Available online at www.CivileJournal.org

Civil Engineering Journal

(E-ISSN: 2476-3055; ISSN: 2676-6957)

Vol. 9, No. 07, July, 2023

The PVD-Accelerated Soil Deposit Consolidation Based on Elliptic Cylindrical Model

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Received 02 March 2023; Revised 11 June 2023; Accepted 18 June 2023; Published 01 July 2023

Abstract

One method to deal with the problem of soft soil is to accelerate consolidation by preloading and prefabricating a vertical drain (PVD). Consolidation analysis was based on a one-dimensional theory that required PVD as an equivalent circular well. Further studies on a simple approximate for consolidated soil were represented by equivalent permeability coefficients, k_{ve}. The equivalent conductivity coefficient is influenced by the soil and PVD permeability coefficients. The formulation of k_{ve} based on the influence area in cylindrical has been applied to a lot of construction projects. According to the comparative analysis of the classical consolidation theory, it is considered that the diameter of the circle is less representative. This study proposed a simple formulation of k_{ve} based on the elliptical assumption of influence area. The k_{ve} was derived based on an equal average degree of consolidation in one dimension, which applied the elliptical coordinate for degree of consolidation in the radial direction. The formulation is based on an elliptical cross-section and a cylindrical coordinate formulation. The validation of this formula is conducted with numerical calculations using 2D FEM. The results show that the consolidation time in the elliptical discharge area is shorter than that in the circular discharge area.

Keywords: Vertical Permeability Equivalence; Consolidation; Influence Area; Radial; Ellipse.

1. Introduction

Soft soil is one of the problems found in the construction world. The high compressibility of soft soil causes damage to road construction and buildings. One method to stabilize soft clay soil is Prefabricated Vertical Drain (PVD). It works effectively and efficiently as a vertical drain to accelerate the consolidation process of soft clay soil.

Prefabricated vertical drain (PVD) installation is cost-effective and easy [1]. It is the axis or channel sheet of a composite geosynthetic system, which consists of a polymer inner core with flow path grooves on both sides and an outer nonwoven geotextile filter jacket. PVD accelerates consolidation by removing water in fine-grained soils [2–9]. Most of the researchers assume the area of influence of the PVD is circular or cylindrical. Several equations were published to calculate the equivalent diameter of the axis channel with different considerations, yielding different results [10]. The most important issue discussed in PVD designs is the determination of the equivalence of the diameter of the tapered drain with the radial drain, which determines the size of the inlet surface [2]. The difference between the rectangular PVD and the circular equivalent diameter quietly affects the flow line [11]. Chai et al. [12] introduced the k_{ve} as a simple method to represent the equivalent vertical permeability of soil and PVD. The 3-dimensional analysis for embankment and PVD-cased designs recommended the equivalent permeability for practice [13].

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http://dx.doi.org/10.28991/CEJ-2023-09-07-08

Considering the real size of PVD [14] analytical solution in elliptical cylinder more realistic instead of cylinder. A study of the elliptical cylinder of PVD derived the average degree of consolidation and assessed the accuracy of the solution. The non-linear consolidation equation based on the elliptical coordinate for vertical drainage, considering the non-linear changes of the permeability and compressibility coefficients of soft soil during the consolidation process, was derived [14]. The difference between the elliptical cylindrical and the cylindrical models is the equivalent average horizontal drainage distances. The analytical solution of vacuum preloading and consolidation with PVD was also delivered by this assumption [15]. This study is proposed to derive the formulation of k_{ev} based on an elliptic cylindrical model. The verification of the formulation is delivered by embankment on soft soil, compared to the cylinder model.

2. Theory of PVD as a Cylindrical Body

2.1. Basic Theory

The fundamental consolidation theory is Terzaghi's one-dimensional consolidation, which can be written as:

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2}; C_v = \frac{k_v}{m_v \gamma_w} \tag{1}$$

where C_v is the vertical consolidation coefficient, u is the excess pore pressure, t is the real-time, z is the position of the soil deposit, k_v is the permeability coefficient in a vertical direction, m_v = the coefficient of vertical compression, and γ_w is the unit weight of water. Equation 1 is derived under the assumption that the dissipation of water is only in a vertical direction.

PVD works to accelerate the settlement under the loading stage due to the primary consolidation of soil. The water flows mainly in the horizontal direction because the spacing between PVD is shorter than the vertical direction flow to porous layer soil. The following radial (horizontal) consolidation theory was proposed by Barron (1948) [16]:

$$\frac{\partial u}{\partial t} = C_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right); C_h = \frac{k_h}{m_v \gamma_w} \tag{2}$$

where C_h is the horizontal consolidation coefficient (m^2/s); k_h = the horizontal coefficient of permeability. The combination of vertical and horizontal flow consolidation is derived from Equation 3 proposed by Carrillo (1942) [17]:

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} + C_h \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{3}$$

The consolidation progress had been estimated by the average degree of consolidation U. The initial pore water pressure distribution and the time factor data were used to calculate the degree of consolidation in the vertical direction. Meanwhile, the degree of radial consolidation was developed by Baron in 1948 [16] as Equation 4:

$$U_h = 1 - exp\left[\frac{-8T_h}{F_n}\right]; T_h = \frac{C_h\, t}{D^2} \tag{4}$$

where: t is time of consolidation (s); D is diameter of unit of PVD influence area (m); F_n is drain distance factor is $\ln(D/d_w)$- ¾; d_w is PVD equivalent diameter.

Generally, PVD is installed in a square or triangular pattern. With the area of influence, the equivalent diameter of a rectangle D is equal to 1.13 S, and D is equal to 1.05 S for a triangular pattern, where S is the spacing between PVDs. The theory of consolidation with radial drainage assumes that the water in the soil flows horizontally in a circular section to PVD. The radial consolidation equation considers the vertical drainage diameter (d_w). This d_w value is the cylindrical diameter of the drainage column if the vertical drainage is made of sand columns. PVD bands typically have a width of 100–120 mm and a thickness of 3–6 mm. The cross-section is not circular; therefore, the equivalent area in circular must be expressed in terms of the equivalent diameter, d_w notation. The equivalent diameter of the PVD is defined as the diameter of the drainage circle, which has the same drainage capability as the PVD (Figure 1).

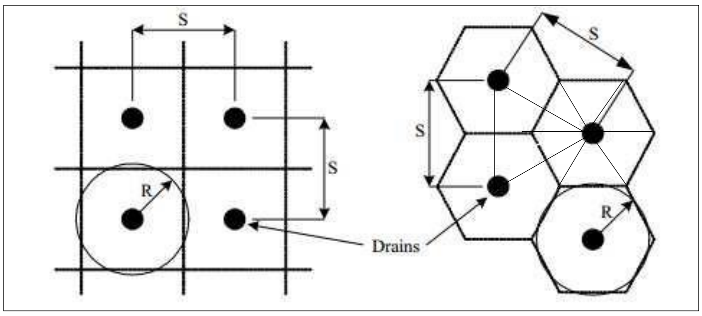
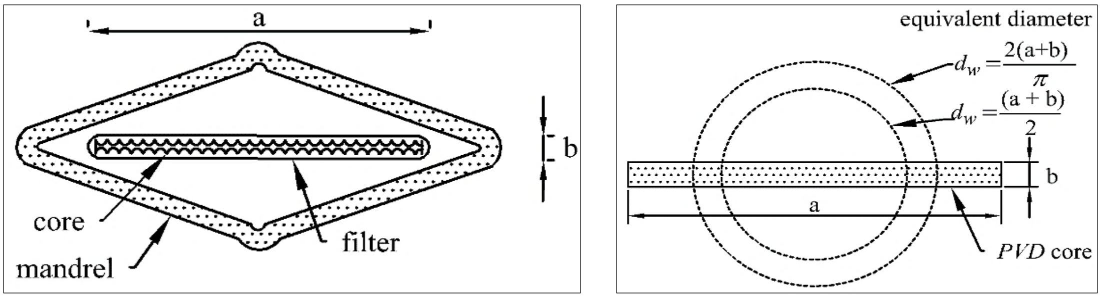


Figure 1. PVD pattern (left: square pattern; right: triangle pattern)

In many conditions, the equivalent diameter (d_w) can be considered independent of the soil layer conditions, soil properties, and the effect of PVD installation, but it depends only on the drainage geometry and configuration. Figure 2 shows the mandrel that is used to install PVD in the soft soil layer. It must be strong enough to prevent bending or buckling. The soft soil would be analyzed in the smear zone due to the disturbed zone.



(a) (b)

Figure 2. (a) Cross-section of PVD and mandrel, (b) PVD Equivalent Diameter

In the design, the equivalent diameter is determined by surface area of the vertical drainage in a circle that is the same as or equivalent to the surface area of the vertical drain as [9] estimate $\pi d_w = 2(a+b)$. Thus, the equivalent diameter is:

$$d_w = \frac{2(a+b)}{\pi} \tag{5}$$

where a is typical width, b is thickness of PVD.

The other researchers proposed d_w in Table 1 based on different considerations. The equations are considered as a throttle to control the drain [18]. The equation was established by considering the cross-sectional area of a PVD. The development of d_w is based on similar discharge in equivalent circular when it is subjected to pressure-controlled pumping conditions [2]. However, a definitive recommendation is available regarding the validity of these equations.

Table 1. PVD diameter equivalent formula

Reference	Equivalent equation
Hansbo [9]	$d_w = \frac{2.(b+\delta)}{\pi}$
Atkinson & Eldred [18]	$d_w = \frac{(b+\delta)}{2}$
Fellenius & Castonguay [19]	$d_w = \sqrt{\frac{4b\delta}{\pi}}$
Long & Covo [10]	$d_w = 0{,}5b + 0{,}7b$
Abuel-Naga & Bouazza [2]	$d_w = 0{,}45b$

2.2. Modelling PVD in Plane Strain Analysis

Modeling the effects of prefabricated vertical drainage (PVD) in element analysis as plain strain has been widely applied. Methods are classified into four groups, which are solid elements, macro elements, one-dimensional (1D) drainage elements, and equivalent vertical hydraulic conductivity (k_{ve}), respectively. In the k_{ve} method, the PVD-improved soil layer can be analyzed in the same way as the unimproved soil. For all methods, the average degree of consolidation in the plane strain analysis is matched with axisymmetric conditions by modifying the hydraulic conductivity of the soil layer and/or discharge capacity or distance of the plane strain channel. The effectiveness of the k_{ve} method has been investigated by a large-scale model test of PVD consolidation and numerical results. The method is acceptable under axisymmetric conditions.

Chai et al. [12] developed an analysis of the degree of consolidation by increasing the mass of the hydraulic conductivity, which provides different formulas for the degree of vertical and horizontal consolidation. Generally, vertical drainage increases the hydraulic conductivity of the subsoil mass in the vertical direction. It is much more logical to determine the vertical hydraulic conductivity, which represents the vertical drainage effect of the natural soil layer and the radial drainage effect due to PVD installation [13]. The solution for the equivalent vertical hydraulic conductivity is as follows:

$$k_{ve} = \left(1 + \frac{2.5l^2}{\mu D^2}\frac{k_h}{k_v}\right)k_v \tag{6}$$

3. Theoretical of PVD as Elliptic Cylindrical

3.1. The Performance PVD in Elliptical Coordinate

According to Huang et al. [14], the shape of PVD is more appropriately considered to be equivalent ellipsoids when it is compared using cylindrical equivalent methods. The width b is much greater than the thickness δ, which is why it is more reasonable for the strip PVD to be equivalent to an elliptical cylinder as Figure 3.

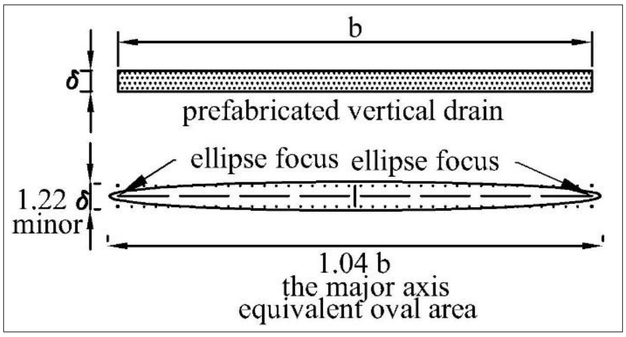


Figure 3. Equivalent elliptical model and PVD

The assumption of the drainage body that the smear zone and influence are all simplified as cylindrical ellipses is as follows:

- Lateral deformation was ignored, and the vertical deformation at each point at the same depth is the same. The strain is defined only by vertical deformation.
- The seepage follows Darcy's flow in a horizontal direction, and pore water pressure is the same in a radial direction.
- The water flow at a cross-section interval time from the soil at any depth z to the drain is equal to an increment in the upward discharge of it.
- The compression behavior of drain, smear zone, and undisturbed soil is similar, while the conductivity coefficients of the three are different.
- The water is released to the upper surface of the unit cell while the boundaries of the other unit cells are closed.
- The radial and vertical seepage can be determined separately based on Carrilo's law, whereas the vertical seepage might be calculated according to Terzaghi's one-dimensional consolidation theory.

The physical model of vertical drain based on an elliptical cylinder is illustrated in Figure 4. Notation of ρw is the diameter of the elliptical cylinder drainage, ρs is the diameter of the confocal elliptical cylinder smear zone, ρe is the diameter of the confocal elliptical influence area, u is the excess pore water pressure in the soil, kw is the permeability coefficient of the PVD, and k_s is the permeability coefficient of the smear zone. The single PVD drainage and the elliptical area equivalence estimate appropriate parameters of drainage range.

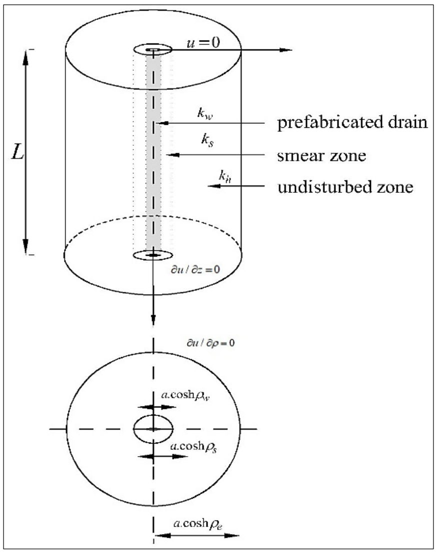


Figure 4. Elliptical equivalent model of PVD

When the PVD is arranged in a triangle pattern:

$$\rho_e = \frac{1}{2} asin\, h\left(\frac{\sqrt{3}}{\pi}\frac{S^2}{a^2}\right) \tag{7}$$

When the PVD is arranged in a square pattern:

$$\rho_e = \frac{1}{2} asin\, h\left(\frac{2}{\pi}\frac{S^2}{a^2}\right) \tag{8}$$

The function *'a sinh'* represents the inverse function of the hyperbolic sine function *'sinh'*. S is space from the center to the center of PVD.

$$a = \frac{1}{2}\sqrt{(1.04b)^2 - (1.22\delta)^2} \tag{9}$$

The formulations for the consolidation of the PVD in elliptic cylindrical coordinates are derived in Equations 10 and 11.

$$\frac{\partial \varepsilon_v}{\partial_t} == m_v \frac{\partial \bar{u}_\rho}{\partial_t} \tag{10}$$

$$\begin{aligned} \frac{k_s}{\gamma_w} &= \frac{1}{a^2(cosh^2\rho - cos^2\theta)}\left(\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \theta^2}\right) = -\frac{\partial \varepsilon_v}{\partial t}; \rho_{w\le}\rho_{\le}\rho_s \\ \frac{k_h}{\gamma_w} &= \frac{1}{a^2(cosh^2\rho - cos^2\theta)}\left(\frac{\partial^2 u}{\partial \rho^2} + \frac{\partial^2 u}{\partial \theta^2}\right) = -\frac{\partial \varepsilon_v}{\partial t}; \rho_{s\le}\rho_{\le}\rho_e \end{aligned} \tag{11}$$

where ε_v is the volumetric strain, u is the radial excess pore water pressure, and $\bar{u}_\rho$ is the average radial excess pore water pressure in the elliptic cylindrical coordinates. These are the basic consolidation equations in the elliptical cylindrical coordinate system. The partial differential equation is obtained by some boundary conditions as Equation 12.

$$\frac{\partial^2 \bar{u}_\rho}{\partial^2 z \partial t} + \lambda \frac{\partial^2 \bar{u}_\rho}{\partial^2 z} + \chi^2 \frac{\partial \bar{u}_\rho}{\partial z} \tag{12}$$

where;

$$\lambda = \frac{2k_h}{m_v \gamma_w F_h a^2 \sinh\rho_e \cosh\rho_e} \tag{12-a}$$

$$\chi^2 = \frac{1}{\gamma_w}\left(\frac{\sinh 2\rho_e}{\sinh 2\rho_w} - 1\right)\frac{2k_h}{F_h a^2 \sinh\rho_e \cosh\rho_e} \tag{12-b}$$

The method of separation of variables is applied to solve by assuming that PVD is ideal drainage to derived simple average 2D consolidation model [14] to reach Equation 13.

$$U_{vr} = U_{(z)} = 1 - exp(-\lambda.t) \tag{13}$$

where:

$$\lambda = \frac{2\pi C_h}{A_e F_n} \tag{13-a}$$

$$A_e = \frac{\pi}{2} a^2 \sinh 2\rho_e \tag{13-b}$$

F_h is average excess pore pressure distribution parameter under elliptical cylinder theory: $\frac{k_h}{k_s} F_{ws} + F_{se}$ and t is time.

3.2. Derive k_{ve} based on Elliptic Cylindrical Model

The equivalence value of the vertical hydraulic conductivity k_{ve} was derived based on the average degree of consolidation under 1D conditions. Carrillo's theoretical solution was used to combine the effects of vertical and radial drainage [20].

$$U_{vr} = 1 - (1 - U_v)(1 - U_h) \tag{14}$$

where; U_{vr} is average degree of consolidation in subsoil with PVD, U_r is average degree of consolidation due to radial drainage, U_v is average degree of consolidation due to vertical drainage.

$$U_v = 1 - exp(-C_d T_v) \tag{15}$$

where; C_d is constant= 3.2 [12].

The U_{vr} was influenced by equivalent vertical permeability coefficient (k_{ev}). The following flow chart (Figure 5) explain how the coefficient was derived.

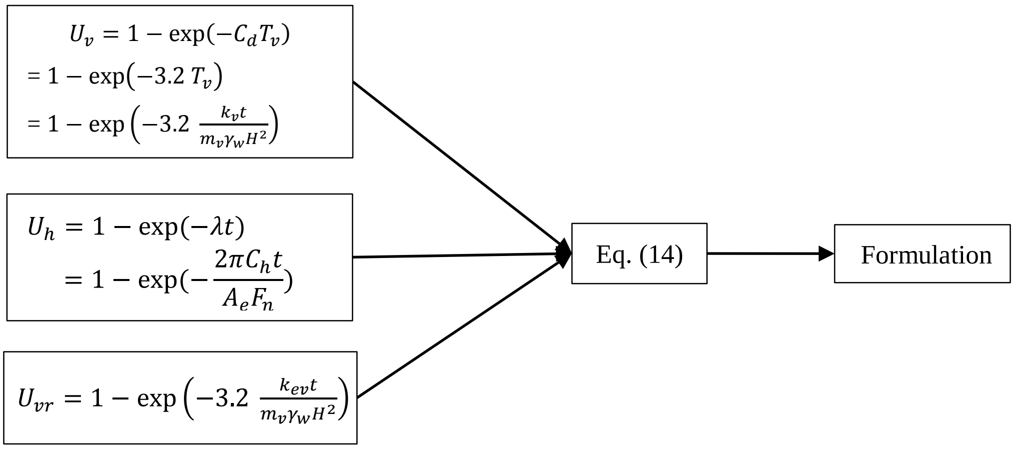


Figure 5. Derivation of equivalent vertical conductivity based on elliptic cylindrical model

The equivalent vertical conductivity can be expressed as

$$k_{ve} = \left(1 + \frac{1.96.k_h.H^2}{A_e.F_h.k_v}\right) kv \tag{16}$$

4. Verification on Formulation

Previous study is recognized have deep soft soil deposits in Gempol-Pasuruan Toll Road, East Java, Indonesia [20]. The deposit consisted of normally consolidated clayed silt in 17 m depth. The index compression was 0.445, natural water content was 50.5%, and organic content was 3.3%. The groundwater level was about 1 m below the ground surface. The subsoil layer under soft clay is clayed silt and gravel from 18.0 m to 24.5 m in depth. The measurement of embankment: breadth = 13.6 m; depth=10.2 m (including a 0.5 m sand mat); slope 1:2. The parameter of soil was in Table 2. The PVDs were installed 17 m in depth with a spacing of 1m in triangle pattern. The area of the improved zone was 37.2 ×17 m. The subsoil below soft soil is porous compare to clayey silt in which two ways of drainage were applied in the PVD-improved zone.

Table 2. Soil layer parameters

Parameter	**Depth (m)**				
	0.0 – 17	**17 – 21.5**	**21.5 - 24.5**	**-**	**-**
Layer	**1**	**2**	**3**	**-**	**-**
Type of soil	**Clayey silt**	**Clayey silt, sand, and gravel**	**Silty Sand**	**Embankment**	**Sand Mat**
Material Model	***Soft Soil***	***Mohr-Coulomb***	***Mohr-Coulomb***	***Hardening Soil***	***Mohr-Coulomb***
Type	**Undrained**	**Drained**	**Drained**	**Drained**	**Drained**
γ_{sat} (kN/m^3)	15.0	18.9	17.3	11.238	21.2
γ_{dry} (kN/m^3)	8.4	14.3	11.4	9.241	16.269
Cohesion (kN/m^2)	35	26.5	15.7	25	29.42
Shear strength (°)	10.5	17	25	20	14
k_x (m/day)	0.00864	0.00864	8.64	0.0864	8.64
k_y (m/day)	0.00864	0.00864	8.64	0.0864	8.64
C_c	0.445	-	-	-	-
C_s	0.0636	-	-	-	-
e_0	2.1	0.8	1.3	-	-
Poisson Ratio (v)	0.5	0.25	0.3	0.2	0.2
E (kN/m^2)	-	28200	210000	-	-

For the case, the improved area was assumed as two conditions; circular cylindrical and elliptic cylindrical models of vertical drain. The hydraulic conductivity of PVDs and soft soil was determined by vertically equivalent conductivity in Equations 6 and 16 and Table 3.

Table 3. Parameter related to the behavior of PVD to determine k_{ve}

Elliptic cylindrical			Circular cylindrical		
Item	**Symbol**	**Value**	**Item**	**Symbol**	**Value**
Diameter of influence area (m)	ρ_e	3.4	Drain diameter (m)	d_w	0.006618
Diameter of drainage (m)	ρ_w	0.047	Unit cell diameter (m)	D	1.05
Diameter of smear zone (m)	ρ_s	0.096	D/d_w	n	15.8
r_e/r_w	n	15.1	Smear zone diameter (m)	ds	0.287
r_s/r_w	s	1.0	Ratio k_h over k_s in field	$(k_h/k_s)_f$	3
Physical parameter	Fa	1.998	Discharge capacity (m^3/s)	q_w	86.4
Cross sectional area (m^2)	Ae	1.948			
Coefficient	λ	0.0247	Equivalent conductivity (m/s)	k_{ve}	0.00915
Equivalent conductivity (m/s)	k_{ve}	0.0101			

The 2D FEM analysis was executed to determine settlement and pore water pressure due to the consolidation process. The total settlement embankment was 2.02 m based on the elliptic cylindrical model in Figure 6. The result of FEM for the circular cylindrical model exhibited almost a similar value of 2.03 m. The maximum excess pore pressure occurred out of the improved area.

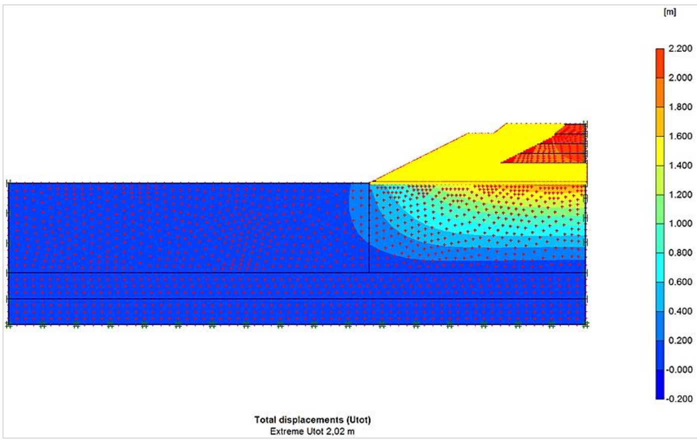


Figure 6. Total displacement in an ellipse for 1 m spacing

Based on the result shown in Figure 7 by comparing the settlement curves of elliptic cylindrical and circularly cylindrical models, the proposed model resulted in a slightly faster consolidation rate and was consistent with the result of excess pore pressure. FEM analysis also resulted pore-pressure dissipation rate at Figure 8 which is insignificant result. The proposed model consumed 167 days to reach a ninety percent degree of consolidation, while the circular model required 174 days.

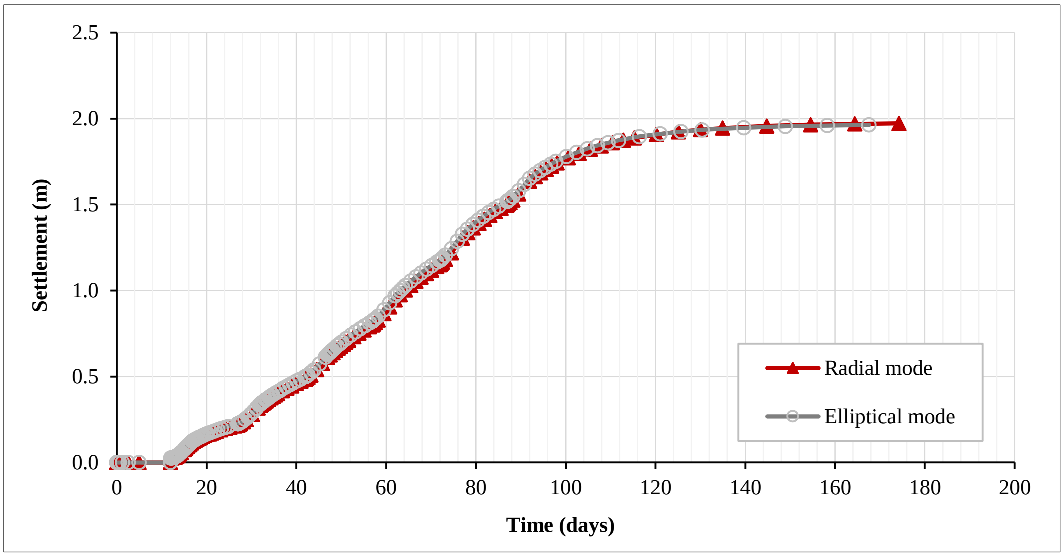


Figure 7. Distance factor on degree of consolidation

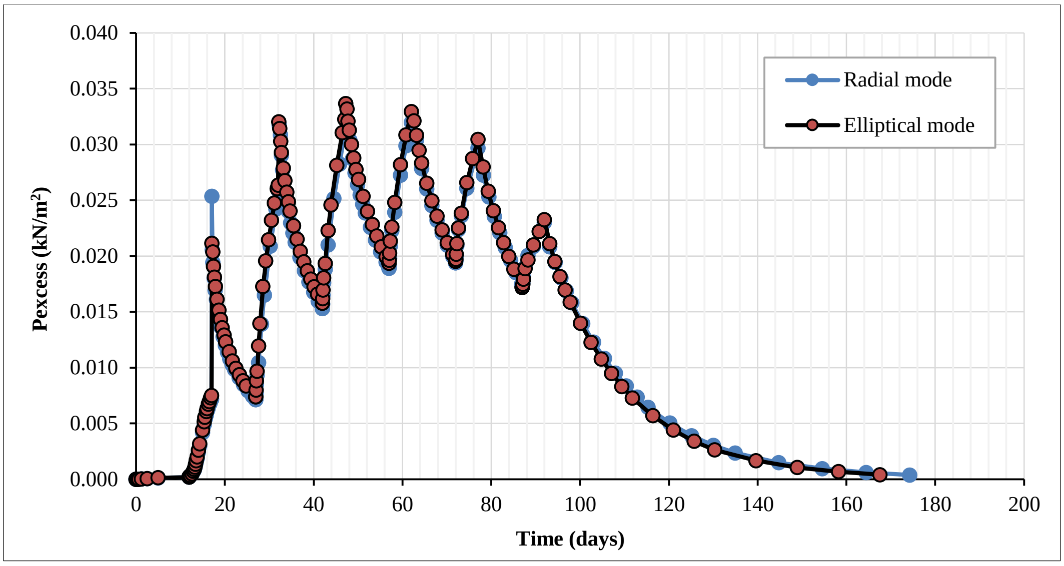


Figure 8. Excess pore pressure variation

5. Conclusion

The degree of horizontal consolidation is expressed in elliptical coordinates, and the degree of vertical consolidation based on the Terzaghi 1D theory and the Carrillo theoretical combination theory derives the formula for the equivalent vertical conductivity coefficient as: $k_{ve} = \left(1 + \frac{1.96.k_h.H^2}{A_e.F_h.k_v}\right) kv$. The analysis applied to the road embankment on soft soil showed that the k_{ve} value for the elliptical cylindrical model, which was 0.0101 cm/sec, was greater than that for the circular cylindrical model, which was 0.00915 cm/sec. The degree of consolidation of 90% was achieved by the elliptical cylindrical model in a shorter time, namely 167 days, but the length of time was not significantly different from that required by the circular cylindrical model to reach the degree of consolidation of 90%, which was 174 days. The circle cylindrical model has been used as a reliable method. Accordingly, the k_{ve} formulation with an elliptical model can also be recommended as a simple approximate method for practical purposes.

6. Declarations

6.1. Author Contributions

Conceptualization, Y.Z., H., and G.R.K.; methodology, Y.Z. and G.R.K.; software, Y.Z. and G.R.K.; validation, Y.Z., G.R.K. and H.; formal analysis, Y.Z., H. and G.R.K.; investigation, Y.Z. and G.R.K.; data curation, G.R.K. and H.; writing—original draft preparation, H. and G.R.K.; writing—review and editing, Y.Z. and G.R.K.; visualization, G.R.K.; supervision, Y.Z. All authors have read and agreed to the published version of the manuscript.

6.2. Data Availability Statement

The data presented in this study are available in the article.

6.3. Funding

The research and publication of this paper have been made possible due to financial support by research grant Brawijaya University, Indonesia.

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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