



Robust Open-Source Solution for Bridge Decrement Estimation for Data with Outliers

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Abstract

Dynamic tests enable assessment of the structure's technical condition and provide information necessary for management and maintenance throughout the object's life cycle. On their basis, the dynamic characteristics of the object are estimated (e.g., the logarithmic decrement). The possible occurrence of atypical features in the obtained signal (e.g. amplitude beat, outliers), as well as the influence of the type of devices and sensors used for measurements, should be considered. If these features are omitted during the analysis, key dynamic characteristics may be evaluated incorrectly. Therefore, this study presents development of a reproducible, universal and robust open-source algorithm for effective estimation of the logarithmic decrement of bridge structures as a reproducible research. Using the presented approach, it is possible to obtain correct results regardless of the signal's specificity and its atypical features, as well as the type of devices used to collect data in the in-situ conditions. Two approaches based on the use of advanced regression models are considered to estimate the logarithmic decrement. These are direct non-linear approximation (*DNAP*) and Hilbert non-linear approximation (*HNAP*). The enriched *HNAP* solution was then implemented as a Python module with a "Signal" class and tested on two independent in-situ examples. The presented approach led to effective and correct estimation of the logarithmic decrement, and proved to be insensitive to the type of bridge, its structural characteristics, atypical features of the obtained signal, and the specificity of the data acquisition techniques. In contrast to methods based on deep machine learning, the presented solution does not require a large learning set representative for a given type of design and works independently of the size of the data sample. As demonstrated in the paper, the solution based on the Hilbert transform allows efficient determination of the damping decrement even in the presence of beat frequencies as well as outlier data. The algorithm works independently of the measurement method, with the necessary functions for preprocessing being implemented in the module itself. The solution has been optimized for improved speed, reliability, and reproducibility of results.

Keywords: Structural Vibrations; Bridge Load Tests; Amplitude Beat; Logarithmic Decrement; Hilbert Transform; Robust Methods; Non-Linear Approximation; Python Programming; Reproducible Research.

1. Introduction

The most frequently used dynamic characteristics of engineering structures, including bridges, are the forms and frequencies of natural vibrations as well as vibration damping parameters. While estimation of the natural frequencies is, in many cases, uncomplicated [1], determination of the logarithmic decrement often requires the use of more complex methods. The results of dynamic tests of structures allow us to assess their condition and often provide critical information for their management and maintenance throughout their life cycle.

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In-situ dynamic load testing and short-term monitoring under service load is usually conducted with the use of temporarily installed, dedicated sensors [2, 3]. On the other hand, systems permanently installed (embedded) on the structure as well as external measuring devices [4] are used in the case of long-term monitoring. The types of measurement techniques and sensors applied in the dynamic bridge tests are presented, e.g., in [5–7]. One of the most popular measurement techniques is the application of microelectro-mechanical (MEMS) accelerometers and inclinometers [6, 8]. Brunetti et al. [9] used the accelerometric measurement results to evaluate the damping characteristics of a railway bridge for Italian high-speed railways. One of the key goals of this study was to evaluate the effect of non-structural secondary elements of the bridge (tracks, handrails, and bearings)-often not included in the design models-on increasing the stiffness and damping response. On this basis, it was also assessed whether the theoretical (specified in the regulations) damping factors have a sufficient safety margin.

Complementary or alternative non-contact data acquisition devices are used. For example, radar interferometry with the use of the IBIS-S® radar was performed in the vibration analysis of a high-speed four-track rail bridge [10]. The results of the measured dynamic response of the bridge under different loading conditions were compared with the results from the Structural Health Monitoring System (SHM) based on the embedded hydrostatic devices. Similar measurement results were obtained with both techniques in relation to all the six analyzed bridge load cases. The main difference between the leveling and radar measurements was the sampling frequency; the high sampling frequency of radar measurements made it possible to determine the maximum displacements caused by the live load.

Vibration measurements can also be made using sensors natively dedicated to other purposes. ATR (Automatic Target Recognition) systems are an example of a remote sensing device recording changes in the position of a target. Sensors embedded in the robotic total stations (ATR) are designed to support automatic surveying measurements. However, after proper marking of the selected structural element of the bridge, they can track changes in the position of the reflector in the vertical plane, thus recording the vibrations of the bridge [11].

The non-contact techniques in the in-situ measurements were also used in the study of a residential building damaged during an earthquake [12]. The dynamic properties of the structure were estimated by measuring displacements using accelerometers, and then analyzing the history of displacements in time recorded using radar interferometry. Alva et al. (2020) [12] show high potential for the use of radar interferometry in the diagnosis of the condition of structures located in seismic areas. The main advantage of this technique is that there is no need to approach or enter the structure to directly install the measuring devices.

Regardless of the application potential of individual vibration measurement techniques, it is worthwhile knowing the dynamic features specific to a given type of engineering structure. A common problem in bridge structures is the amplitude beat phenomenon [13, 14]. It occurs when two or more structural elements have similar, but not identical, vibration frequencies. This phenomenon leads to an impairment of the durability of the bridge due to the faster wear of its structural elements and, consequently, even to a potential failure. This issue is directly related to the problem of correct estimation of the logarithmic decrement, which is the key characteristic used in the assessment of the dynamic response of bridges. Nakutis & Kaškonas (2011) [15] show that in the case of close spectral components in the vibration data, the selection of the optimal filter parameters is difficult, and the selection of the non-optimal filter parameters may have a significant impact on errors in the estimation of the damping decrement. A modified method of estimating the damping decrement was proposed, which-in relation to the presented data-does not require bandpass filtering and allows for amplitude damping to be taken into account. The level of errors in the estimation of the logarithmic decrement estimated with the use of the proposed, modified method is comparable to the values given in the publications on the subject of vibrations of bridge structures.

It must be, however, emphasized that Nakutis & Kaškonas (2011) [15] indicate that the proposed methodology has not been tested in relation to the signals with a phase shift, in the case of greater diversification in the amplitude components, or in the presence of additive noise. Moreover, there may be more atypical features of the signal. Those features are especially important in the vibrations of road bridges, e.g. during mandatory load testing allowing certain bridge structures for service [16–19]. These are, for example, the following features:

- Existence of several (and not only two) main components of vibrations [14, 20];
- Irregularities in the sampling frequency - e.g. in case of measurements with robotic total stations;
- High damping (damping closer to the critical) - leading to severe limitations in the observation of several amplitude beat cycles;
- Existence of a large number of outliers in a not very numerous time series (observations with high leverage in the regression model - e.g. interferometric data [21]);
- Existence of an erroneous additive constant - e.g. for amplitudes determined by numerical integration from acceleration measurements made with accelerometers;
- Larger standard deviations for one of the directions of vibration, e.g. zenith direction for global navigation satellite system (GNSS) receivers.

The problem of analysis of bridges is also complicated by the fact that, depending on the type of structure of the object, its response to static loads and dynamic excitation is different, as well as the nature of the radar profile [22]. Combined with the large number of indicated possible sources of irregularities occurring in the signal, as shown above, the number of combinations of data cases we can deal with during analysis is very large. At the same time, the aspect of time also plays an important role in the diagnosis of bridge structures. Conducting acceptance testing of structures generally takes place just before the structure is placed in service. In the case of diagnostic tests of existing structures, on the other hand, it is crucial to assess the condition of the structure as soon as possible in order to take it out of service before it poses a danger to road users or to restore traffic on the closed structure once the results of the analysis show that there is no danger.

All this results in the necessity of using such methods of measurement data analysis that will not be laborious, time-consuming and expensive. Thus, these methods must be characterized by high accuracy of the results, including being resistant to the previously mentioned factors in order not to show false alarms as to the assessment of the condition of the structure. With literature research and own experience, the authors identified the problem of lack of effective and universal tool for processing data representing free vibrations of engineering objects. The research goal the team set for themselves was to develop algorithms, and consequently a working solution, that has the following characteristics:

- Do not require a large learning dataset typical of the structure,
- The solution is to be universal, i.e. it does not require transfer-learning procedures for different types of structures,
- They will work independently of the signal amplitude, so for a given bridge structure it will be possible to determine the damping decrement during test experiments related to the test load in different load schemes,
- The solution has to enable signal filtering in time domain (free vibration extraction) as well as filtering in the frequency domain,
- The solution is to be resistant to outliers and eliminate their influence on the calculation result automatically,
- The solution is to enable calculation of the damping decrement also for the signals in which the phenomenon of beat frequency occurs,
- The solution is to be resistant to the occurrence of outliers regardless of the statistical distribution of these errors,
- The solution is to have an open source code and be optimized in terms of calculation speed so that it can be used directly during the dynamic load testing and short-term monitoring under service load,
- The solution is to be able to work on computers with limited resources such as Raspberry Pi.

Therefore, this paper concerns the development of a universal and robust open-source procedure for effective estimation of the logarithmic decrement of mechanical vibrating systems, with particular emphasis on bridges—providing correct results regardless of the specificity and atypicality of a given signal recorded in the in-situ conditions with the use of different data acquisition devices and techniques.

The presented approach is distinguished by its high efficiency regardless of the type of bridge structure, and thus allows obtaining correct results for different responses of the structure to dynamic forcing, and thus for different nature of the radar profile. Moreover, the solution is robust to both the anomalies present in the signal and the type of equipment and data acquisition techniques, effectively eliminating the potential problems signaled above.

An important element of the presented approach is that we need a small amount of data - data from a single measurement is sufficient to perform the analysis. This solution does not require calibration based on data from measurements of other objects of the same construction, nor does it require an extensive measurement database (as is the case with machine learning algorithms, for which large data sets are the basis for network training).

Data analysis also does not require data preprocessing or data cleaning, which for many currently available solutions is an indispensable stage of analysis, generating additional time and financial burden. For the presented solution, all necessary operations were implemented within the presented Python module.

The paper is organized in the following way: Section 2 presents the problems of data acquisition equipment issues and the specifics of bridge in-situ vibration signals. It also presents issues related to the adopted linear and nonlinear regression procedures. Section 3 discusses the methodology adopted and the computer implementation in the form of a Python module with a custom signal class developed. Section 4 presents the application of the computer implementation on real measurement data. It considers measurements performed on bridges of varying construction, where data were collected using different techniques and the resulting signal was characterized by different atypical features. Conclusions and plans for further research are presented in Section 5.

2. Problem Description

2.1. Issues Regarding Data Acquisition Devices

As mentioned in the introduction section, the issue of the bridge vibration data analysis can be split into two groups of general topics. The first group of problems concerns the specificity of data obtained from particular types of measuring devices, and related advantages and disadvantages of particular acquisition techniques. Therefore, the authors concisely include the key remarks on this issue in Table 1. It illustrates most of the aspects which were taken into account in the development of the computational implementation presented in this study, so that it could be effectively applied regardless of the data acquisition technique.

Table 1. Common techniques of bridge vibration measurements and their features

Measurement method	Measured quantity	Advantages	Disadvantage	Comments
radar interferometers	line of site or sight disp.	Multiple points can be measured at once with high accuracy. Measurement compatible with [23] 3D data in a relative coordinate system.	Calculating projection displacement needed. Difficulty in interpreting a radargram, e.g. for rail steel truss bridges [23] non-compliance for engineering structures.	Relatively expensive method.
accelerometers	acceleration	No line of sight restrictions. Very high frequency.	Limited accuracy in determining the vibration amplitude. One point at a time.	Many accelerometers can work independently at many points at once, practically in continuous mode.
Global Navigation Satellite System	3D position with time stamp	3D data in the absolute coordinate system accurately identified in time	Limited accuracy. Satellite line of sight needed.	The vertical z coordinate is about 2.5 times less accurate (than x and y)
Total station with Automatic target recognition	vertical angle changes	Data can complement the radar measurement.	Single point measurement only in the vertical plane with a limited sampling rate.	Non-uniform sampling - the LSSA algorithm must be used for spectral analysis.
Potentiometric Displacement Sensors (PDS)	Vertical displacement	High accuracy	Only one direction of measurements	Difficult setup depending on the type of bridge crossing. Rel. inexpensive technique.

The second group of problems regards the specificity of the dynamic response of the bridge under inspection, which is discussed in more detail in the following section.

2.2. Specificity of the In-Situ Bridge Vibrations Signals

Real structures such as road bridges are systems with an infinite number of dynamic degrees of freedom. In numerical modelling, a simplification by means of discretization of continuous mass distributions into the system of equivalent concentrated masses is used. As a result, usually a complex system of matrix equations is obtained [24]. However, additional simplification can be made in the vibration analysis of a specific measurement point on the object (e.g. in the mid-span cross section of the bridge). The analogy to a physical system with one dynamic degree of freedom – in this case to a classical damped oscillator - is often used. The justification for this approach is presented in e.g. [18].

Finding a function representing such a simplified physical system requires finding solutions to the following equation:

$$\frac{d^2x}{dt^2} + \frac{1}{\tau} \frac{dx}{dt} + \omega_0^2 x = 0 \quad (1)$$

Here, x is vertical displacement in [mm], t is time [s], ω_0 is angular frequency [rad/s]. Solutions of (Equation 1) are sought by finding function parameters in the form:

$$x = Ae^{-\beta t} \cos \omega t \quad (2)$$

Here, A is amplitude [mm], β is damping factor. Consequently, it is possible to determine the logarithmic decrement since its value can be determined as follows:

$$\Lambda = \beta T \quad (3)$$

Here, T is period [s]. Equations 1 and 2 represent a theoretical problem of a classical damped oscillator. They are a composite of a periodic function and an exponential function (Figure 1-a) [25, 26]. Calculation of the frequency spectrum is based on the component of the periodic function, while calculation of the damping parameters, including the logarithmic decrement, is obtained from the envelope analysis.

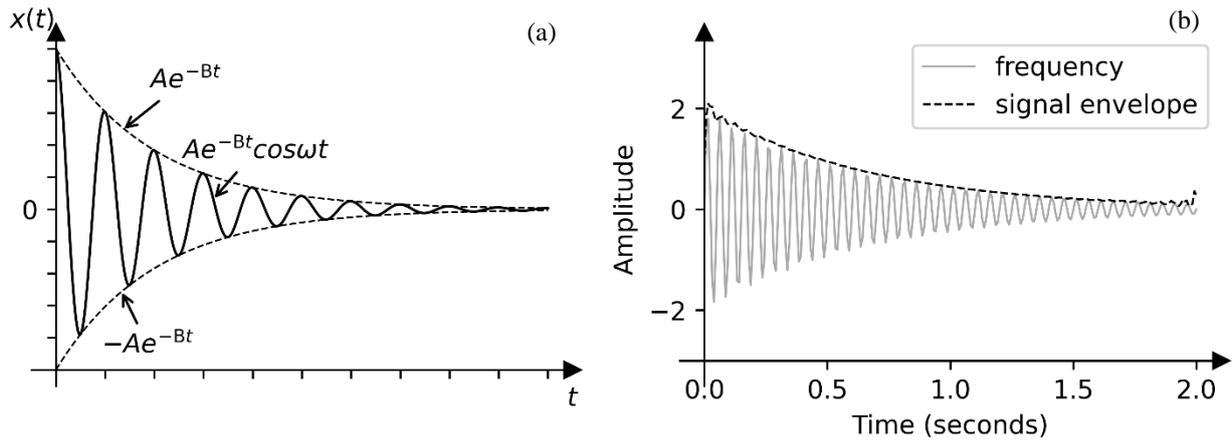


Figure 1. Theoretical model of a damped oscillator; b) Filtered signal from the in-situ acquisition and its Hilbert transform [26]

Contrary to the basic theoretical model, the signal from the in-situ measurements is a combination of many components together with measurement noise. It should also be borne in mind that at the stage of data acquisition, the damping function is not explicitly represented in the obtained data (Figure 1-b).

Thus, there are two basic ways to correctly estimate the bridge’s logarithmic decrement basing on the in-situ data. In the first approach, hereinafter referred to by the authors as direct non-linear approximation (DNAP), the discrete data form the on-site acquisition is approximated by a non-linear function consistent with the equation (Equation 2). Based on this, it is possible to calculate the decrement from the parameters of the approximating function. In the second approach, the approximation is applied to the signal’s envelope, obtained with the use of the Hilbert Transform, hereinafter referred to as Hilbert nonlinear approximation (HNAP). Both methods (DNAP and HNAP) are based on the use of advanced regression models.

2.3. Adopted Procedures of Linear and Non-Linear Regression

Regardless which of the described methods of decrement estimation is used (DNAP or HNAP), the procedure involves performing a linear or non-linear regression. The regression technique requires finding the minimum of the loss (cost) function known from the machine learning algorithms. In case of the standard least squares fit solution, the optimal parameters of the regression model are found on the basis of the residual values defined as the difference between the variable value and its estimation from the model. For a two-dimensional problem (a digital discrete signal is processed in the time and amplitude domain) n data points (x_i, y_i) are analyzed, where x_i is the independent variable, and y_i is the dependent variable (an observation or measurement result). As a result:

$$v_i = y_i - f(x_i, a) \tag{4}$$

Here, v_i is residual, x_i is the independent variable, y_i is the dependent variable (observation or measurement result), $f(x_i, a)$ – linear regression model. Looking for a solution which meets the condition:

$$S = \sum_{i=1}^n v_i^2 = \min. \tag{5}$$

Here, S is sum of squared residuals. In general, taking into account the weight matrix, the solution has the form:

$$S = \sum_{i=1}^n p v_i^2 = \min. \tag{6}$$

Here, p is weight matrix. However, the above solution has a fundamental problem - the second derivative of the loss function is constant (Figure 2).

$$\frac{d^2}{d(v^2)} S(v) = p = \text{const} \tag{7}$$

In consequence, all observations (i.e. discrete values of measured displacements) receive the same weight p - independent of the residual values. This excludes the possibility to automatically eliminate the impact of the outliers in the form of the optimal regression model. Moreover, it causes a potential error in a subset of observations which affects the entire variance-covariance matrix of the adjustment model.

For the above reason, the procedure has to adopt another loss function in which the influence of the residual value from a given, individual observation is taken into account in the regression model. Examples of such functions are presented in Figure 3. The first one (Figure 3-a) solves the problem of the outliers in a radical manner, that is, any

observation which has a residual coefficient greater than the adopted criterion is eliminated from the solution. The second loss function (Figure 3-b) is a solution which gradually reduces the influence of the observation as the residual coefficient increases.

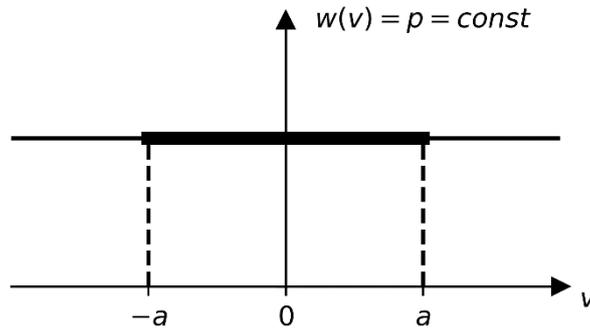


Figure 2. Second derivative of the objective (loss) function assumes the same weight to all observations in the least squares method, regardless of the value of v residuals in the regression model

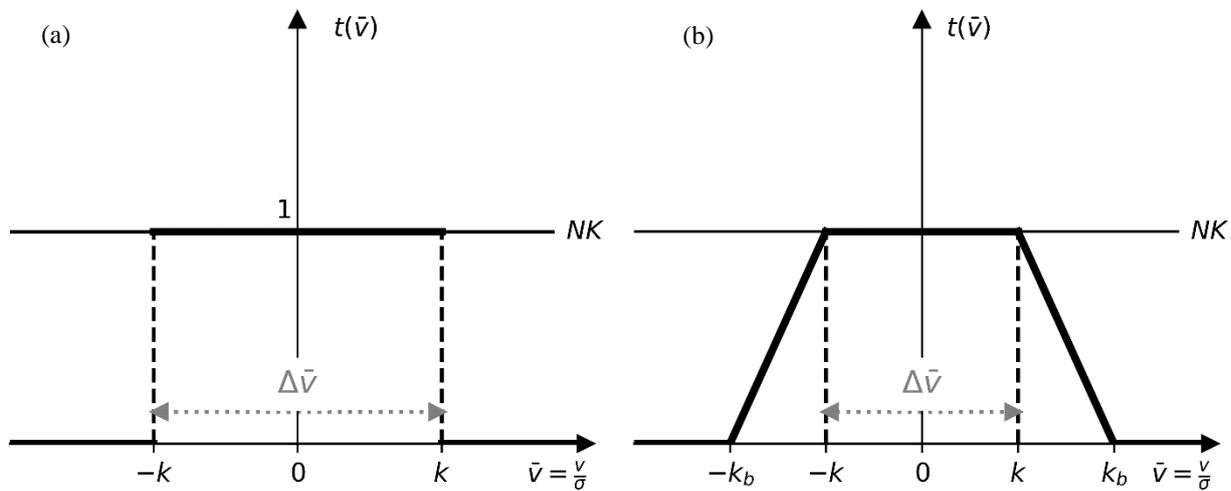


Figure 3. Examples of robust linear regression models

Notwithstanding, the main issue of this study is proper assessment of damping in the vibrations of bridge structures - which is a highly non-linear phenomenon. Thus, in the main solution, the least-squares of a nonlinear problem are assumed and so, the non-linear regression method is applied.

In general, the adequate equation takes the form:

$$JX = K + V \tag{8}$$

Here, J - Jacobian matrix, X - matrix of approximate values of the unknowns of the model, K - linearized equations, V - coefficient vector. In the first step, the algorithm assumes the expansion of the observational equations into Taylor Series in order to obtain the Jacobian coefficient of the linearized equations:

$$J = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_m}{\partial x_1} & \dots & \frac{\partial F_m}{\partial x_n} \end{bmatrix}, x = \begin{bmatrix} dx_1 \\ \vdots \\ dx_n \end{bmatrix} \tag{9}$$

$$K = \begin{bmatrix} L_1 - F_1(x_1, x_2, \dots, x_n) \\ \vdots \\ L_m - F_m(x_1, x_2, \dots, x_n) \end{bmatrix}; V = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \tag{10}$$

With approximate values, the first approximation of the result is made based on the following transformation:

$$X = (J^T J)^{-1} J^T K = N^{-1} J^T K \tag{11}$$

The equation including the weight matrix W takes the form:

$$WJK = WK \tag{12}$$

$$X = (J^T W J)^{-1} J^T W K = N^{-1} J^T W K \tag{13}$$

As a result, corrections to the approximate values of the unknowns are determined (matrix X). After adding the corrections, successive adjustments are made iteratively until the values of the corrections in the n-th iteration are smaller than the adopted convergence criterion. If the matrix of unknowns J is correctly determined, and the approximate values of the unknowns in the first iteration allow for a convergent computational process, the desired result of the parameters of the model is obtained in a finite number of iterations.

2.4. DNAP

In the direct non-linear approximation (DNAP), the discrete data form the on-site acquisition is directly approximated by a non-linear function consistent with the equation (Equation 2). In this case, the entire form of the complex function is estimated in one step, including the part describing the periodic vibration and the part describing the exponential form. It is not an optimal solution for several reasons. Firstly, it is more demanding in terms of determining the initial values of the solution, and since it is an iterative method, it is a mandatory stage. As a result, the algorithm based on this approach is less versatile. Numerous authors' experiences in the proof load-testing of road bridges show that the calculated parameters of the approximating function are not suitable for correct description of the dynamic response of the bridge structure under investigation. This is for example the case during the analysis of several consecutive free vibrations recorded after departure of test-loading vehicles moving previously through the bridge with different speeds (e.g. 10, 30, and 50 km/h). For each experiment, the obtained parameters of DNAP function will be different (which is not desired). Secondly, in this method, an essential damping factor β is obtained with the same accuracy and uncertainty as all the other parameters of equation (Equation 2), which are not as crucial in the discussed problem of the logarithmic decrement estimation. Thirdly, such a solution, although applicable, will be ineffective in case of the presence of amplitude beat in the dynamic response of a given bridge. The discussed situation is not particularly unique and occurs often, for example, in analyses of the suspension bridges. Additionally, the DNAP procedure can be sensitive to the presence of atypical features of the acquired signal such as existence of outliers, irregularities in the sampling frequency, high damping, which are often encountered in vibration testing of bridge structures. This drawback is especially visible when non-linear approximation is done with the use of standard least squares approach without implementing robust algorithms. To illustrate the influence of the non-linear regression strategy choice, Figure 4 highlights some of these problems on exemplary simulated data in the authors' dedicated public Python module [27].

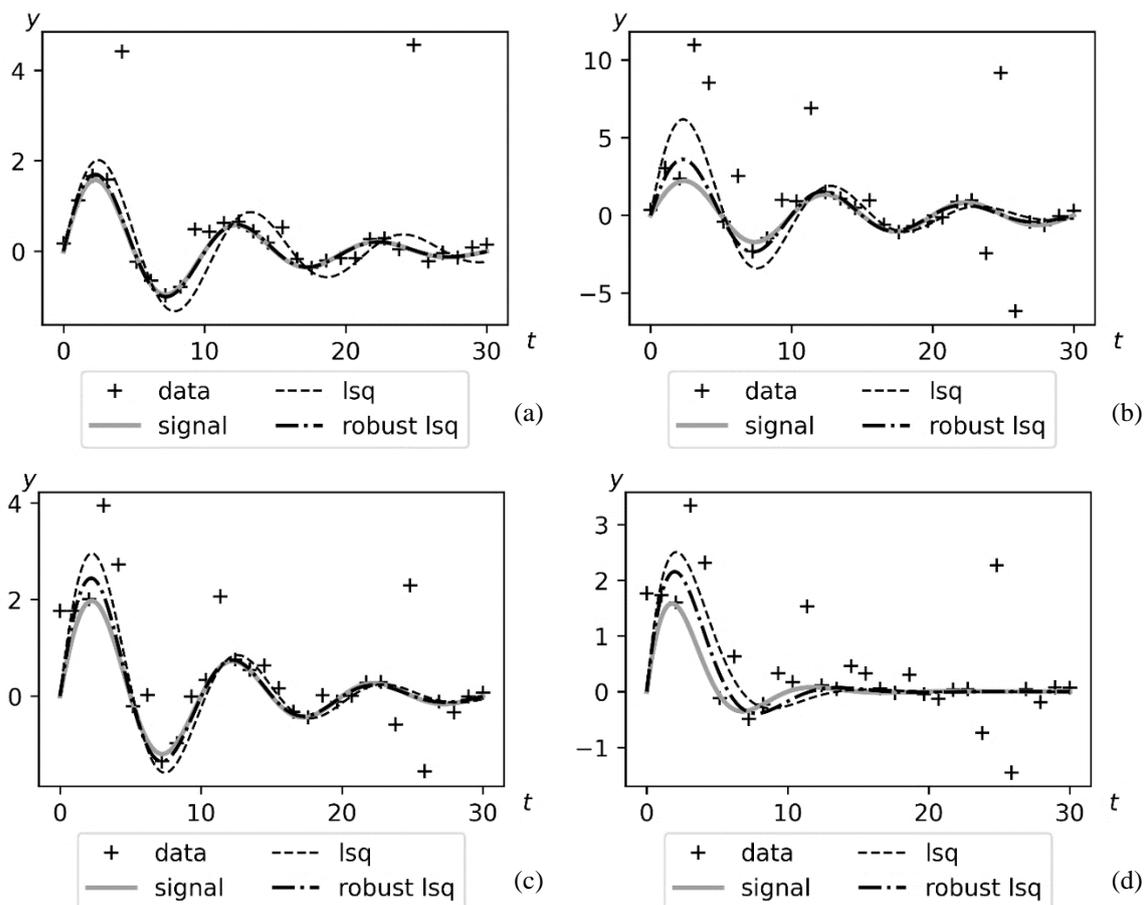


Figure 4. Examples showing the sensitivity of the DNAP approach and the impact of using appropriate nonlinear regression strategy (simulated signal)

Due to the above mentioned and graphically highlighted remarks (Figure 4), in the authors' final computational implementation, HNAP strategy was chosen with adoption of a robust non-liar regression algorithm.

2.5. HNAP

In the HNAP approach, the approximation does not refer directly to the signal from the in-situ measurements, but to its envelope obtained from the Hilbert transform [28]. The form of this approximation function is simpler than in case of DNAP and is consistent with formula (Equation 14):

$$x = Ae^{-\beta t} + c \quad (14)$$

Here, c is additional approximation parameter (equals 0, after translation by mean value of the signal).

Due to the use of the Hilbert transform, it is possible to directly determine the envelope and, on its basis, to determine the value of damping coefficients using nonlinear regression method. Alternatively, linear regression can also be fitted to the resulting data after subjecting them to the natural logarithm operation (approach not used in this study). Basic limitation of the described approaches is high sensitivity of the solution to measurement errors and the values of the envelope at the beginning and at the end of the discrete signal. These values are often disturbed due to the specificity of the digital transformation procedure [28]. However, this problem can be solved by a proper time domain cut-off of the resulting Hilbert envelope in the computer implementation. Figure 5 shows two examples of HNAP application on the simulated signal.

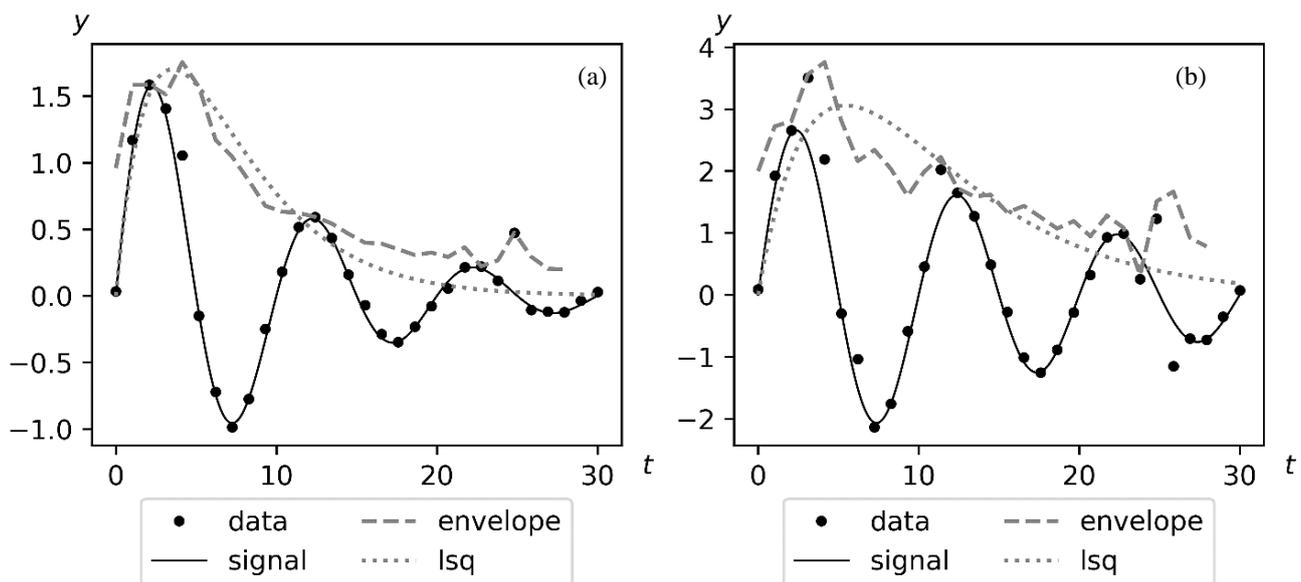


Figure 5. Example of the HNAP application on the simulated signal

3. Methodology and Computational Implementation

To enable the use of the developed computational implementation by other research teams, the solution was implemented as a Python module with a "Signal" class (Figure 6) and made publicly available [27] as reproducible research. The class uses the *NumPy* library and two *SciPy* library modules [26]. The "Signal" class has five initial attributes: the name, the vector of the coefficients of equation (Equation 14), the time vector representing the points in time at which the signal was sampled, the amplitude vector representing the displacement values and the signal envelope representing the Hilbert transform. The automated method of the class's constructor initializes the attributes as *NumPy* arrays (except for the "name" attribute).

The Signal class has the following methods:

- *fun* method which returns a vector of the difference between the observations and the estimated model,
- *load_data* method which takes as an attribute the path to the data file to be analyzed and returns the calculated vector of time and displacements (or accelerations) to a specific object (instance) of the class. computation is performed in such a way as to avoid the influence of computational artifacts at the beginning and end of the signal,
- *compute_sampling_spacing* method, which computes key parameters of sampling features of the input signal from the in-situ device. It returns sampling mean value, sampling median, and sampling standard deviation,

- *compute_period method*, which is responsible for computing period features of the structural response. It returns mean period, median period as well as standard deviation of periods
- *fourier_trans* method responsible for frequency domain representation,
- *lowpass_filter* and *bandpass_filter* methods, which were implemented to allow filtering of the signal with a low-pass and a band-pass filter, respectively,
- *compute_envelope* which is a method providing the results of the Hilbert transform application,
- *lsq_results* method which is responsible for determining the coefficients of equation (Equation 14) (in particular, the damping coefficient β) using the simple least squares method,
- *softL1_results* method which is responsible for determining the coefficients of equation (Equation 14) (in particular, the damping coefficient β) using the robust least squares method with Mean Absolute Error (MAE) weight function,
- *huber_results* which is responsible for determining the coefficients of Equation 14 (in particular, the damping coefficient β) using the robust least squares method with Huber weight function.

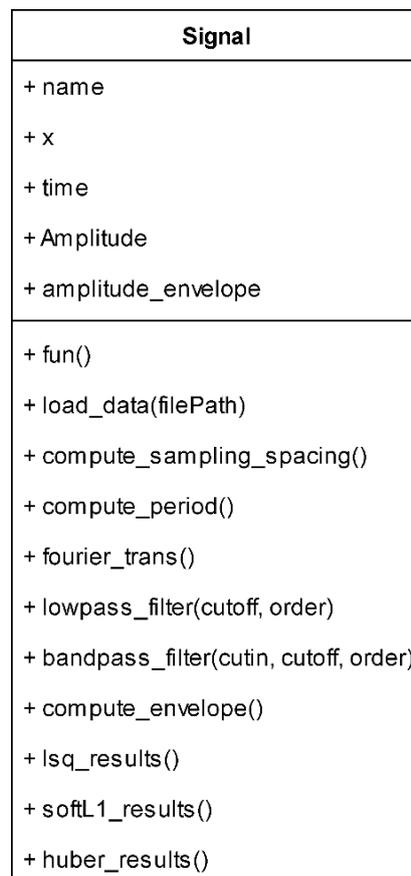


Figure 6. UML *Signal* class diagram

The “Signal” class workflow (Figure 7) starts with pre-processing the data to obtain a subset of time series representing free vibrations of the tested bridge object. The second step is digital filtering. This stage of calculations enables determination of the logarithmic decrement for a specific vibration frequency with the greatest possible reliability and accuracy. Despite the use of bandpass filtering, the signal may not be close to the theoretical representation of a damped oscillator. The reason is possible occurrence of the outliers or amplitude beat. In order to limit the influence of these factors on the final result of the estimation, in the next step the signals envelope is calculated with the use of Hilbert transform. It is known from the mathematical model that it has the form which can be approximated with an exponential function. As mentioned previously, the data may contain errors or measurement artifacts so non-linear regression is performed using the robust methods.

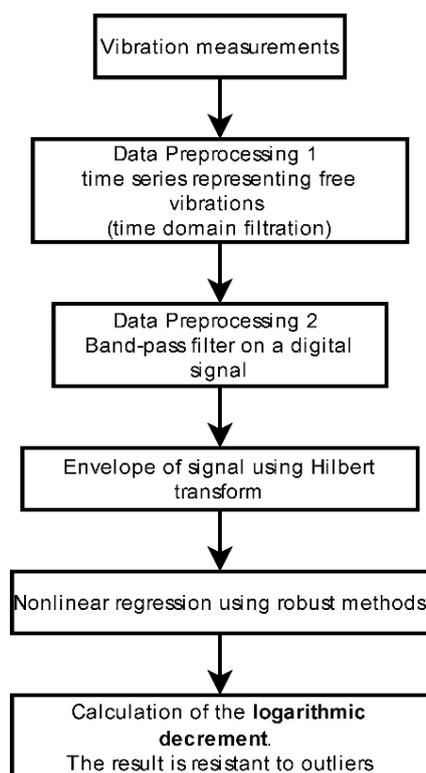


Figure 7. Signal class workflow

The presented, computational algorithm is developed mainly for observations made in accordance with ISO 4866 [23], therefore, a time series including vibration amplitudes represented as a digital signal is processed (discretely sampled data). For this reason, the developed open-source Signal class also includes methods of filtering the signal in the frequency domain. The key difficulties in determining a reliable and accurate value of the logarithmic decrement have been resolved as follows: (I) The problem related to the initial parameters describing the analogy to the damped oscillator and the potential presence of the amplitude beat has been solved by proper usage of the Hilbert Transform; (II) The problem of the observations significantly deviating from the mathematical model (including measurement errors) has been solved with adequate application of robust non-linear regression techniques.

The presented computational solution was validated against two real-life examples. Both are related to the estimation of the bridge damping characteristics during their dynamic load testing. The authors' procedure turned out to be effective in proper estimation of the logarithmic decrement for data obtained from an existing road arch bridge (example 1), which is characterized by the presence of the amplitude beat, as well as a post-tensioned, two span, double girder bridge (example 2) in which the variability of the amplitude and damping values was significant.

Basing on the theoretical analyses and practical in-situ validations presented in this paper, it can be stated that the presented procedure facilitates efficient dynamic analyses of variety of bridge structures and measurement techniques applied for data acquisition. In particular, the procedure turned out to be resistant to such phenomena as amplitude beat, outliers, high damping as well as irregular sampling rate, which are often the main problem of proper estimations and assessments. The research methodology is shown in Figure 8.

4. Application of Computational Implementation to Real In-Situ Bridge Vibration Data

Due to the variability of the amplitude value in regard to free vibrations of bridge structures, determination of the logarithmic decrement, directly by definition, is often burdened with a very large standard deviation. The presented solution minimizes the impact of three basic difficulties: above mentioned variability of the amplitude caused by the measurement inaccuracy of the given data acquisition device, variability caused by the amplitude beat, existence of the outliers at the beginning and end of the signal resulting from the change in the representation of an analogue mechanical signal into a discrete, digital one.

To highlight the application potential of the presented computational solution two real-life, in-situ bridge examples were analysed. In both, the main goal was to estimate the damping characteristics of free vibrations after departure of dynamic test loading. To prove that the proposed method is versatile and robust, the data acquisition in the chosen examples was performed with different measurement techniques. Furthermore, the amplitude beat and other abovementioned features were also present in the acquired data.

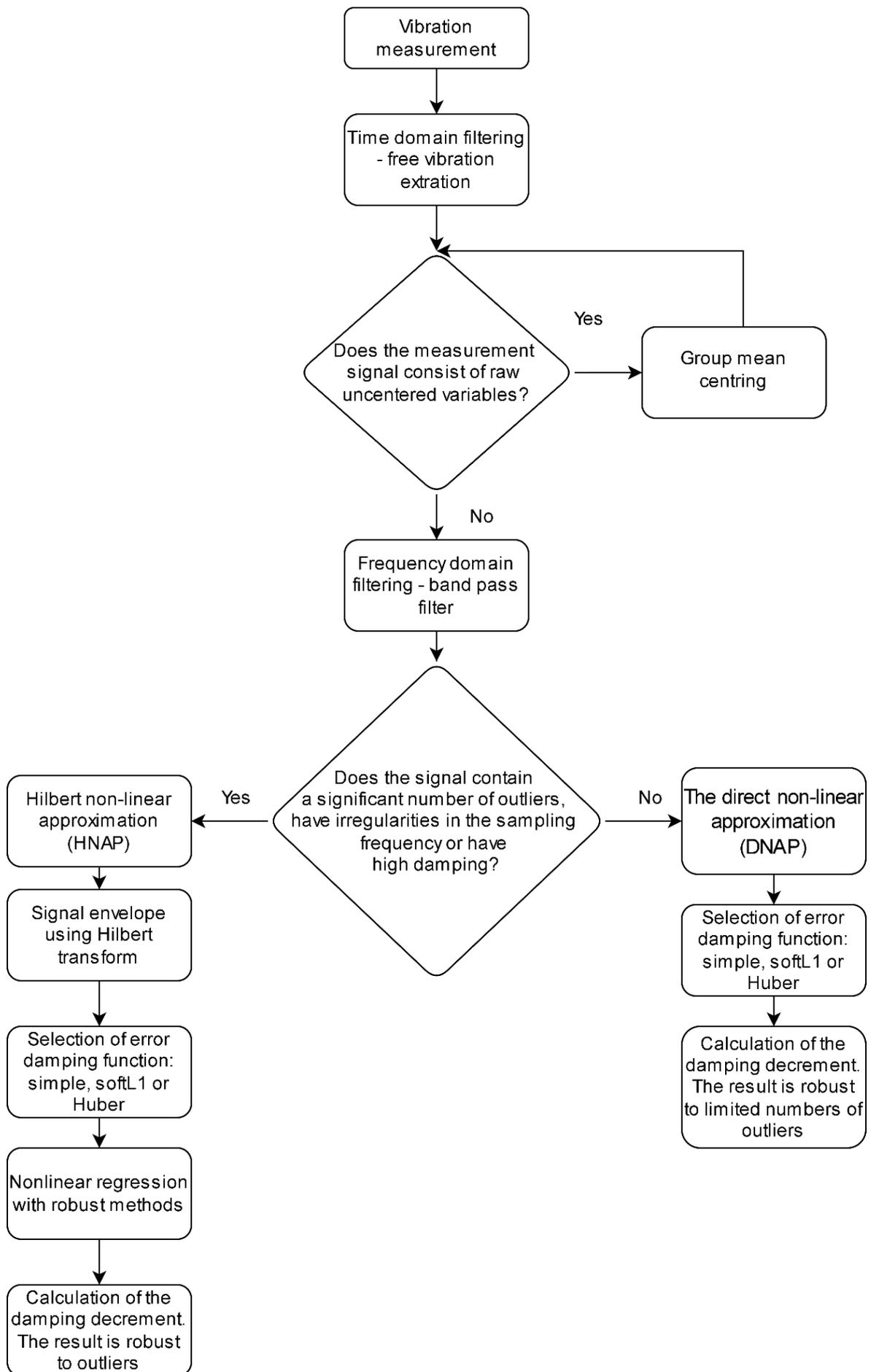


Figure 8. Flowchart of the research methodology

First example concerns analyses of a road arch bridge spanning over 60 m. It consists of two steel arch girders with box cross sections and reinforced concrete deck. Data acquisition was performed with the use of radar interferometry technique. Figure 9 presents the final results of implementation of the presented Python computational solution. As can be noted - despite many difficult features in the acquired data - combination of proper time and amplitude filtration with conjunction of the Hilbert transform and non-linear robust approximation, leads to proper estimation of free vibrations damping characteristic with satisfying level of regression characteristics (available for reader in detailed manner in [27]).

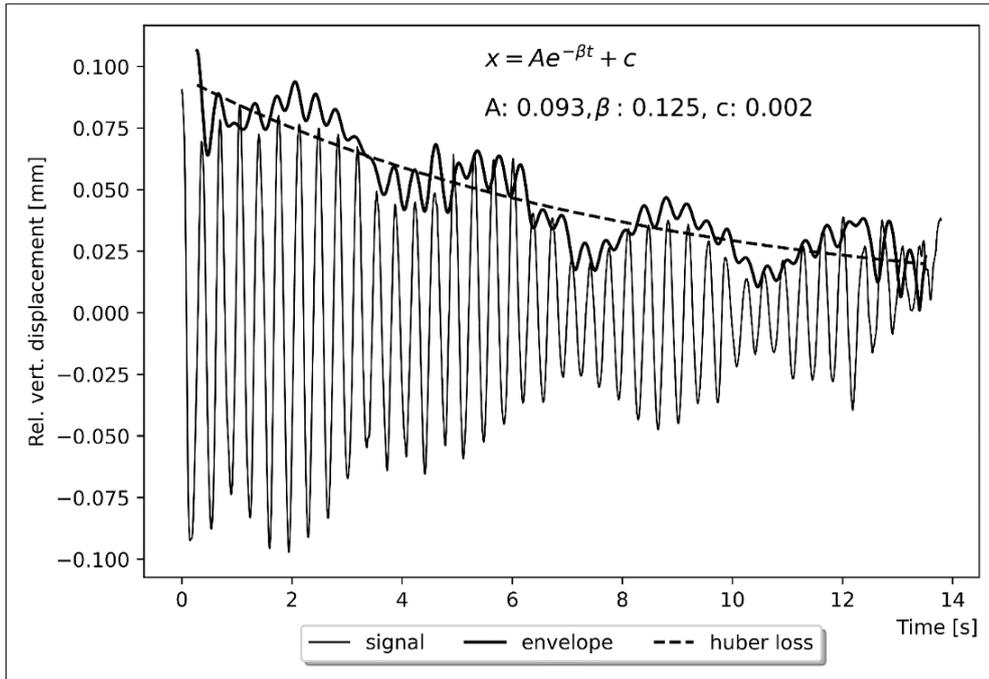


Figure 9. Exemplary application of the proposed solution regarding free vibrations of a road arch bridge

Second example regards a post-tensioned, two span, double girder bridge. In contrary to the previous example, the data was acquired by means of Potentiometric Displacement Sensors (PDS), the damping coefficient of the structure was high, thus the amplitude values were decreasing rapidly (Figure 10). Despite different (almost opposite) characteristics of the dynamic response and different measurement technique, the proposed strategy of signal analysis again proved to be a correct and robust in obtaining desired damping information (example 2 in [27]).

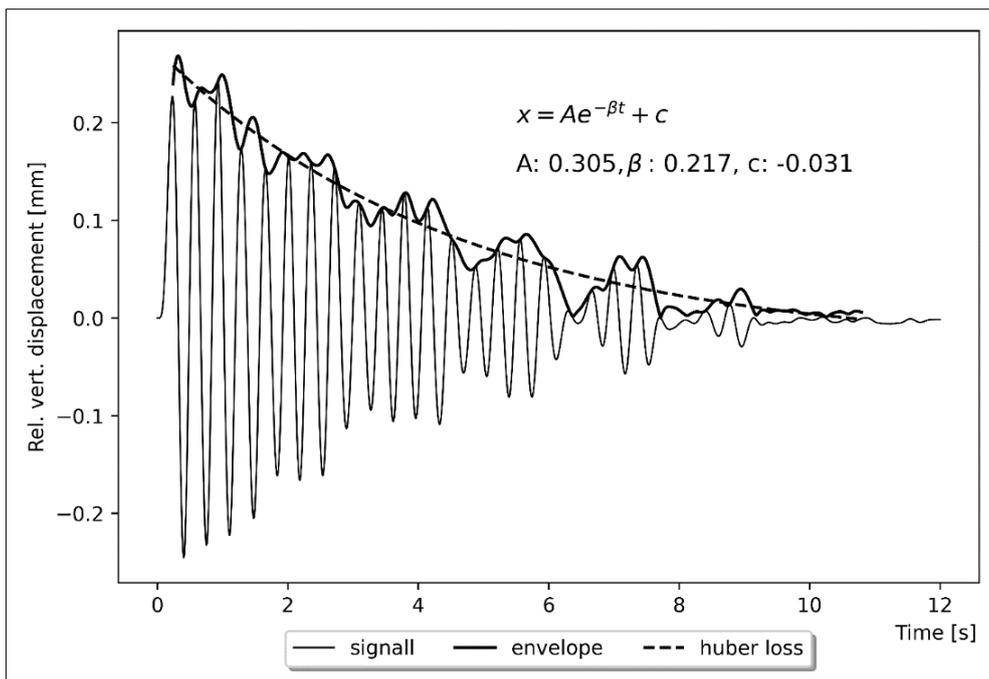


Figure 10. Results of the implementation of the proposed computational procedure regarding a double-girder, post-tensioned road bridge

5. Discussion

In-situ bridge vibration data can be difficult to analyze, especially in the aspect of proper and robust logarithmic decrement estimation. This feature is identified as crucial in the assessment of bridge dynamic response e.g. during dynamic proof load testing. Thus, the key goal of this study was to develop a reliable, open-source reproducible computational solution, which would lead to correct estimation of this feature, regardless of possible existence of the outliers, amplitude beat, atypical, digital signal features or data acquisition technique.

On the basis of the conducted analyses (in total, about 4000 sets of measurement signals were analyzed; they are available to reviewers together with the code repository [27]), the following features of the presented solutions were found.

The approximation of the measurement signal by a nonlinear function proposed in the DNAP solution is characterized, in principle, by limited robustness to the number and size of outliers (Figure 4). This method will be suitable if the signal is characterized by the following attributes:

- The number of outlier observations is limited;
- The signal, apart from the limited number of outliers, is not characterized by other attributes that make the analysis difficult, e.g. there is no beat frequency or frequency drift;
- The signal is sampled continuously, i.e. the standard deviation of the sampling frequency is small;
- The signal does not have the character of critical damping.

Using the method based on nonlinear approximation, we accept the fact that all parameters of the function are determined simultaneously with equal weight, even though the attenuation coefficient is the most important for us (e.g. in contrast to the phase shift of the whole signal). In this method, it is not possible to remove a systematic factor for the damping factor (it can be only achieved indirectly by Group Mean Centering at the first stage of data preprocessing).

Effectively, this method correctly filters only rare outliers in the data set (Figure 4-a). Dense distribution of outliers at the beginning of the data set (where the vibration amplitude is large) or their uniform distribution in the whole range of values in dataset results in less accurate calculations of damping – Figures 4-b and 4-c. In particular, the method will perform poorly for reinforced concrete bridges where the probability of critical damping is high – Figure 4-d.

In the second method, HNAP, the outlier data problem is significantly reduced. An example how algorithm works for two types of structures is shown in Figures 9 and 10. It is worth to point out that even the systematic error present in the signal (frequency rumble) does not affect the estimation of the damping factor. The authors point out that the quality of the results can be further improved in this method by applying more precise bandpass filtering criteria on the input signal, at data preprocessing step 2 – Figure 7.

Moreover, after performing the Hilbert transform, four computational options are available to the user for obtaining the attenuation decrement:

- Nonlinear estimation of the envelope, which is an exponential function using the nonlinear least squares method;
- Nonlinear envelope estimation as an exponential function using robust nonlinear least squares method - examples in Figures 9 and 10;
- Linear envelope estimation with linear least squares method - requires calculation of logarithm from Hilbert transform data;
- Envelope estimation.

As a result, the developed solution does not assume the use of large learning sets, is resistant to measurement errors of different statistical distribution and eliminates systematic errors in the measurement set. As can be seen in source code algorithm have been optimized for fast calculations but can be applied to SHM base on IoT architectures.

The problems presented in this paper are an extension of research conducted by the authors so far. The problem of data acquisition with the use of non-contact measurement technologies characterized by the occurrence of atypical features in the signal obtained was presented in Kohut et al. (2012) study [29]. Owerko (2013) [30] presents the results of estimating the damping parameters with two independent algorithms. The analyses were based on data acquired with a radar system during dynamic loading of the bridge. A characteristic feature of the analyzed measurement data was the recording of strong measurement noise. The analyses conducted showed that regression for a large amount of data is characterized by a large scatter of estimated values. These analyses resulted in conclusions regarding the amount of data necessary to determine the complete form of vibration in real conditions as an indication in planning subsequent tests. The influence of the type of structure on the nature of measurement data obtained by ground-based radar interferometry was discussed in Owerko (2014) study [22]. On the other hand, analyses presented in [31] have shown that damage detection based on DSF is not sufficient for bridge structures and requires the use of complementary algorithms.

Also the results of tests carried out on various types of bridge structures, presented in the literature review, indicate that the data post-processing leading to the logarithmic decrement estimation should be performed with the use of robust regression methods. Moreover, the algorithm should not depend on the initial values of the regression model. It is particularly important in the examined issue, because for different types of bridge structures the ranges of the occurring vibration amplitudes, dominant frequencies and damping parameters can vary significantly.

As a result of the authors' experience with previous analyses, as well as a review of available methods, and in particular their limitations, the development of a reproducible, versatile, and robust open-source algorithm for efficient estimation of the logarithmic decrement of bridges was undertaken. This solution addresses current challenges in the analysis of measured data of structures under dynamic loading.

An alternative approach to the presented algorithm, and a current trend in research, is the use of machine learning to analyze signals recorded by various measurement methods (including detection of atypical signal features). In principle, the results of the computations performed so far are characterized by high efficiency. However, as Zhang & Lei (2021) [32] points out for anomaly types such as outlier data, there is still a possibility to improve the recognition accuracy. This indicates that the sensitivity of deep learning is not always sufficient for the analysis of measured signals.

The problem of classifying time series based on trained models is that these models are typically trained on sets that include data with well-defined features. Attempting to use the trained network to evaluate a problem based on data with other features (e.g., for a different type of structure, other atypical features present in the data) may not provide satisfactory results because the model was designed for completely different data. Hence, each occurrence of data with different characteristics makes it necessary to update the model or create from scratch a model dedicated to a specific case [33], which does not make this approach universal and robust.

The labor- and time-intensive nature of data preparation and the need for appropriate computer equipment to train the network are also important considerations. Due to the problem of the number of factors affecting the quality of the data (e.g. type of construction, anomalies, data acquisition conditions, data acquisition equipment), repeatedly pointed out in this paper, it is necessary to involve an expert who will manually label the collected data - for the case of supervised learning. Only such prepared data, divided into training, validation and test data sets can be input data for neural network. Another aspect is the training of neural network itself - selection of appropriate model architecture and training parameters, and refining it to obtain satisfactory results is often a long and expensive process.

An example data preprocessing procedure for computer vision and deep learning-based data anomaly detection is described by Bao et al. (2019) [34]. The resulting survey data was preprocessed, resulting in over 333,000 samples used for anomaly detection, and the network training time alone took 6 hours. On the other hand, Zhang & Lei (2021) [32] use accelerometer data, split into fragments, giving a total of over 28,000 data. In comparison, for data from ground-based interferometric radar measurements used to detect vehicle crossings of a bridge structure, the training subset alone was over 47,000 samples (where the entire collection was divided in a ratio of 70:15:15 into training, validation, and test subsets) [35].

An alternative approach is to use unsupervised learning. Mao et al. (2021) [36] presented results using GANs and autoencoders for anomaly detection. This approach saves time due to the lack of data labeling. Despite the achieved accuracy of 94% or up to 90%, the authors point out that atypical data sometimes showed similar features in the time domain as data without typicality and were treated as typical, which reduced the detection accuracy. An increase in the quality of the monitored data can be guaranteed by combining the proposed method with the ARIMA model. This confirms the time and labor-intensive nature of anomaly detection in the analyzed time series using machine learning.

6. Conclusions

Based on the literature survey and the authors' own experience, the authors identified the problem of the lack of an effective and universal tool for processing data representing free vibrations of engineering structures.

This paper presents the development of a reproducible, universal, and robust open-source algorithm for efficient estimation of the logarithmic decrement of bridge structures as a repeatable test. Two approaches based on the use of advanced regression models were considered to estimate the logarithmic decrement. The presented computational solution was validated on two real examples. Both of these examples concern the estimation of damping characteristics of bridges during their dynamic load tests. The procedure developed by the authors was successful in correctly estimating the logarithmic decrement for data obtained from an existing road arch bridge, which is characterized by the presence of amplitude rumble. The second example analyzed was a two-girder, two-span, prestressed concrete bridge in which the variation in amplitude and damping values is significant.

Based on the theoretical analysis and practical in-situ validation presented in this paper, it can be concluded that the presented procedure facilitates effective dynamic analysis of a variety of bridge structures using different measurement techniques applied for data acquisition. In particular, the procedure proved to be robust to phenomena such as amplitude

beat, outliers, high damping, and irregular frequency rate. These phenomena are often a major problem in correct estimation and evaluation.

The authors plan to further develop the presented computational repository by (I) extension of the `load_data` method to automate handling of the input files coming directly from widely used measuring instruments; (II) automation in regard to time domain filtration to obtain appropriate subsets for damping analysis during structure-free vibrations; and (III) publication of the developed modules and classes in the form of a library, web services, or web application. Subsequently, it will be possible to expand the solution with a child class adapted to the specificity of steel railway bridges and accelerometer data support.

In addition, the authors plan to adapt the solution's source-code in a form which facilitates its use for the recipients with limited computational background (e.g., in the more accessible form of a Jupyter Notebook) and the use of methods allowing for spectral analysis (also for devices sampling unevenly in time). The stability of the frequency response in time (e.g. using a spectrogram analysis) with the attenuation parameters of a given frequency is also under consideration for future steps.

6.1. Strengths and Limitations

Among the main strengths of the presented universal and robust open-source procedure for efficient estimation of the logarithmic decrement of oscillating mechanical systems, with particular emphasis on bridges, one can primarily distinguish the possibility of obtaining correct results regardless of:

- The specificity and typicality of the in-situ recorded signal;
- Selection of the type of equipment and data acquisition techniques used;
- The type of structure of the object, which determines its response to static and dynamic loads and the nature of the radar profile;
- Lack of necessity for data preprocessing (necessary operations are performed within the implemented Python module);
- Work on small data sets;
- Speed of implementation.

The main limitations of the presented approach are related to:

- The necessity of basic knowledge of the Python language and having appropriate software in order to use the open procedure provided in Owerko et al. (2021) [27];
- Lack of automatic handling of input files coming directly from commonly used measuring devices.

7. Declarations

7.1. Author Contributions

Conceptualization, T.O. and P.O.; methodology, T.O, P.O. and K.T.; software, T.O. and P.O.; validation, P.O. and T.O.; formal analysis, P.O. and K.T.; investigation, T.O.; resources, T.O. and P.O.; data curation, K.T.; writing—original draft preparation, K.T.; writing—review and editing, P.O.; visualization, P.O. and K.T.; supervision, P.O.; funding acquisition, T.O. All authors have read and agreed to the published version of the manuscript.

7.2. Data Availability Statement

Publicly available datasets were analyzed in this study. This data can be found here: [27].

7.3. Funding

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7.4. Conflicts of Interest

The authors declare no conflict of interest.

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