

1. Introduction

One of the oldest impressive structural systems, domes which are formed from single or multi-layer bar elements, have geometrical curvature in longitudinal direction and ordinate. These structures are utilized to cover the large span like exhibitions, worships, stadiums and large halls. These specific structures create an unobstructed inner space and are so economical in material usage [1]. Being lighter compared to more conventional structural forms, high degree of redundancy, suitable and adequate stiff and consistent performance in load bearing made spatial domes particular and strategic structures that present very ideal utility in essential situations like destructive earthquakes or when it is needed to find a vast and safe shelter in urgent conditions. Research on progressive collapse began in 1968 after Ronan Point apartment destruction but incident of world trade towers in 2001 actuated researchers to evaluate of important buildings performance to abnormal loads and progressive collapse. These magnificent structures have always attracted engineers and designers because of their low weight, appropriate stiffness, ideal seismic performance, amazing beauty, covering

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of vast spaces without utilizing interior supports and etc. In recent years, many researchers have studied the collapse of spatial structures specially reticulated domes. It was believed that multilayer spatial structures do not cause concern about progressive collapse due to multiplicity of members and the high degree of redundancy. As a result, most researches have been conducted on single-layer domes. Comparing presented methods by GSA and UCF, the alternate path method is a more capable and more realistic method among researchers. This method evaluates the resistance and stability of the structure against progressive collapse by removing a member which informs the designer about resistance potential of structure against collapse. The alternate path method has been formed redundancy of structure. Availability of the alternate load-bearing elements and also alternate paths for load transition from applied position to a resistance point, describe concept of redundancy. Numerous studies about progressive collapse of building structures against the absence of enough research about spatial structures in spite of their vast functions is obvious. Researchers call large spatial structures symbolic buildings carrying high social and economic importance which offer various grounds to research and studying encounter by abnormal loads [2].

Sun and et al. in 2012 evaluated structure progressive collapse process with static and dynamic behaviour modelling involving fire condition in steel structures [3]. Feng in 2009 presented his studying on a 20 story building by applying both material and geometric nonlinear behaviours [4]. Low and et al. in 2013 did numerical modelling to predict collapse process in tall reinforced concrete buildings in strong earthquakes based on FEM*. Using IDA† and with concentrate on their near-collapse behaviour [5]. Shen and et al. (2015) [6] applied collapse seismic analysis in ordinary buildings that their lateral loading system had been elected non-ductile concentric braced frame‡. With progressive collapse dynamic modelling, Iriban and et al. (2011) [7] evaluated impact of both material and time of column elimination parameters in progressive collapse of a multi-storey reinforced concrete frame. Pertained to bridges, Jen and et al. (2013) [8], Kiyaminf and et al. (2015) [9], and Miyachi and et al. (2009) [10] studied progressive collapse in a stone arcing bridge, multi span bridge in Jozho city of China and two truss bridge with the span length of 200 meters; respectively.

In distinct types of buildings and bridges, progressive collapse issue has been evaluated widely with various methods and appliances by considering widespread and effective parameters in structure performance. But studying this topic in spatial structures which deals with increasing development in these structures has become more serious and significant recently. First serious investigation in this way and consequently presenting the most prevalent use of progressive collapse potential evaluation method namely Load Alternate Path Method§, was conducted by Smith in 1988 for first time in evaluation of double layer grids. While this method has been utilized for framed structures already [11]. Diverse studies and researches in progressive collapse of spatial structures have been performed by various researchers afterward. [12-16].

Recently in 2019, Li-min Tian et al. [17] tested a commonly used reinforcing technology on four substructures that were abstracted from a single layer spatial grid structure. They proposed a modified optimization method for two typical failure mechanisms. Before it, Rezaia and Torkzadeh (2015) [18] have been described usage of collapse constraints in the context of progressive failure. Li-Min Tian et al. (2018) [19] evaluated an experimental study on the anti-collapse mechanisms of long-span single-layer spatial grid structures. Their experimental results, including the load-displacement responses, sequences and modes of failure, and strain measurements were analysed, and the anti-collapse mechanism was examined. In addition, they conducted a numerical simulation for a single member derived from the test specimens. Four substructure experiments were conducted for a single layer spatial grid structure to investigate the typical failure mechanisms [20]. Based on their experimental results, a numerical simulation using multi-scale technology was calibrated. In order to prevent typical failures, they proposed a novel method including kinked steel pipe reinforcement and extra member reinforcement. It validated by a numerical simulation. Vitaliy et al. (2017) [21] calibrated three finite element deletion strategies for use in modeling fracture and material separation in spatial steel structures. They compared these methods for their capability to predict the location of fracture initiation and the direction of fracture propagation. Each strategy is based on micromechanical fracture behavior of the material and is independent of the overall structure type. Ye and Qi (2017) [22] simulate the complicated mechanical behaviour that occur in the collapse process of structures, extended DEM to study the continuum structures. Their proposed method is applied to the collapse simulation process of single-layer reticulated dome models. Compared with the shaking table test, it is observed that the simulation results including the collapse process and the fracture location of joints, agree well with the experimental phenomenon.

1.1. Progressive Collapse Concept

Collapse, an ultimate limit state when structure is subjected to abnormal loads which one of the most famous of them is progressive collapse. "Disproportionate" phrase is next to "progressive" term. In general, progressive collapse means diffusion of a wide and chain collapse that is caused by damage to fairly small part of structure. Domino collapse, the

* Finite element method

† Incremental dynamic analysis

‡ C.B.F.

§A.P.M.

most prevalent type of progressive collapse which its progress causes ultimate overturning in the structure and Gaussian curvature change in the spatial domes. In other hand, disproportionate failure is a structural collapse which its intensity or magnitude doesn't have any proportion with intensity or magnitude the cause of collapse. Disproportionate collapse can be progressive or non-progressive (immediate).

In spite of different meanings, progressive and disproportionate collapse terminologies often used in place of each other because disproportionate collapse mostly occurs in progressive manner. In addition, if a disproportionate collapse triggers successive failures in a big part of the structure, this event eventually leads to progressive collapse [18].

1.2. Alternate Path Method (APM) Concept

The A.P. method is utilized to reach to adequate resistance against progressive collapse in structural system. The threat independent methodology, type of excitant event doesn't have significance in functional process of A.P method since this method considers response of structural system when excitant event destroyed structure critical members. In this condition, alternate paths exist for load bearing if one of the structure members is removed from functioning. In general, this method is used to evaluate progressive collapse potential and controls the conditions which a structure can or can't compensate the eliminated member effect. This technique can be used for new structures design and capacity controls of available structures. [4].

2. Theoretical approach

The equilibrium equation of a finite element system for a nonlinear general problem is as follows:

$$R_i - F_i = 0 \quad (1)$$

That in which, R_i is consequence of external forces in step i and F_i is the vector corresponding to the internal forces. When an analysis involves nonlinear conditions dependent on the path or time-dependent phenomena, the appropriate response is obtained through incremental step-by-step approach. In this approach, it is assumed that the answer is known in step i and it is unknown in the step of $i + 1$. Δi is an increment that is chosen appropriately. Will have:

$$R_{i+1} - F_{i+1} = 0 \quad (2)$$

$$F_{i+1} = F_i + \Delta F_{i+1} \quad (3)$$

In that, ΔF_{i+1} is the increment of internal forces and is defined in terms of the Tangential stiffness matrix K_{ui}^t as follows:

$$\Delta F_{i+1} = K_{ui}^t \cdot \Delta U_{i+1} \quad (4)$$

In which ΔU_{i+1} is nodal displacement increment vector.

The tangential stiffness matrix is the same as the stiffness parameter in linear analysis, except that it depends on forces and partial displacements. This matrix should be formulated based on the latest information of updated structure, the work done by the incremental changes in the displacement, both of the first and upper order sentences, and the initial forces of the members. We will have the following equations:

$$K_{ui}^t \cdot \Delta U_{i+1} = R_{i+1} - F_{i+1} \quad (5)$$

$$U_{i+1} = U_i + \Delta U_{i+1} \quad (6)$$

During the analytical process, we follow all the partial of the structure from the initial mood to the final mood. This means that a Lagrangian formulation is chosen for the problem. In the Lagrangian incremental method, the equilibrium of the structure in step i is expressed using the principle of virtual displacements. In an elastic analysis, the material characteristic matrix, is constant and the total stress is calculated from the total strain. But in the inelastic analysis, total stress at time t , is dependent to stress and strain history. To overcome the divergence problem at the ultimate point and follow the paths of equilibrium and passing from critical to post-critical points, the arc-length method is the most effective method for tracking the equilibrium path. The main idea of this method is the introduction of a load factor that increases or decreases the amount of applied loads. The equation governing the nonlinear problem in this strategy in increment $i+1$ is as follows:

$$\lambda_{i+1} \cdot R - F_{i+1} = 0 \quad (7)$$

In which the λ_{i+1} is an indeterminate load factor (scalar) and R is the load vector on the structure. This vector can include any kind of loading, but it is constant throughout the calculation of the response. The basic assumption in the analysis is that the load vector changes proportionally at the time of the computation of the response and its value is controlled by the coefficient of load. One of the most commonly used methods for arc-length process is the Riks method. In this method, load increment is controlled by a constraint equation to cause the repeat path follow a path that is perpendicular to the tangent to the repeat initiation point.

3. Collapse Modeling and Analysis

Double layer spatial domes have more rigidity compared to single layer domes. This issue has caused the assumption that double layer structures are resistant to applied damages and don't need special investigation. This is while that this spatial structures are at risk of progressive collapse due to production and design errors, over loading, extreme local loads, inappropriate maintenance, length of thin members, unfit connections and etc. In this paper, collapse behavior studying of double layer spatial domes in special load and support conditions have been conducted based on similar idea. In dome structures to discover snap-throw event conditions, researchers have presented diverse types of loadings [23] which among them three critical loading cases that can simulate real conditions for progressive collapse occurrence have been selected in evaluation of structure collapse performance. Design of structure has been performed for these loads and two perimeteral and meridional support conditions. Static analysis of collapse has been done utilizing alternate path (AP) method and with considering both material and geometrical nonlinear behaviour for a double layer Diamatic dome using finite element software ABAQUS. For nonlinear equilibrium equations solving, Arc-length Riks method has been utilized. For collapse analysis, the process of below has been done by ABAQUS software in a step by step manner:

- For each member effective modes of buckling have been obtained from Eigenvalues buckling analysis. Initial imperfection modeling equal to 0.001 times the length of the member has been extracted from linear combination of these modes.
- After applying the imperfection and by considering large deformations, behavior of compressive members has been taken from nonlinear analysis and so ideal stress-strain diagram of the members has been traced. Full Elastic-Plastic bilinear behavior of steel has been used for tensile members behavior modeling (Figure 4).
- Critical members of the structure have been specified by linear static analysis and based on maximum compressive force in each model. Three critical members have been determined for each model (Figure 8).
- Geometrical and material nonlinear static analysis have been conducted on every six intact models. The undamaged model is the structure without critical member elimination.
- By elimination of each critical member, nonlinear static analysis with AP method has been performed in every model (Figures 9 to 14).

To overcome divergence problem in limited point in nonlinear static analysis, the Riks Arc-Length method has been used to follow the equilibrium paths and pass critical points to post-critical points. In this method a constraint equation controls the load increments.

4. Experimental Study

A double layer Diamatic spatial dome with external span of 20 m and height of 5 m has been studied (Figure 1) and its configuration has been conducted using FORMIAN software (Figure 2).

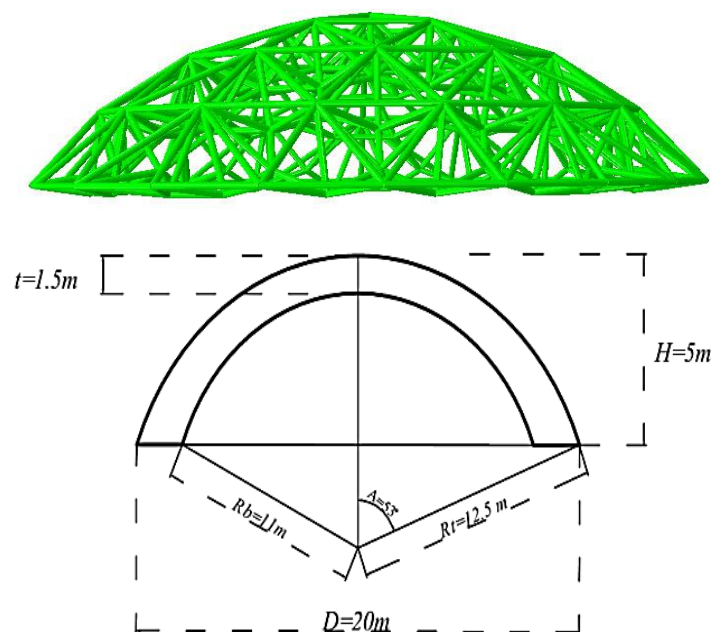


Figure 1. Section of the dome

This structure has been evaluated in both perimeter and meridional support conditions which each of them is under three critical-load conditions (Figure 3). Diverse models properties of the structure has been presented in Table 1. To equate the conditions, one type of hollow tubular section in five different lengths adaptable with British Standard (B.S) has been utilized in every six models according to Table 2. Supports have been located, in perimeter support conditions at every nodes of last ring of bottom layer of the dome and in meridional support conditions, at two nodes of last ring of bottom layer in two sides of top layer meridian lines. In meridional support conditions compared to perimeter support conditions 10 supports are eliminated. The models with perimeter or peripheral supports and the models with meridional supports are identified by DP* and DM† phrases respectively.

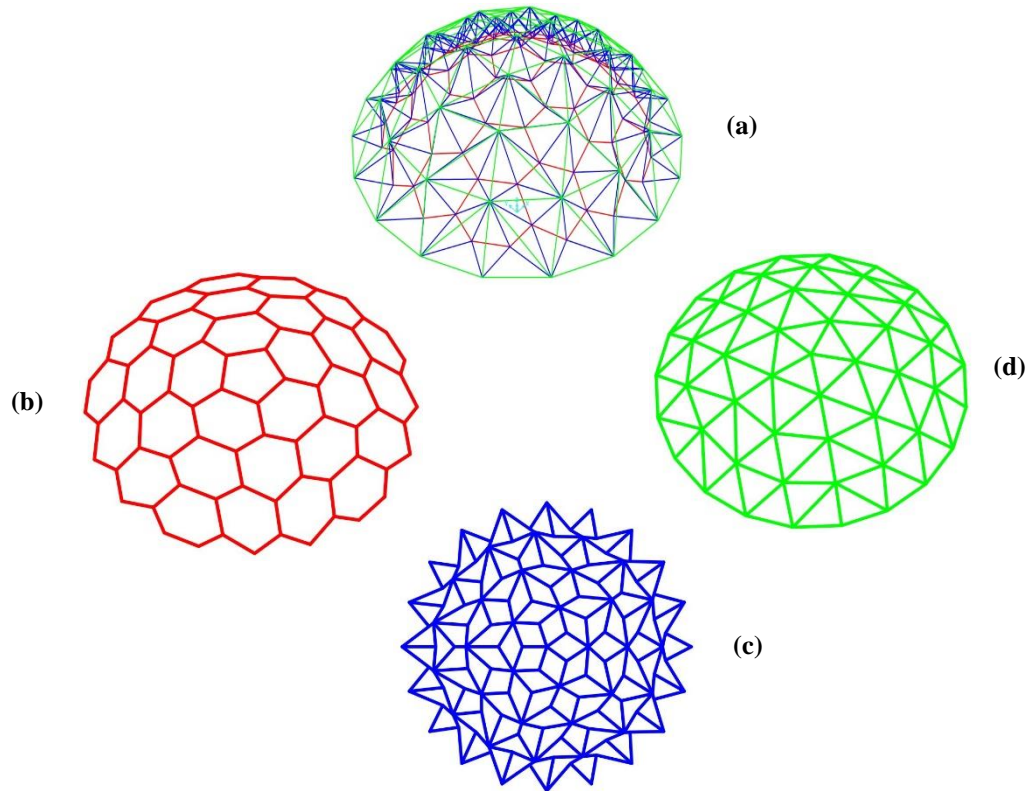


Figure 2. Configuration of the dome: a-Assembled dome; b-Bot layer; c- Web layer; d- Top layer

Three critical-load conditions that have used which consist of:

- A vertical concentrated load on crown node that applied to structure increasingly (sum of the dead and snow loads).
- Constant concentrated loads on all the nodes of dome (dead load "D") and increasing concentrated load on the crown node (snow load "S").
- Constant concentrated loads on all the nodes of dome (dead load) and increasing concentrated load on the middle node of a sector of the top layer of the dome (snow load).

All the loads are applied to structure at the same time.

Table 1. Analysis models properties

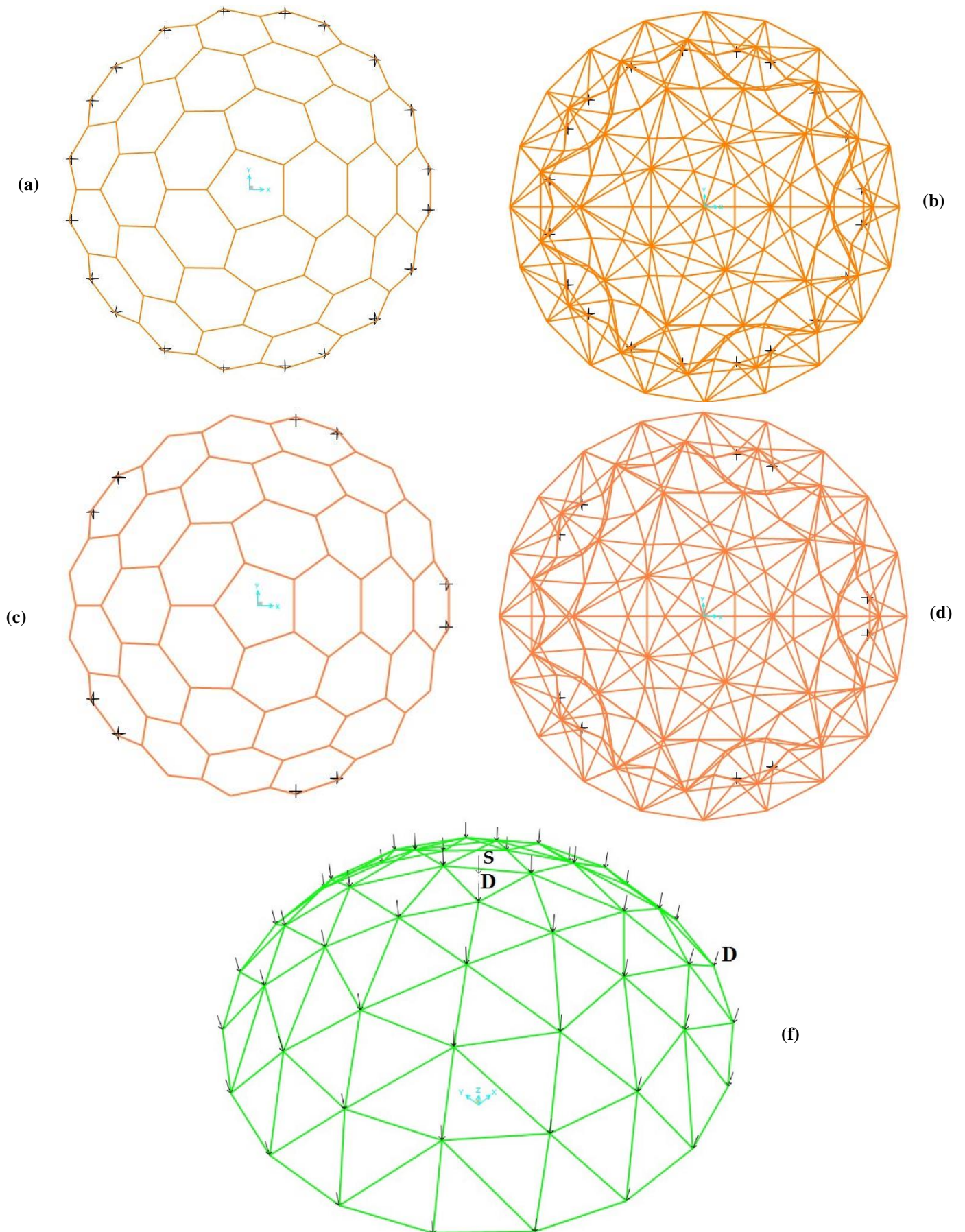
Title of the model	The support conditions	The Loading conditions
DPC	Perimeter	D* & S** at crown node
DMC	Meridional	D & S at crown node
DPUC	Perimeter	D at whole nodes & S at crown node
DMUC	Meridional	D at whole nodes & S at crown node
DPUN	Perimeter	D at whole nodes & S at mid node
DMUN	Meridional	D at whole nodes & S at mid node

D*: Dead load; S**: Snow load

* Dome with Perimeter Supports

† Dome with Meridional Supports

Section sufficiency control for all cases have been conducted in SAP 2000 software according to limited design LRFD criteria of AISC-360-10 code while considering the reduce resistance factor $\phi=0.9$ for both compressive and tension modes.



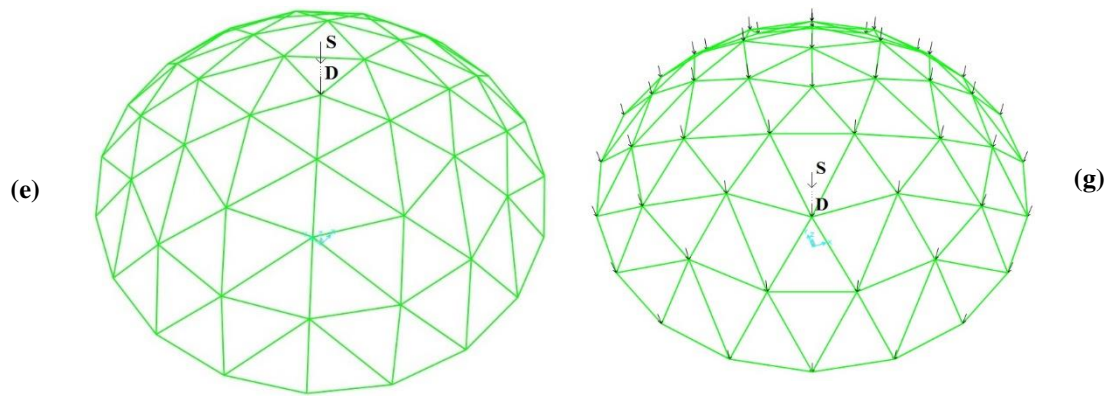


Figure 3. Support and Loading conditions of the dome: (a) Bot layer plan in DP mode; (b) The dome plan in DP mode; (c) Bot layer plan in DM mode; (d) The dome plan in DM mode; (e) Concentrated loading at the crown node; (f) Uniform dead load at whole & dead and live concentrated load at the crown node; (g) Uniform dead load at whole & dead and live concentrated load at the mid node.

The gravity load combinations of ASCE-07 code are used for load cases. Gravity acceleration is equal to 9.81 m/s^2 and St-37 with presented properties in Table 3 is used for material.

Dead and snow loads are equal to 50 kgr/m^2 and 200 kgr/m^2 respectively and gravity load combinations are: $(1.4D, 1.2 D + 1.6 S)$

Table 2. Properties of the structure members

Type	Profile (mm)	The dome layer	Section area (mm^2)	Slender factor (L/r)
I	CHHF 273 × 16	Top	12918.23	32.95
II	CHHF 273 × 16		12918.23	38.45
III	CHHF 273 × 16		12918.23	21.97
IV	CHHF 273 × 16	Web	12918.23	16.48
V	CHHF 273 × 16		12918.23	27.46
III	CHHF 273 × 16	Bot	12918.23	21.97
IV	CHHF 273 × 16		12918.23	16.48

Table 3. Steel material properties

Young's modulus E (Kgr/m^2)	Mass density ρ (Kgr/m^3)	Yield stress F_y (Kgr/m^2)	Ultimate stress F_u (Kgr/m^2)
2.1×10^{10}	7850	2.4×10^7	3.6×10^7

5. Collapse nonlinear static analysis

The Stress-strain behavior of the members presented in Figure 4.

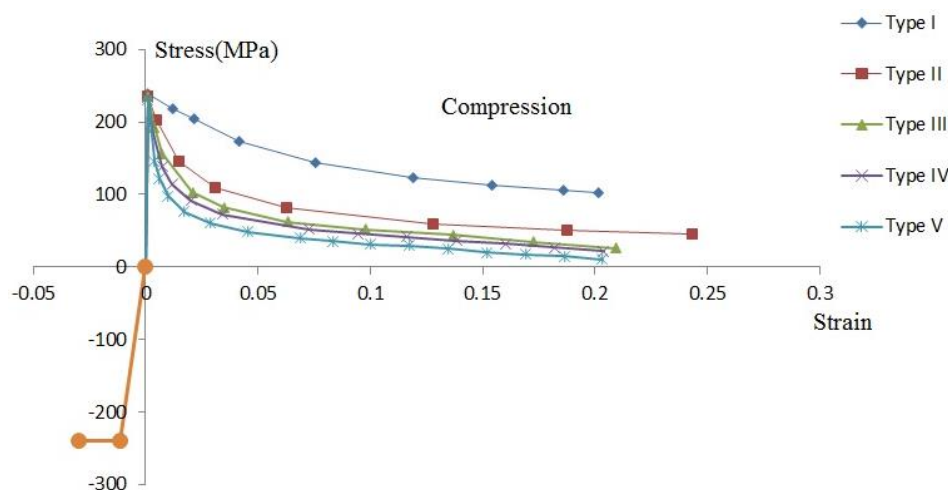


Figure 4. Stress-strain behaviour modelling of the structure members

A linear analysis has been conducted in each load condition and a critical member has been chosen in every layer of the dome based on maximum compressive stress. The results presented in Figures 5 to 8.

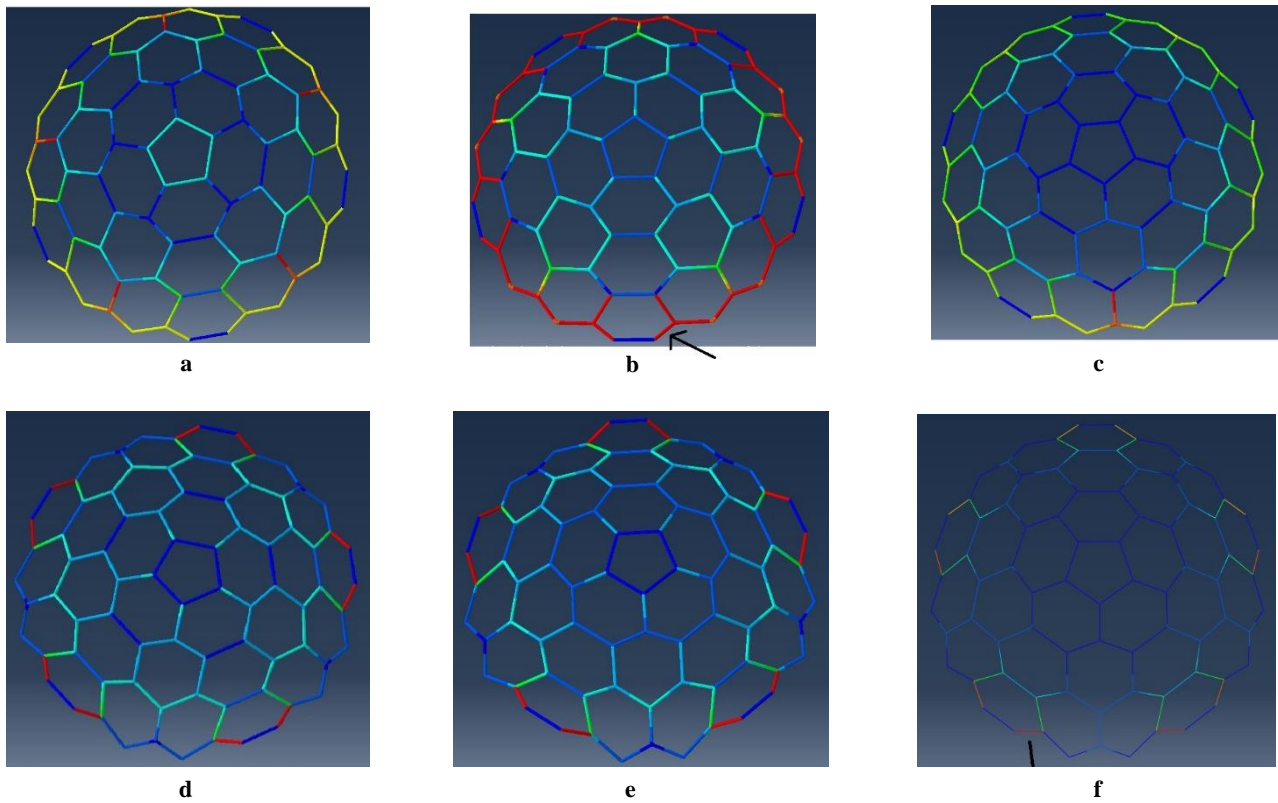


Figure 5. Linear static analysis for bot layer under different loading in Table 1: a. DPC- b. DPUC- c. DPUS- d. DSC- e. DSUC- f. DSUS

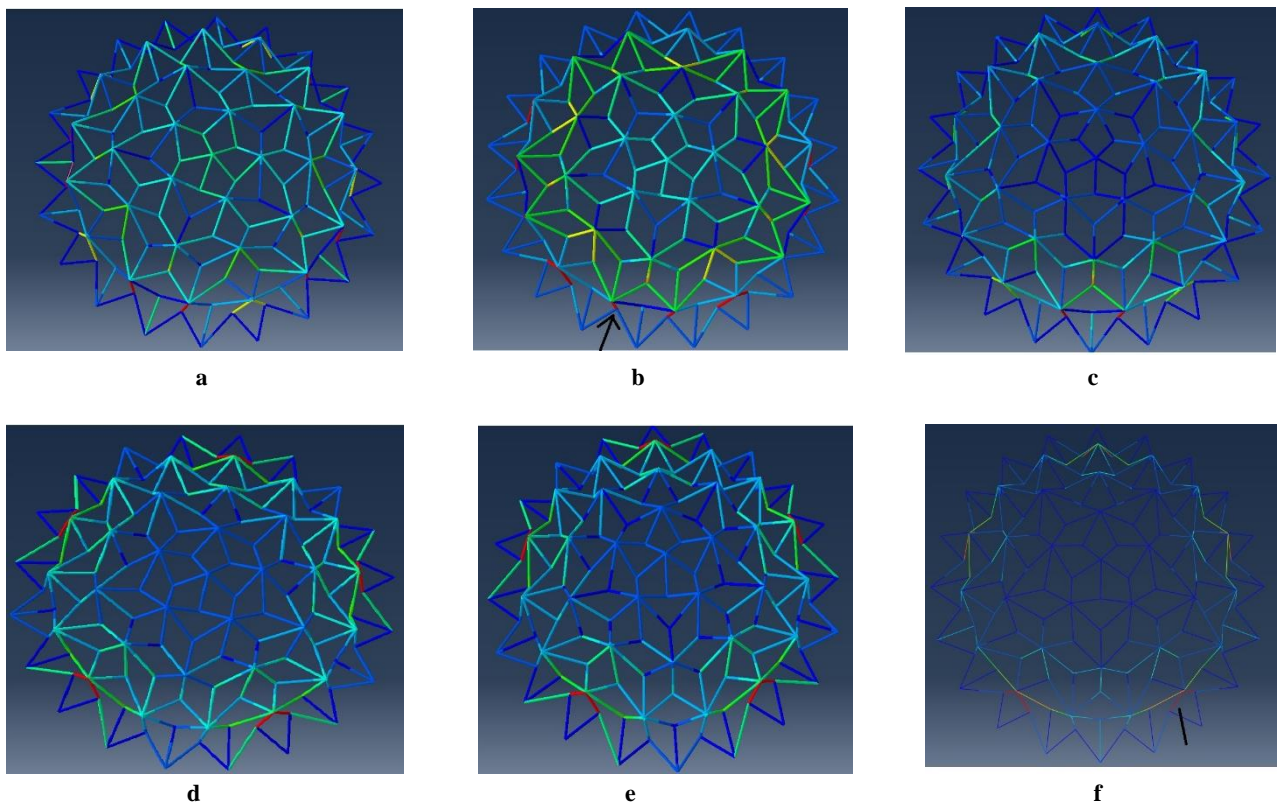


Figure 6. Linear static analysis for web layer under different loading in Table 1: a. DPC- b. DPUC- c. DPUS- d. DSC- e. DSUC- f. DSUS

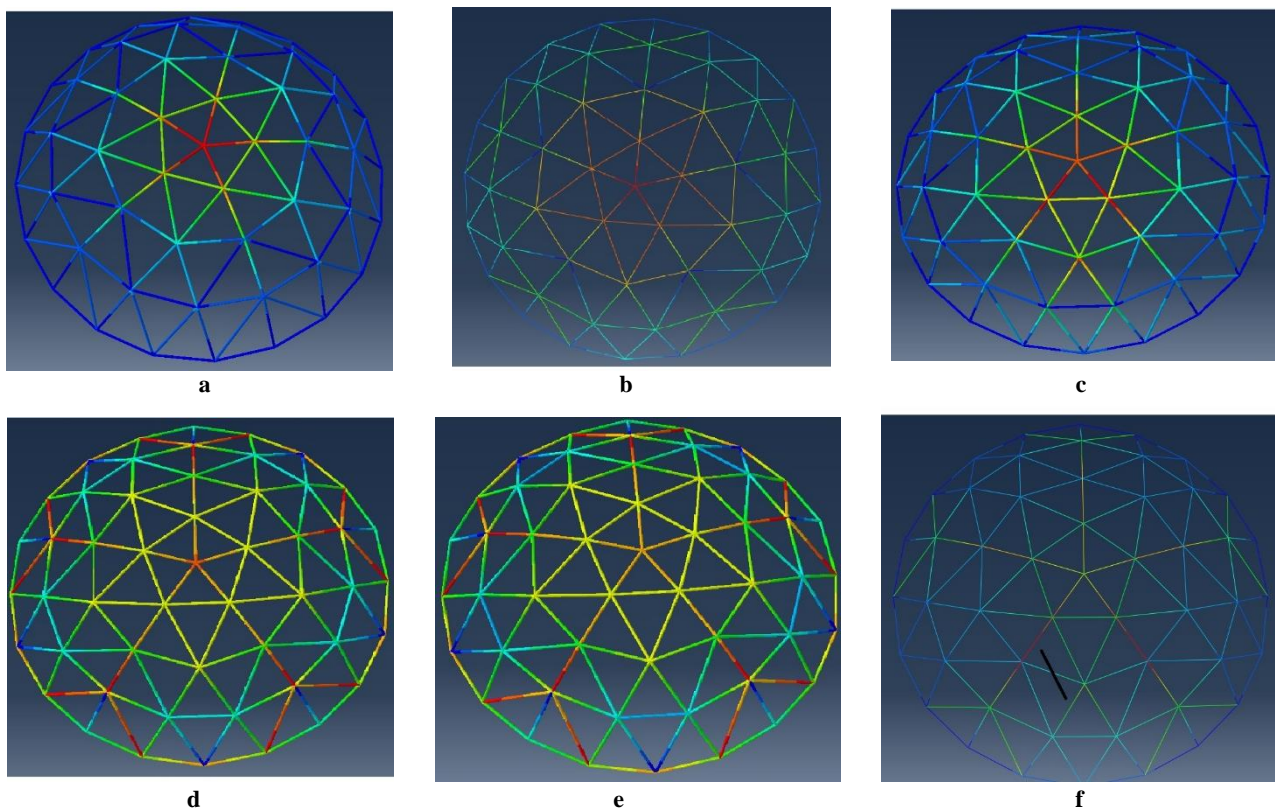


Figure 7. Linear static analysis for top layer under different loading in Table 1: a. DPC- b. DPUC- c. DPUS- d. DSC- e. DSUC- f. DSUS

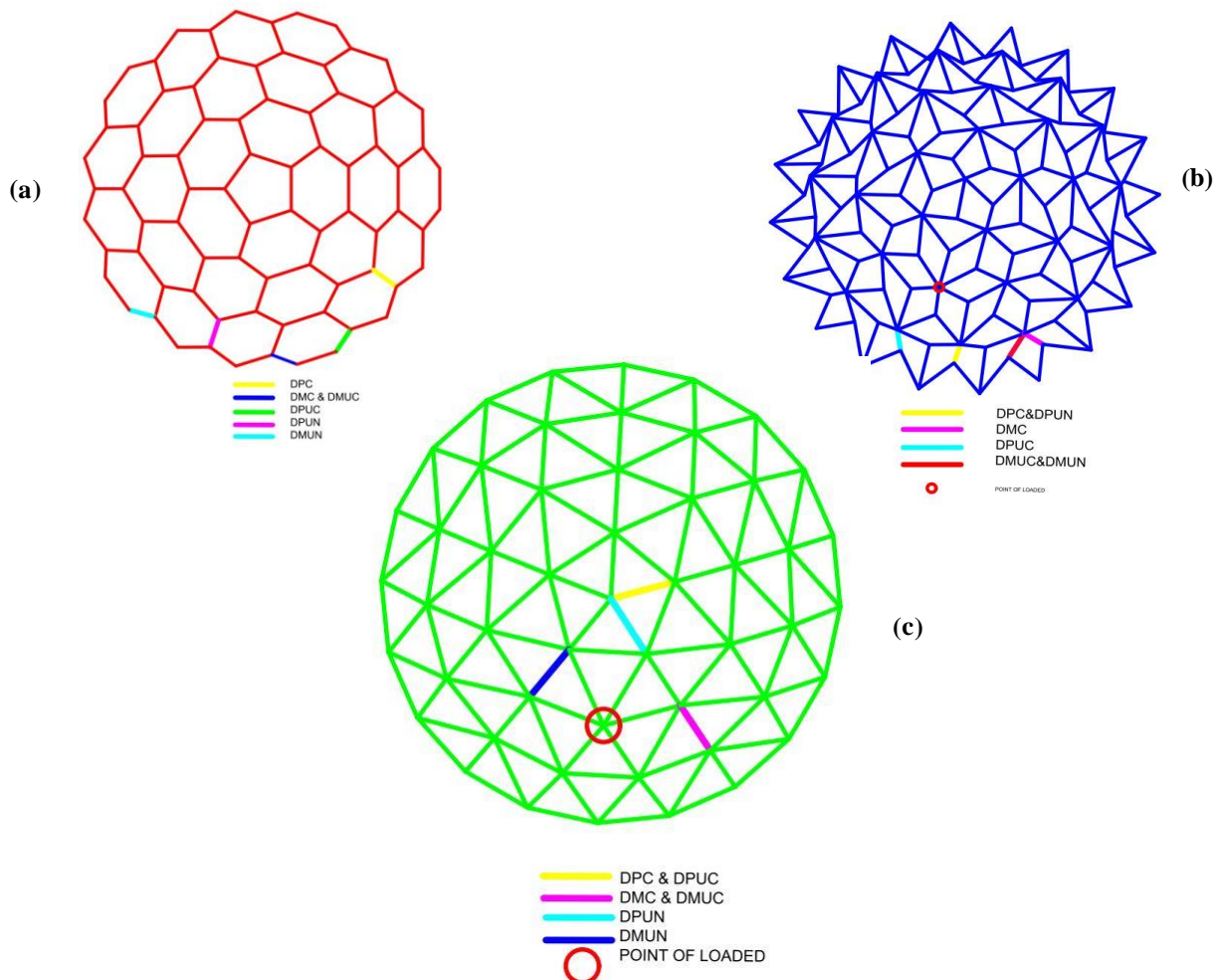


Figure 8. The critical members of each model: a-Bot layer; b-Web layer; c-Top layer

After elimination of each critical member, nonlinear static analysis with AP method has been performed in every model (Figures 9 to 14). Behavior of each model has been presented in continue:

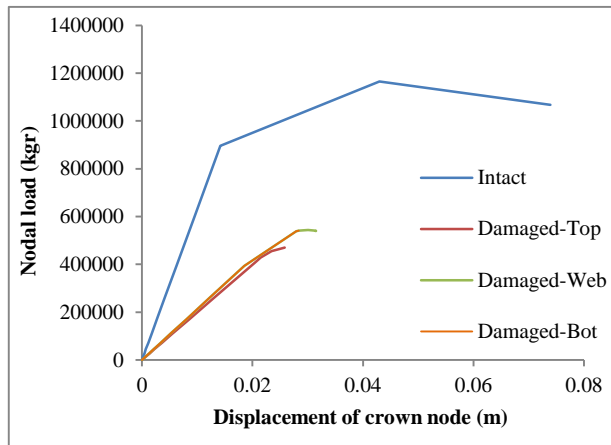


Figure 9. Load-displacement behaviour of DPC model

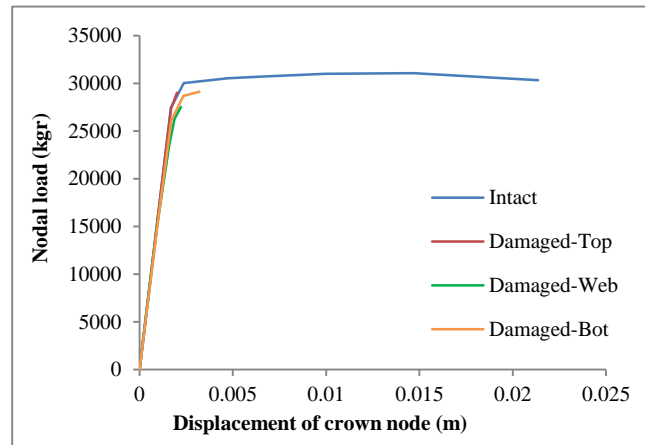


Figure 10. Load-displacement behaviour of DMC model

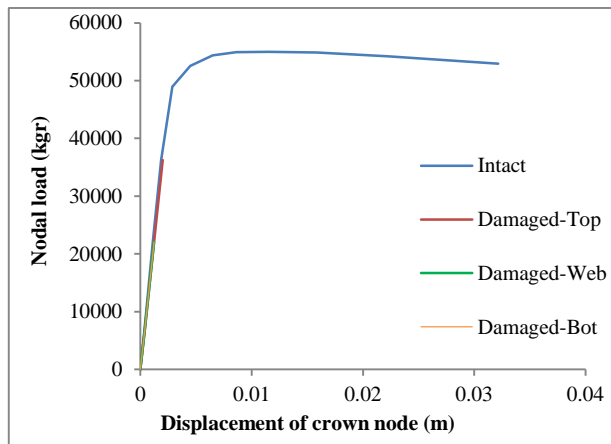


Figure 11. Load-displacement behaviour of DPUC model

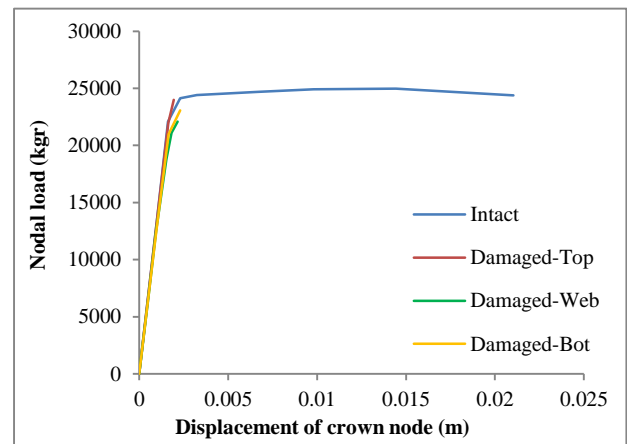


Figure 12. Load-displacement behaviour of DMUC model

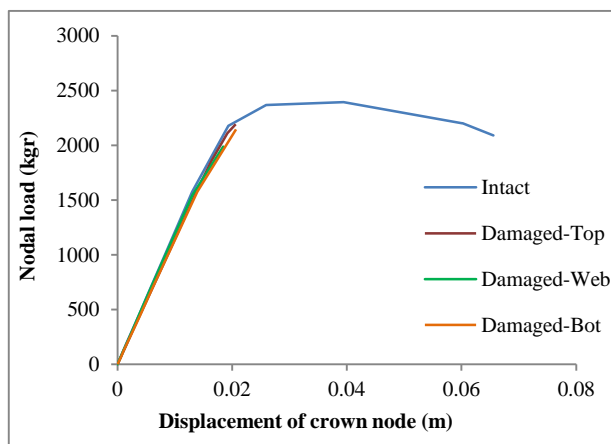


Figure 13. Load-displacement behaviour of DPUN model

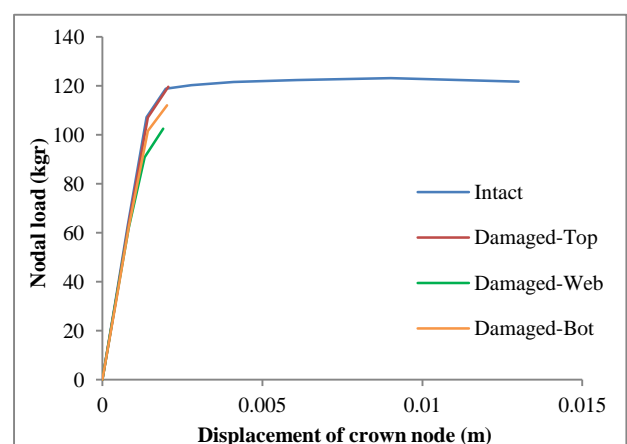


Figure 14. Load-displacement behaviour of DMUN model

Outcomes of computations of collapse analysis have been mentioned in Table 4. Parameters that presented in this table are:

Δ_1 : Deformation of the intact structure in first collapse level;

Δ_2 : Deformation of the intact structure in ultimate collapse level;

P_1 : Collapse load of the intact structure in first collapse level;

P_{TOP} : Collapse load of the damaged structure with elimination of the critical member in the top layer;

P_{WEB} : Collapse load of the damaged structure with elimination of the critical member in the web layer;

P_{BOT} : Collapse load of the damaged structure with elimination of the critical member in bot layer;

μ : Ductility;

α : Residual redundant factor;

$$\mu = \frac{\Delta_2}{\Delta_1} \quad (8)$$

$$\alpha = \frac{P_i}{P_1} ; \quad i: \text{TOP; WEB; BOT} \quad (9)$$

First collapse level is the beginning point of buckling of the first compressive set and ultimate collapse level is defined by beginning point of progressive collapse which causes general collapse in the structure.

Residual redundant factor defines redundancy effect in the structure and present remained loading capacity in the damaged structure [24].

A redundant structure has been defined as the structure that its additional structural capacity or reserve resistance of it allow to carry loads more than the expected design loads because of considering singular capacity of its members. Redundancy is the function of singular properties of load bearing members while these properties themselves are function of topology, continuity and ductility of the structure. Also redundancy is a structural attribute in the global system of the structure. It must be mentioned that redundancy isn't just obtaining resistance. Structural redundancy topic is usually investigated in related to the fail-safe systems.

Table 4. Final result of collapse nonlinear analysis

Model	Nonlinear analysis results of intact structure			Nonlinear analysis results of damaged structure			Residual redundant factor (α)			Ductility (μ)
	Δ_1 (m)	Δ_2 (m)	P_1 (kgr)	P_{TOP} (kgr)	P_{WEB} (kgr)	P_{BOT} (kgr)	TOP	WEB	BOT	
DPC	0.0142	0.0430	895437	428628	393529	393529	0.480	0.440	0.440	3.020
DMC	0.0017	0.0147	27454.5	27342.7	26274.3	26101.8	0.995	0.960	0.950	8.730
DPUC	0.0029	0.0115	48932.8	36283.4	22087.5	22087.5	0.740	0.450	0.450	4.040
DMUC	0.0016	0.0144	22080.6	21987.3	21102.9	20991.4	0.995	0.960	0.950	8.910
DPUN	0.0193	0.0394	2177.55	2110.35	1546.57	1574.58	0.970	0.710	0.720	2.040
DMUN	0.0014	0.0090	107.203	107.034	90.9189	101.614	0.998	0.850	0.950	6.550

6. Conclusion

In this research, buckling and stability of compressive members were computed to evaluate double layer spatial dome performance in progressive collapse and the structure's behavior was investigated based on redundancy criterion. Force absorption potential and pass from progressive collapse has been studied by presentation an evaluation factor called Residual redundant factor. Investigation has indicated that redundancy factor is an appropriate criterion to survey resistance of multilayer structures with numerous of members. This factor can be used in primary calculation of the structure in which there is assurance from non-progression of collapse apparently.

Proportional to geometric and loading conditions, behavior modeling in compressive members has been formed and it was abstained from usage a unique behavior for all of members. This issue increase accuracy of modeling and validity of results. The proposed method is applied to the collapse simulation process of double-layer dome models.

The special support and loading conditions which were applied to the dome can utilize for obtain most critical mode of design.

By linear static analysis of the dome, the critical members were identified after modeling the compressive members' behavior and the bilinear ideal behavior of the steel was utilized for tensile members. Using non-linear static analysis, the structure's behavior was evaluated in six cases of the load and support after critical members eliminated from the structure. With comparing in the models' behavior and regarding presented results of the calculations in Table 4 it can be stated that despite high degrees of redundancy and enough rigidity, progressive collapse may occur in double layer spatial domes due to buckling of some members under abnormal local loading. Even in lateral support conditions ductility and load bearing therefore resistance to collapse had been decreased over 50%. Collapse of the first member did not cause to collapse of whole the structure in any of models. In same support conditions, Diamatic double layer domes are more sensitive to local loading in crown that given the importance of this issue when we attend to snow area of the dome in the upper ring. In same concentrated loading conditions, decline in number of the supports resulted in

decrease ductility and damage bearing capacity in spatial dome so massively that amount of this reduce may be quadrant. Although in same support conditions, applying a uniform load at all of the top layer nodes of the dome do not have any effect on the dome collapse parameters. Comparing the ductility and residual redundant factor, it can be observed that the layers of double layer spatial dome are more vulnerable in perimeter support conditions in comparing to meridian support conditions. Investigating the collapse paths in Diamatic domes, it has been specified that beginning of collapse is at the applied load position in perimeter support conditions and damage spread through meridian lines in whole the structure. So in constant with loading conditions, decrease number of supports caused the collapse of the path begins from remained supports and moves to upward and crown node.

This study is the way for further refining investigation of the collapse process and entire-process design of the collapse of the dome and also for providing a new numerical analysis process.

7. Conflicts of Interest

The authors declare no conflict of interest.

8. References

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