A Macroscopic Traffic Model Based on the Safe Velocity at Transitions

Zawar Hussain Khan [1*], Thomas Aaron Gulliver [1], Waheed Imran [2]

[1] Department of Electrical and Computer Engineering, University of Victoria, PO Box 1700, STN CSC, Victoria, BC, Canada.

[2] National Institute of Urban Infrastructure Planning, University of Engineering and Technology, Peshawar, Pakistan.

Abstract

The increasing volume of vehicles on the road has had a significant impact on traffic flow. Congestion in urban areas is now a major concern. To mitigate congestion, an accurate model is required which is based on realistic traffic dynamics. A new traffic model is proposed based on the conservation law of vehicles which considers traffic dynamics at transitions. Traffic alignment to forward conditions is affected by the time and distance between vehicles. Thus, the well-known Lighthill, Whitham, and Richards (LWR) model is modified to account for traffic behavior during alignment. A model for inhomogeneous traffic flow during transitions is proposed which can be used to characterize traffic evolution. The performance of the proposed model is compared with the LWR model using the Greenshields and Underwood target velocity distributions. These models are evaluated using the Godunov technique and numerical stability is guaranteed by considering the Courant, Friedrich, and Lewy (CFL) condition. The results obtained show that the proposed model characterizes the flow more realistically, and thus can provide better insight into traffic behavior for use in controlling congestion and pollution levels, and improving public safety.

Keywords: Macroscopic Traffic Flow; Inhomogeneous Flow; Reaction Distance and Time; Safe Velocity.

1. Introduction

The economic growth of a country is affected by the road infrastructure. Congestion wastes significant time and degrades this growth. The pollution due to the corresponding emissions causes serious health problems and reduces the quality of life. Smart traffic infrastructure is required to mitigate this congestion and address the corresponding issues. Effective traffic management requires realistic traffic characterization to predict infrastructure use [1]. This paper considers the time and distance between vehicles for alignment to forward conditions. When a stimulus is perceived, a driver reacts during the reaction time and then aligns to forward vehicles during the transition time. The reaction distance is covered during the reaction time, whereas the transition distance is covered during the transition time. Safe time includes the transition and reaction times required for safe alignment. This can be considered the minimum time needed to avoid accidents. The safe distance is covered during the safe time and includes the reaction and transition distances. The equilibrium velocity distribution corresponds to a homogeneous traffic flow with no transitions. This distribution depends on the vehicle density as well as driver behavior and road conditions [2].

The safe distance and time as well as the maximum density and velocity affect the transition behavior of traffic. Thus, a traffic flow model based on these parameters can be used to investigate traffic behavior and forecast traffic

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* Corresponding author: khanuvic@gmail.com

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conditions, thus helping to mitigate congestion and reduce pollution. For example, real-time information can be stored in roadside units for communication to nearby vehicles to warn of congestion ahead and lower the potential for accidents. Suggestions can be provided to drivers to adjust their speed and/or take alternate routes.

Velocity v and density ρ determine the macroscopic spatial and temporal traffic evolution. Density is the number of vehicles per unit length and traffic flow $q(\rho)$ is the product of density and velocity. Lighthill and Witham (1995), and Richards (1956) [3, 4] proposed the LWR model which assumes changes in flow are small and traffic alignment is instantaneous [5]. This model ignores inhomogeneous traffic flow behavior. However, an inhomogeneous traffic flow can occur whenever traffic conditions vary between locations on a road [6], and this will affect the velocity and density.

The Payne (1971) model considers vehicle conservation when there is no acceleration and includes driver presumption and relaxation when acceleration does occur. Presumption is driver anticipation to a stimulus ahead while relaxation is traffic alignment to this stimulus [7]. Whitham (2011) independently characterized traffic based on similar assumptions [8], so it is called the Payne-Whitham (PW) model. This model assumes large changes in flow do not occur and traffic variations are smooth [9]. Due to this inadequate characterization, velocity and density evolution can be unrealistic when the changes are large [5, 10]. This can occur in situations such as the sudden application of brakes in anticipation of an accident.

Del Castillo et al. (1994) improved the PW model by considering driver presumption and reaction time for small transitions in velocity and density [2]. A concavity condition was employed which requires a decrease in flow for a large density and vice versa. Further, the traffic flow versus density is assumed to be concave both spatially and temporally, and a decreasing density is needed to make the behavior realistic. Daganzo (1955) showed that the PW model can have negative velocities at abrupt changes in density. This is because traffic will occupy empty spaces which means movement opposite to the direction of flow, resulting in the velocity being negative (which is fluid-like behavior). It has been argued that traffic is anisotropic, i.e. forward traffic conditions contribute to changes in flow and this flow is not influenced by rearward vehicles [9]. Thus, an anisotropic model was developed based on driver interaction which considers both forward and rearward traffic conditions. To smooth variations in the traffic flow, the LWR model employs diffusion (viscosity) terms based on the velocity and density which are $\frac{\partial^2 v}{\partial^2 x}$ and $\frac{\partial^2 \rho}{\partial^2 x}$, respectively. However, these terms can create negative velocities at discontinuities. The Del Castello et al. model may also produce unrealistic results at discontinuities [11]. This is because the anticipation and reaction time in this model are too large.

Markos (1998) [13] countered the arguments in Del Castillo et al. (1994) and Daganzo (1955) [2, 9] and suggested that the average velocity can be employed for vehicles in a macroscopic flow. Differences in velocity are due to microscopic inhomogeneous traffic conditions so changes in velocity can occur anywhere in the flow. Further, macroscopic traffic models are a simplification of microscopic traffic flow which is affected by vehicle sizes, distances between vehicles, and driver behavior, and thus these factors should be considered. In addition, negative flow can be avoided by allowing only positive velocities.

Aw and Rascle (2000) characterized traffic evolution based on the arguments of Daganzo (1955) to overcome the deficiencies of the PW model [14]. Their model consists of two coupled equations. The first characterizes conservation of vehicles and the second determines acceleration. With this model, driver presumption is a monotonically increasing function of density. However, changes in velocity are ignored so the acceleration can be high for a large density. Berg, Mason, and Woods (BMW) [15] proposed a car-following model based on headway. Headway is the distance between vehicles required for alignment and is given by $\zeta = \frac{1}{\rho}$. It is small when the density is large which results in greater interactions between vehicles. The BMW model employs a diffusion (viscosity) term based on acceleration, but it ignores the time and distance required for alignment.

Traffic models should consider driver physiology [16]. Further, the velocity during alignment to forward conditions should be included Khan et al. (2020) [17]. Driver reaction has been considered to provide more realistic behavior than existing models [18, 19]. Further, realistic parameters were used to better characterize traffic [20]. Thus, the LWR model is improved in this paper by including the transition behavior of traffic. The safe distance and safe time are considered based on the anticipated velocity, and the flow is slow with a large safe distance. Further, the traffic density distribution differs according to the safe distance and has a larger variance when this distance is smaller. The change in this distribution during a transition depends on the velocity changes required to achieve a homogeneous flow and maintain the safe distance. Transitions occur because of traffic bottlenecks, ramps, and traffic control lights, and result in an inhomogeneous traffic flow. Conversely, if a transition does not occur, the flow should be homogeneous.

The rest of this paper is organized as follows. Section 2 presents the LWR and proposed traffic models. The Godunov scheme is employed in Section 3 for numerical evaluation of these models. A comparison of the LWR and proposed models is presented in Section 4. Finally, some concluding remarks are given in Section 5.

2. Traffic Flow Models

Traffic flow dynamics should be considered in developing traffic flow models. Figure 1 shows the steps in this development. First, a model framework is defined based on qualitative statements and traffic observations. Second, the related traffic characterization literature and physical laws are used to obtain a model. Then, the performance of the model is evaluated numerically. These results are used to modify the model as required to obtain an acceptable traffic characterization [18]. The LWR and proposed traffic models are presented below.
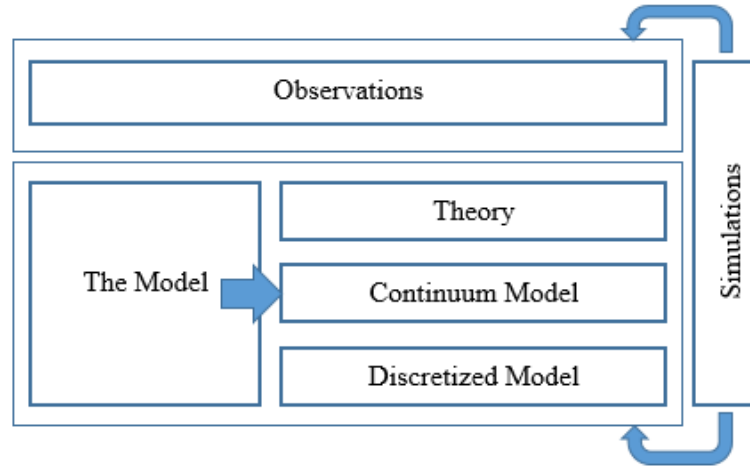

Figure 1. The steps in the development of a traffic flow model

The LWR model is based on the principle of conservation of matter and is given by Lighthill and Witham (1995), and Richards (1956) [3, 4]:

$$(\rho)_t + \big(\rho v(\rho)\big)_x = 0, \tag{1}$$

where ρ is density and $v(\rho)$ is the equilibrium velocity distribution. The subscript t denotes partial derivative with respect to time and x denotes partial derivative with respect to space. This model assumes vehicle conservation on a road so there are no exits or entrances. A smooth traffic density is also presumed. Traffic following the equilibrium velocity distribution results in a homogeneous flow. This distribution is determined by the density distribution which characterizes traffic behaviour on a very long (infinite length), ideal road [21]. An ideal road does not have any disturbances to the traffic flow. The LWR model assumes vehicles adjust their velocity in zero time which is unrealistic. As a consequence, this model cannot be used to evaluate traffic behaviour during transitions. Further, velocity adjustments made during a transition result in an inhomogeneous flow, which is not possible with the LWR model [5].

A new traffic model is proposed which incorporates traffic behaviour during transitions. Traffic adjusts to the equilibrium velocity distribution according to the anticipated change in velocity [27]. This change can be characterized as an acceleration [16] given by:

$$a(\rho) = \frac{v(\rho)^2 - v_a^2}{2d_s}, \tag{2}$$

where v_a is the average velocity at the transition, d_s is the safe distance and t_s is the safe time. $v(\rho)$ is the velocity distribution that traffic adjusts to when a transition occurs. For the LWR model, this distribution [22] can be expressed as:

$$v(\rho) = a(\rho)t_s, \tag{3}$$

and substituting this in (1) gives:

$$(\rho)_t + (\rho a(\rho)t_s)_x = 0. \tag{4}$$

Now substituting (2) in (4) results in:

$$(\rho)_t + \left(\rho \left(\frac{v(\rho)^2 - v_a^2}{2d_s}\right)t_s\right)_x = 0. \tag{5}$$

Several equilibrium velocity distributions have been proposed in the literature [23]. The Greenshields distribution [24] is widely used and is given by:

$$v(\rho) = v_m\left(1 - \frac{\rho}{\rho_m}\right), \tag{6}$$

where ρ_m is the maximum density and v_m is maximum velocity. This indicates that density and velocity are inversely related. The Underwood distribution [25] is also commonly employed and can be expressed as:

$$v(\rho) = v_m \exp\left(\frac{\rho_m}{\rho}\right). \tag{7}$$

This is an exponential velocity distribution based on density. Substituting (6) into (5) gives:

$$(\rho)_t + \left(\rho\left(\left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)^2 - v_a^2\right)\frac{t_s}{2d_s}\right)_x = 0. \tag{8}$$

The safe velocity is $v_s = \frac{d_s}{t_s}$ so (8) can be written as:

$$(\rho)_t + \left(\left(\left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)^2 - v_a^2\right)\frac{\rho}{2v_s}\right)_x = 0, \tag{9}$$

so the traffic flow during a transition is:

$$q(\rho) = \left(\left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)^2 - v_a^2\right)\frac{\rho}{2v_s}. \tag{10}$$

This indicates that vehicles maintaining a large safe distance will have slow transitions and few interactions between vehicles, whereas a small safe distance will result in fast transitions and many interactions between vehicles. If there is no transition, $v_a = 0$ can be assumed so the traffic flow becomes:

$$q(\rho) = \left(\left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)^2\right)\frac{\rho}{2v_s}. \tag{11}$$

Thus, the flow with the equilibrium velocity distribution depends on the safe velocity which is not accounted for in the LWR model. If the safe velocity is reduced by a factor β, then $v_s(\rho) = \frac{v(\rho)}{\beta}$, which gives

$$q(\rho) = \frac{\beta}{2}\left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)\rho, \tag{12}$$

and using (6) results in:

$$q(\rho) = \frac{\beta}{2}\rho v(\rho). \tag{13}$$

Then from (13) and (9) we have:

$$q(\rho) + \frac{\beta}{2}\left(\rho v(\rho)\right)_x = 0, \tag{14}$$

which shows that the traffic flow increases as the safe velocity is decreased. The traffic flow reduces to the LWR model flow with $\beta = 2$ as substituting $v_s(\rho) = \frac{v(\rho)}{2}$ in (11) gives:

$$q(\rho) = \left(v_m\left(1 - \frac{\rho}{\rho_m}\right)\right)\rho, \tag{15}$$

and using (6) results in:

$$q(\rho) = \rho v(\rho). \tag{16}$$

Then combining (15) and (16) with (9) gives the LWR model:

$$(\rho)_t + \left(\rho v(\rho)\right)_x = 0. \tag{17}$$

Unlike the LWR model, the proposed model can account for both homogeneous and inhomogeneous traffic flows.

3. Performance Evaluation

Consider a road divided into N equidistant segments and M equal duration time steps. The total length is x_N so a segment has length $h = x_N/N$, and the total time duration is t_M so a time step is $k = t_M/M$. The average traffic density ρ and flow $q(\rho)$ are evaluated for the nth road segment denoted $x_{n-\frac{h}{2}}$ to $x_{n+\frac{h}{2}}$ over time t_m to t_{m+1} using the technique developed by Godunov [26]. The number of vehicles present in the nth segment at time t is given by:

$$l_n(t) = \int_{x_{n-\frac{h}{2}}}^{x_{n+\frac{h}{2}}} \rho(x,t)dx, \tag{18}$$

so the traffic flow in this segment at time t is:

$$\Delta l_n(t) = q\left(\rho\left(x_{n-\frac{h}{2}},t\right)\right) - q\left(\rho\left(x_{n+\frac{h}{2}},t\right)\right). \tag{19}$$

The traffic flow in the nth segment during the time interval (t_m, t_{m+1}) is then:

$$l_n(t_m + 1) - l_n(t_m) = \int_{t_m}^{t_{m+1}} \Delta l_n(t)dt = \int_{t_m}^{t_{m+1}} q\left(\rho\left(x_{n-\frac{h}{2}},t\right)\right) - q\left(\rho\left(x_{n+\frac{h}{2}},t\right)\right) dt, \tag{20}$$

and using (18), this has the form:

$$\int_{x_{n-\frac{h}{2}}}^{x_{n+\frac{h}{2}}} \rho(x,t_m + 1)dx - \int_{x_{n-\frac{h}{2}}}^{x_{n+\frac{h}{2}}} \rho(x,t_m)dx = \int_{t_m}^{t_{m+1}} q\left(\rho\left(x_{n-\frac{h}{2}},t\right)\right) - q\left(\rho\left(x_{n+\frac{h}{2}},t\right)\right) dt. \tag{21}$$

The average density at time step m for the nth segment is:

$$\rho(n,m) = \frac{1}{h}\int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \rho(x,t_m)dx, \tag{22}$$

and the corresponding flow is:

$$\rho(n,m) = \frac{1}{k}\int_{t_m}^{t_{m+1}} q\left(\rho\left(x_{n-\frac{h}{2}},t\right)\right) dt. \tag{23}$$

Substituting (22) and (23) into (21) gives:

$$\rho(n,m + 1) - \rho(n,m) = \frac{k}{h}\big(q(n,m) - q(n + 1,m)\big), \tag{24}$$

For the LWR model, $q(\rho) = \rho v(\rho)$, and for the proposed model $q(\rho)$ is given by (9). The traffic flow has initial density distribution $\rho_0(x)$ at $t = 0$, and this is used to determine the initial average densities. For the time interval (t_m, t_{m+1}) set:

$$\rho(x,t) = \rho(n,m) \quad \text{for} \quad x_{n-\frac{1}{2}h} < x < x_{n+\frac{1}{2}h}. \tag{25}$$

To account for both increasing and decreasing flows, $q\big(\rho(x,t)\big)$ is approximated as:

$$q\big(\rho(x,t)\big) = \begin{cases} q\left(\min\big(p(n-1,m),\rho(n,m)\big)\right), & \text{if } \rho(n-1,m) \leq \rho(n,m) \\ q\left(\max\big(p(n,m),\rho(n-1,m)\big)\right), & \text{if } \rho(n-1,m) > \rho(n,m). \end{cases} \tag{26}$$

For numerical stability, the Courant-Friedrichs-Lewy (CFL) condition is applied so that the maximum distance traffic covers during a time step is not greater than h so that

$$|q'(\rho)|_{\max} \times k < h, \tag{27}$$

where $|q'(\rho)|_{\max}$ is the maximum rate of change at $t = 0$ given by:

$$\max\left(\frac{q(\Delta\rho)}{\Delta\rho}\right) = \max\left(\frac{q(\rho(n,0)) - q(\rho(n-1,0))}{\Delta\rho}\right). \tag{28}$$

According to the CFL condition, the Courant number γ should be less than 1, so to ensure convergence [26], γ is set to 0.5 which from (27) gives:

$$k = 0.5 \times \frac{h}{|q'(\rho)|_{\text{max}}}. \tag{29}$$

4. Simulation Results

The simulation parameters are summarized in Table I. Traffic is observed over a period of three seconds while traversing a road from −20 m to 200 m. The road begins at −20 m so that the traffic can begin uniformly distributed about 0. The length of road considered is $x_M = 220$ m with $M = 450$ so that $h = 0.489$ m. The maximum velocity is $v_m = 30$ m/s and the maximum normalized density is 0.2, i.e. 20% of the road is occupied. The initial traffic density distribution is $\rho_0(x) = 0.09 \exp\left(\frac{-x^2}{50}\right)$ so that the density is uniformly distributed around 0. This distribution is used for both the LWR and proposed models. The initial density interval is set to $\Delta\rho = 0.0004$ to evaluate $|q'(\rho)|_{\text{max}}$, and this is used in (29) to determine k. Non-periodic boundary conditions are employed so that vehicles can move beyond the 200 m point. The traffic density evolution on the road over time is determined using the Godunov technique presented in Section 3. Two average transition velocities, $v_a = 0$ m/s and 10 m/s, are considered with the equilibrium velocity distributions (6) and (7) and $v_m = 30$ m/s.

Table 1. Simulation Parameters

Name	Parameter	Value
Average transition velocity	v_a	0, 10 m/s
Equilibrium velocity distribution	$v(\rho)$	Greenshields and Underwood
Maximum velocity	v_m	30 m/s
Initial density distribution	$\rho_0(x)$	$0.09 \exp\left(\frac{-x^2}{50}\right)$
Length of road	X	220 m
Number of road steps	M	450
Segment length	h	220/450 = 0.489 m
Safe velocity	v_s	10, 20 m/s
Maximum normalized density	ρ_m	0.2
Initial density interval	$\Delta\rho$	0.0004
LWR model time step with the Greenshields distribution	k	0.0067 s
LWR model time step with the Underwood distribution	k	0.0015 s
Proposed model time step with the Greenshields distribution, $v_a = 10$ m/s, $v_s = 20$ m/s	K	0.0102 s
Proposed model time step with the Greenshields distribution, $v_a = 0$ m/s, $v_s = 20$ m/s	K	0.0091 s
Proposed model time step with the Underwood distribution, $v_a = 10$ m/s, $v_s = 20$ m/s	K	0.000256 s
Proposed model time step with the Underwood distribution, $v_a = 0$ m/s, $v_s = 20$ m/s	k	0.000254 s
Total simulation time	t_M	3 s

Figure 2 shows the traffic density evolution with the LWR model at 0 s, 1.5 s, and 3 s. The target to align to forward conditions is the Greenshields distribution. The initial density is shown in blue. At 1.5 s, traffic spans from 18 m to 64 m while the maximum density is 0.060. At 3 s, the traffic spans from 50 m to 110 m and the maximum density is 0.042.
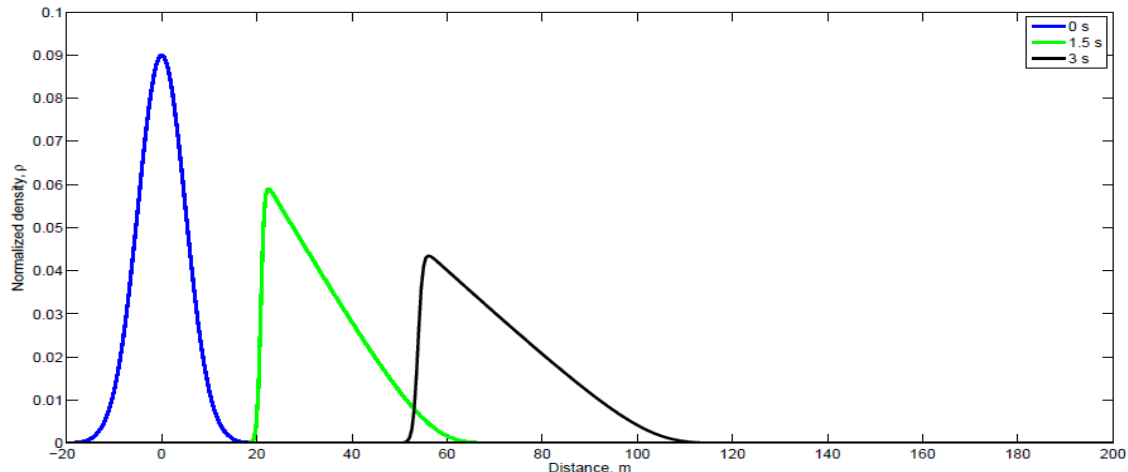

Figure 2. Traffic density evolution with the LWR model when the target equilibrium velocity distribution is the Greenshields distribution

Figures 3 and 4 show the proposed model traffic density evolution for safe velocities $v_a = 20$ m/s and 10 m/s, respectively, with $v_a = 10$ m/s. The safe distances for these velocities are then 20 m and 10 m, respectively. With this model, traffic adjusts from $v_a = 10$ m/s to the Greenshields distribution which is the target equilibrium velocity distribution. The results in these figures show that traffic moves slower with a 20 m/s safe velocity compared to a 10 m/s safe velocity. At 1.5 s, the traffic in Figure 3 spans from 23 m to 83 m, whereas in Figure 4 it spans from 65 m to 155 m. Thus, the traffic density has a greater variance at a lower safe velocity, so this velocity has a significant effect on traffic behaviour. This variance is greater than that with the LWR model shown in Figure 2. The average distance covered is higher at a lower safe velocity as vehicles maintain a smaller safe distance. Figure 5 shows the proposed model density evolution from $v_a = 0$ m/s to the equilibrium velocity distribution with a safe velocity of 20 m/s. There are no significant differences between Figures 3 and 5, which indicates that the average transition velocity has little effect on traffic behaviour.
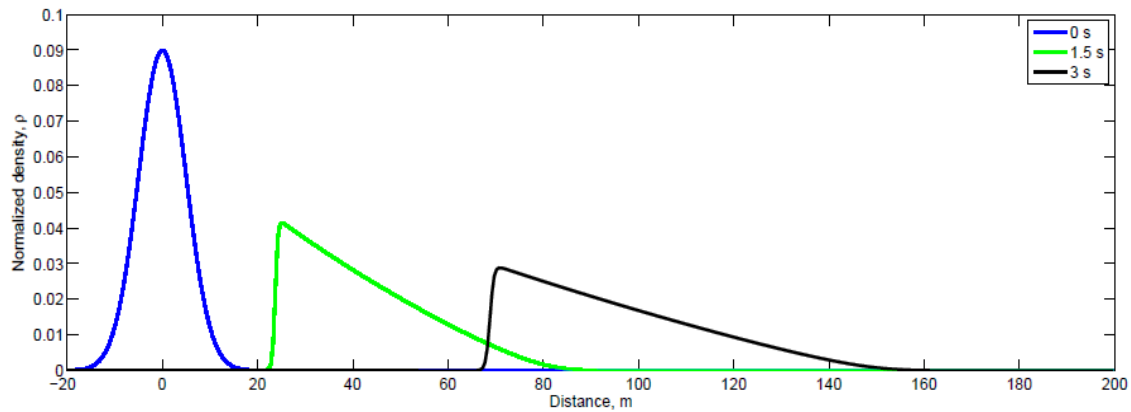

Figure 3. Traffic density evolution with the proposed model for $d_s = 20$ m, $v_s = 20$ m/s, $v_a = 10$ m/s, and $t_s = 1$ s. The target equilibrium velocity distribution is the Greenshields distribution.
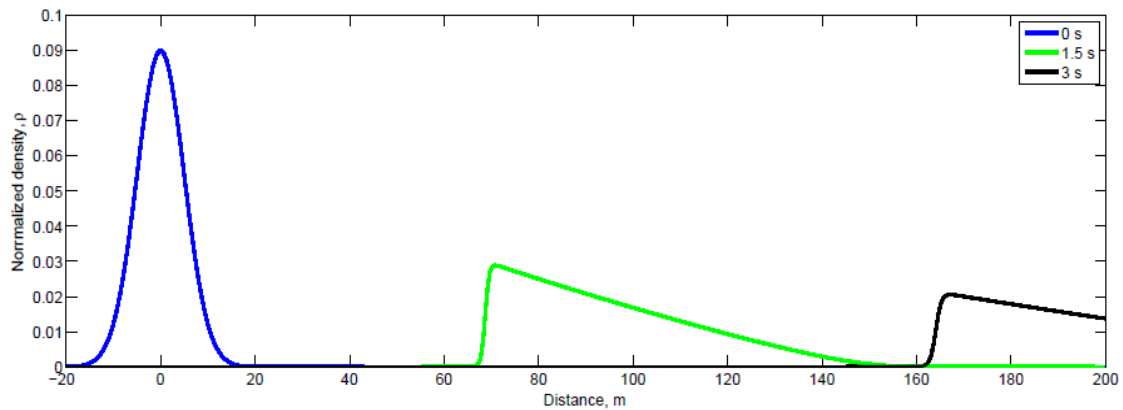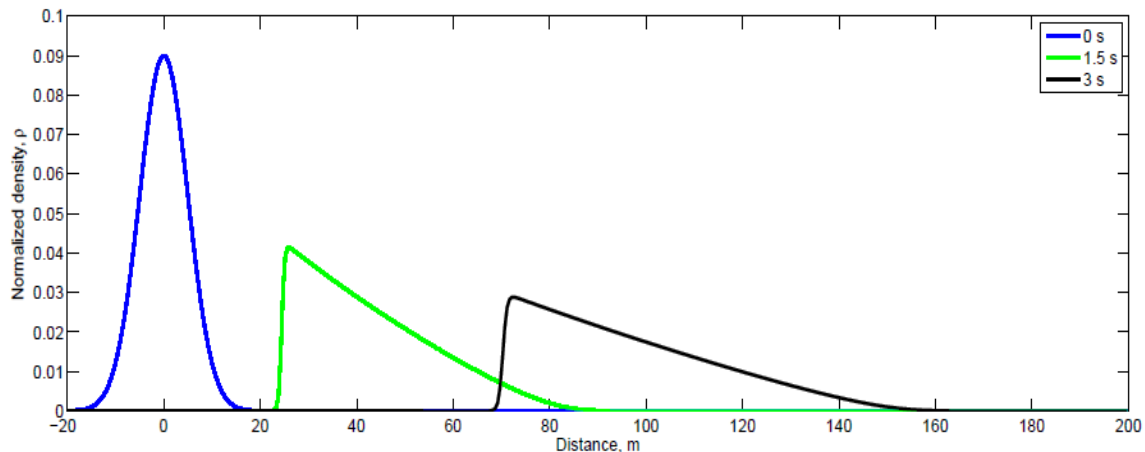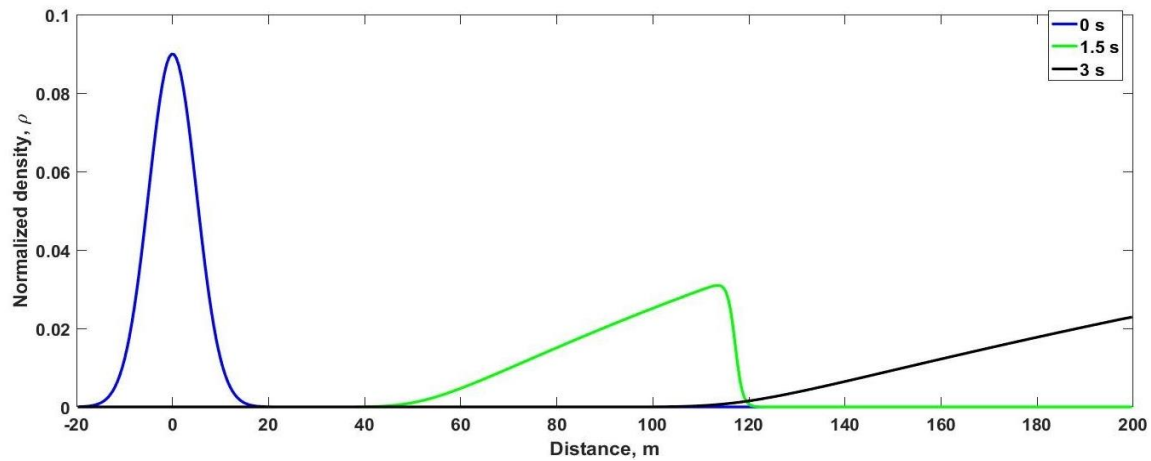

Figure 4. Traffic density evolution with the proposed model for $d_s = 10$ m, $v_s = 10$ m/s, $v_a = 10$ m/s, and $t_s = 1$ s. The target equilibrium velocity distribution is the Greenshields distribution

Figure 5. Traffic density evolution with the proposed model for $d_s = 20$ m, $v_s = 20$ m/s, $v_a = 0$ m/s, and $t_s = 1$ s. The target equilibrium velocity distribution is the Greenshields distribution

Figure 6. Traffic density evolution with the proposed model for $d_s = 20$ m, $v_s = 20$ m/s, $v_a = 0$ m/s, and $t_s = 1$ s. The target equilibrium velocity distribution is the Underwood distribution
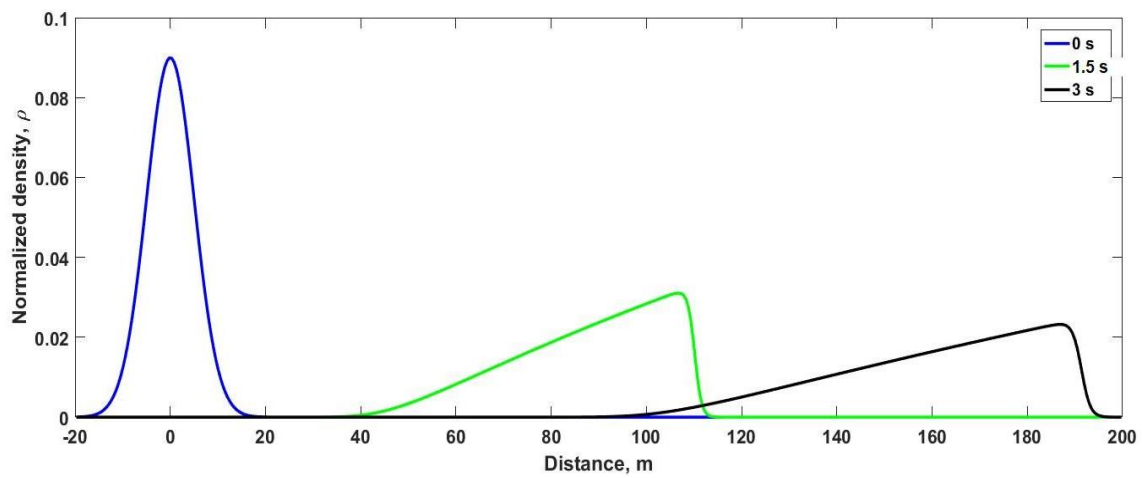

Figure 7. Traffic density evolution with the proposed model for $d_s = 20$ m, $v_s = 20$ m/s, $v_a = 10$ m/s, and $t_s = 1$ s. The target equilibrium velocity distribution is the Underwood distribution

Figures 6 and 7 show the proposed model traffic density evolution for $v_a = 0$ m/s and 10 m/s, respectively, at 0 s, 1.5 s and 3 s. The target equilibrium velocity distribution is the Underwood distribution and the safe time is 1 s. With $v_a = 0$ m/s, no transition occurs, but with $v_a = 10$ m/s, traffic aligns to the forward conditions. At 1.5 s, the traffic in Figure 6 spans from 50 m to 120 m, whereas in Figure 7 it spans from 40 m to 110 m. At 3 s, the traffic in Figure 6 spans from 110 m to beyond 200 m, whereas in Figure 7 it spans from 100 m to 190 m. Thus, the density variance is larger with no transition than with $v_a = 10$ m/s. Figures 3 to 7 show that the traffic evolution with the proposed model and the Greenshields or Underwood target equilibrium velocity distributions is smooth.
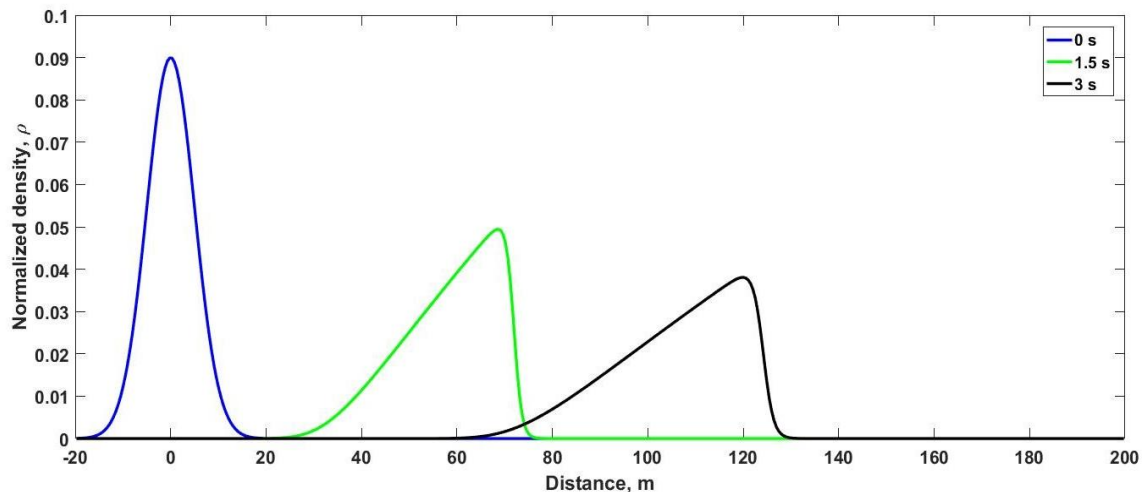

Figure 8. Traffic density evolution with the LWR model when the target equilibrium velocity distribution is the Underwood distribution

Figure 8 shows the traffic density evolution with the LWR model at 0 s, 1.5 s, and 3 s. The target equilibrium velocity distribution is the Underwood distribution. At 1.5 s, the traffic spans from 30 m to 75 m whereas with the Greenshields distribution it spans from 18 m to 64 m as shown in Figure 2. At 3 s, the traffic span with the Underwood distribution is from 70 m to 130 m, whereas with the Greenshields distribution the span is from 50 m to 110 m. Thus, the density variance is greater with the Underwood distribution.

The traffic behaviour of the LWR and proposed models varies greatly as shown in Figures 2 to 8. Figures 3 to 7 show that with the proposed model, the density varies according to the target velocity distribution and safe velocity at transitions. The density with the LWR model only varies according to the target velocity distribution and ignores the conditions at transitions. Thus, the LWR model can only characterize traffic moving with the equilibrium velocity distribution whereas the proposed model also considers the traffic conditions at transitions such as the safe velocity and safe time. These are important parameters and so the proposed model provides a more realistic characterization of traffic behaviour.

5. Conclusion

The LWR model only considers homogeneous traffic flow conditions. Thus, it cannot characterize variations in the flow, in particular transitions when abrupt changes in the density occur. To overcome this drawback, a model was developed which incorporates changes in the velocity during transitions based on the safe time and safe distance. The LWR and proposed models were evaluated under different traffic conditions and with two equilibrium velocity distributions. The results obtained show that the proposed model provides a more realistic characterization of traffic behaviour. Thus, it is a better choice for traffic management to reduce fuel consumption and improve public safety and air quality. The proposed model can be employed in connected vehicles and roadside units to mitigate traffic congestion. It can be extended by including lateral and forward distance headways to improve traffic flow prediction.

6. Declarations

6.1. Author Contributions

6.2. Data Availability Statement

No new data were created or analyzed in this study. Data sharing is not applicable to this article.

6.3. Funding and Acknowledgements

6.4. Conflicts of Interest

The authors declare no conflict of interest.

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