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Numerical Investigation of Stress Block for High Strength Concrete Columns

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Abstract

This paper is intended to investigate the stress block for high strength concrete (HSC) using the finite element model (FEM) and analytical approach. New stress block parameters were proposed for HSC including the stress intensity factor (α_1) and the depth factor (β_1) based on basic equilibrium equations. A (3D) finite element modeling was developed for the columns made of HSC using the comprehensive code ABAQUS. The proposed stress parameters were validated against the experimental data found in the literature and FEM. Thereafter, the proposed stress block for HSC was used to generate interaction diagrams of rectangular and circular columns subjected to compression and uniaxial bending. The effects of the stress block parameters of HSC on the interaction diagrams were demonstrated. The results showed that a good agreement is obtained between the failure loads using the finite element model and the analytical approach using the proposed parameters, as well as the achievement of a close agreement with experimental observation. It is concluded that the use of proposed parameters resulted in a more conservative estimation of the failure load of columns. The effect of the stress depth factor is considered to be minor compared with the effect of the intensity factor.

Keywords: High Strength Concrete; Stress Block; Column; ABAQUS; Finite Element Model.

1. Introduction

A column is one of the most critical members of a framed structure, the failure of which could lead to a catastrophic failure of the whole structure. In recent years, high strength concrete (HSC) columns have been widely used in major construction projects, especially, in high-rise buildings. The advancement in concrete technology and the development of new types of mineral and chemical admixtures have enabled the production of concrete with compressive strength exceeding 150 MPa. HSC could lead to smaller member sizes for compression members and therefore provide considerable savings associated with material costs and a reduction of dead loads. Moreover, due to the superior durability of HSC [1], a considerable reduction of the maintenance efforts and an increase in the service life of the structure can be attained as compared to the normal strength concrete (NSC).

The increasing use of HSC has led to concern over the applicability of current design codes and standards. Although, the latest ACI code provides uniaxial bending interaction curves for up to $f'c$ = 83 MPa, the curves are based on the stress block for normal concrete. Recent research indicates that the behavior of HSC is different from NSC in many aspects. The shape of the stress–strain relation of HSC differs from that of NSC [2].

Equivalent rectangular stress blocks have been proposed for HSC by either different standard codes or researchers.

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The stress block parameters of CSA A23.3 [3], NZS 3101 [4], Ibrahim and MacGregor [5] Bae and Bayrak [6] as published in ACI SP-293-05, or Ozbakkaloglu and Saatcioglu [7], is recommended for HSC as shown in Table 1.

Table 1. Equivalent rectangular stress block parameters for HSC

Standard / Researcher	Stress Block Parameters for HSC, $f'c$ (MPa)
CSA A23.3 [3]	$\alpha_1 = 0.85 - 0.0015\, f'_c \geq 0.67$ $\beta_1 = 0.97 - 0.0025\, f'_c \geq 0.67$
NZS 3101 [4]	$\alpha_1 = 0.85 - 0.004(f'_c - 55)$ $0.85 \geq \alpha_1 \geq 0.75$ $\beta_1 = 0.85 - 0.008(f'_c - 30)$ $0.85 \geq \beta_1 \geq 0.65$
Ibrahim and MacGregor [5]	$\alpha_1 = 0.85 - 0.00125\, f'_c \geq 0.725$ $\beta_1 = 0.95 - 0.0025\, f'_c \geq 0.70$
Bae and Bayrak [6]	$\alpha_1 = 0.85 - 0.004(f'_c - 70)$ $0.85 \geq \alpha_1 \geq 0.67$ $\beta_1 = 0.85 - 0.004(f'_c - 30)$ $0.85 \geq \beta_1 \geq 0.67$
Ozbakkaloglu and Saatcioglu [7]	$\alpha_1 = 0.85 - 0.0014(f'_c - 30)$ $0.85 \geq \alpha_1 \geq 0.72$ $\beta_1 = 0.85 - 0.0020(f'_c - 30)$ $0.85 \geq \alpha_1 \geq 0.67$
Oztekin et al. [8]	$k_1 = -0.0012 f'_c + 0.805$ $k_3 = -0.002 f'_c + 0.964$ $k_1 k_3 = -0.002 f'_c + 0.762$

Investigation of stress block parameters for high strength concrete has been performed by some researchers. ACI code [2], however, suggests stress block parameters for normal concrete only. Many stress-strain relationships for unconfined high strength concrete are found in the literature. Those relationships were developed based on experimental work conducted by other researchers. Several stress-strain relationships are found in the literature, but four of them are selected to adapt stress block for high strength concrete.

Lately, many researchers have suggested a stress block of HSC beams and columns. Al-Kamal [9] proposed a triangular stress block of HSC beams having a concrete strength over of 55 MPa. The proposed stress block was validated using 52 tested singly reinforced HSC beams. It was mentioned that the modified rectangular stress block to find the axial and flexural strength of the HSC column was not likely investigated. Tran et al. [10] developed the parameters of a stress block for geopolymer concrete (GPC) made of fly-ash and slag which, has a compressive strength of concrete up to 66 MPa. Rectangular stress-block were proposed based on previously developed curves of GPC materials and two analytical concrete stress-strain models. Ma et al. [11] carried out theoretical analysis and experimental tests to study the confined columns under compression and bending moment. In the analytical solution, an equivalent stress block parameter was developed for a confined HSC section in which the compression stress distributions were investigated to find the suitable equivalent stress block parameters through using different neutral axis locations. Hasan et al. [12] experimentally investigated the behavior of HSC and steel fibre HSC column specimens. The interaction diagrams for specimens were developed based on the equivalent rectangular stress suggested by CSA A23.3-2014 [13]. Baji and Ronagh [14] carried out statistical study of the concrete rectangular stress block factors. The Monte Carlo Simulation was used to study the influence of improbability of the stress block factors on the flexural strength of beams. It is found that any reliability analysis is very sensitive to the change in the statistical properties of concrete stress block factors. Yang et al. [15] developed a general model of an equivalent stress block that can be used for both light weight concrete and HSC. Peng et al. [16] proposed equivalent rectangular concrete stress block factors to design reinforced concrete elements in flexural by incorporating the effects of the strain gradient. Khadiranaikar and Awati [17] developed an equivalent stress block factor for different concrete strength through the testing of plain concrete columns, RC eccentrically loaded columns, and beams.

In this paper, new parameters for stress blocks were developed based on equilibrium equations to modify and extend the current code design curves of HSC columns. Those new parameters will be used to develop interaction diagrams for columns which were tested experimentally as found in the literature. The interaction diagrams were validated against experimental data from literature and the results obtained from finite element modelling. In addition, the effects of the stress block parameters of HSC on the interaction curves were studied. A mathematical code was developed with the capability of creating design charts similar to those provided by ACI for NSC.

2. Stress-Strain Relationship of HSC

Stress-strain relationships were developed in the literature based on the experimental work done on HSC. One those models was developed and modified by Hognestad [18]. It is called the modified Hognestad model for HSC. The modified Hognestad model is given in Equation 1 by:

$$f_c = f_c'\left(\mathrm{k}\frac{\epsilon_c}{\epsilon_{cu}} - (\mathrm{k}-1)\left(\frac{\epsilon_c}{\epsilon_{cu}}\right)^2\right) \tag{1}$$

Where; f_c: concrete stress corresponding to normal strain ϵ_c, $f'c$= standard cylinder strength.

$$\mathrm{k} = 2 - \frac{f_c'(\mathrm{MPa})-40}{70} \quad \text{for } 60\ \mathrm{MPa} \le \mathrm{f}_c' \le 94\ \mathrm{MPa} \tag{2}$$

$$\epsilon_{cu} = [2.2 + 0.015(f_c'(\mathrm{MPa}) - 40)] \times 10^{-3} \quad \text{for } 60\ \mathrm{MPa} \le \mathrm{f}_c' \le 94\ \mathrm{MPa} \tag{3}$$

Popovics [19] found a stress-strain relationship in Equation 4 for concrete subjected to compression:

$$f_c = \frac{\epsilon_c}{\epsilon_0}\frac{n\,f_c'}{(n-1)+(\frac{\epsilon_c}{\epsilon_0})^n} \tag{4}$$

Where;

$$\epsilon_0 = 735(f_c')^{0.25} * 10^{-6} \tag{5}$$

$$n = 1.0 + 0.058 f_c' \tag{6}$$

Carreira and Chu [20] proposed another stress strain relationship in Equation 4 for concrete subjected to compression with the parameters given in Equations 7 to 9:

$$\epsilon_0 = (0.71 f_c' + 168) * 10^{-5} \tag{7}$$

$$n = \frac{1}{1 - \frac{f_c'}{\epsilon_0 E_{it}}} \tag{8}$$

$$\mathrm{E}_{it} = 0.0736\ \mathrm{w}^{1.51}(f_c')^{0.3} \tag{9}$$

Kumar [21] found another stress-strain relationship in Equation 10 for concrete under compression:

$$f_c = \frac{\epsilon_c}{\epsilon_0}(n f_c')/((n-1) + (\frac{\epsilon_c}{\epsilon_0})^{nk}) \tag{10}$$

Where;

$$\mathrm{E}_{ci} = 3320\sqrt{\mathrm{f}_c'} + 6900 \tag{11}$$

$$n = 0.8 + \frac{\mathrm{f}_c'}{17} \tag{12}$$

$$\epsilon_0 = \frac{n f_c'}{(n-1)E_{ci}} \tag{13}$$

$$k = \begin{bmatrix} \mathrm{k} = 1 & \epsilon_c \le \epsilon_0 \\ k = 0.67 + \frac{f_c'}{62} & \epsilon_c > \epsilon_0 \end{bmatrix} \tag{14}$$

Mertol [22] proposed a stress-strain relationship in Equation 15 for high strength concrete, it is given by:

$$\mathrm{f}_c = \frac{\epsilon_c}{\epsilon_{c0}}\frac{n\mathrm{f}_c'}{(n-1)+(\frac{\epsilon_c}{\epsilon_{c0}})^{n*k}} \tag{15}$$

$$n = 0.310 \times 0.145\mathrm{fc} + 0.78 \tag{16}$$

$$k = 0.10 \times 0.145\mathrm{fc} + 1.20 \tag{17}$$

$$\epsilon_{c0} = 0.0033 - 2 \times 0.145 \times 10^{-5} \times f'_c \quad (18)$$

$$\epsilon_{cu} = 0.0038 - 4 \times 0.145 \times 10^{-5} \times f'_c \quad (19)$$

3. Problem Formulation and Methodology

The objectives of this research work were accomplished following the research approach given by the flowchart shown in Figure ure 1.

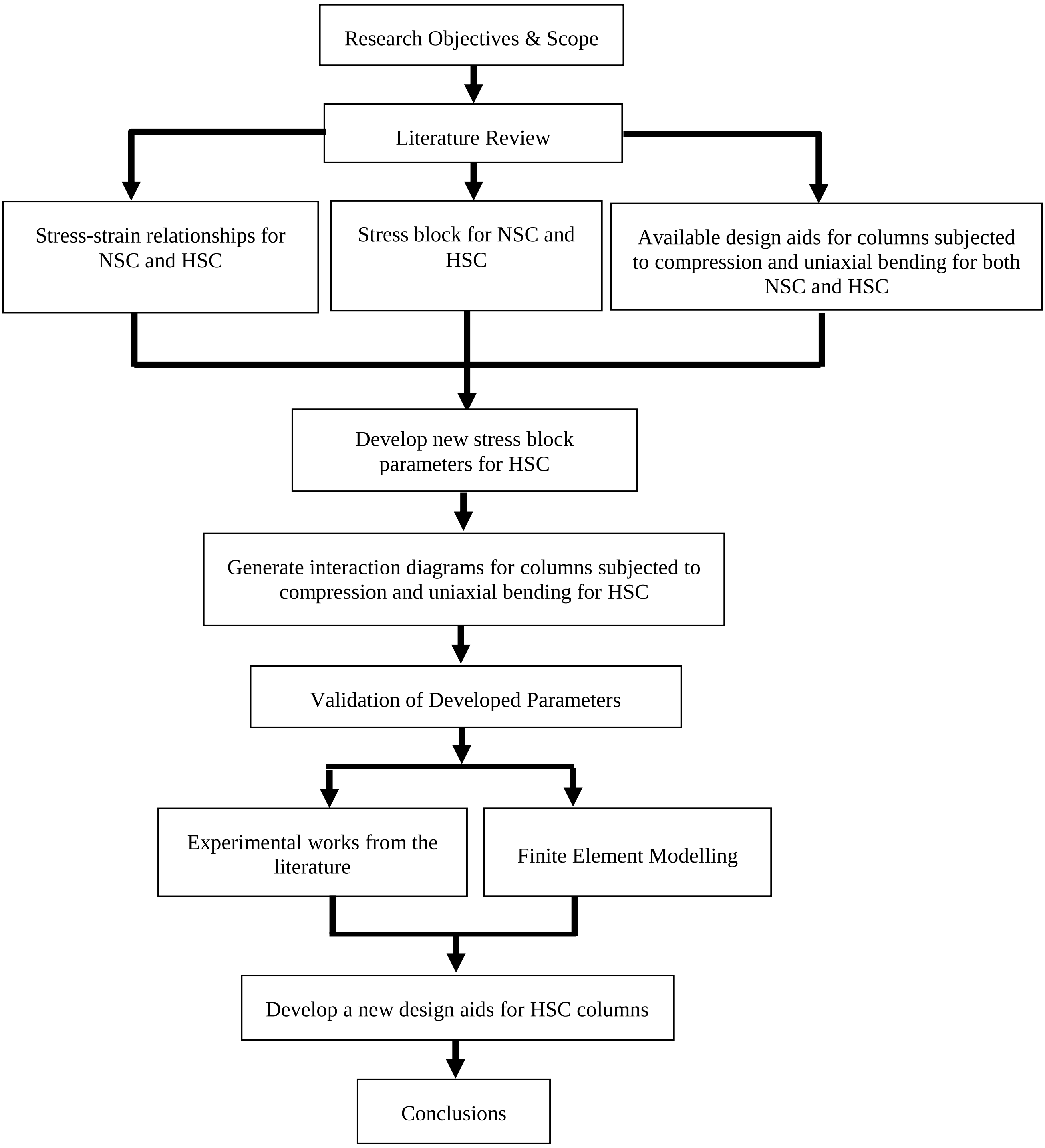

Figure 1. Research methodology flowchart

The actual stress-strain relationships for HSC developed by other researchers such as Hognestad, Popovics, and Carreira & Chu were utilized to find the best representations for stress block parameters (α_1, β_1) based on static equilibrium equations. The generalized stress block and equivalent rectangular stress block are shown in Figure 2. These parameters are derived based on basic equilibrium Equations 20 to 21 as follows:

$$\int_0^{x_0} f_c \, d\epsilon_c = \alpha_1 \beta_1 f'_c \epsilon_{cu} \tag{20}$$

$$\int_0^{x_0} f_c \epsilon_c \, d\epsilon_c = \alpha_1 \beta_1 f'_c \epsilon_{cu} (1 - 0.5\beta_1) \epsilon_{cu} \tag{21}$$

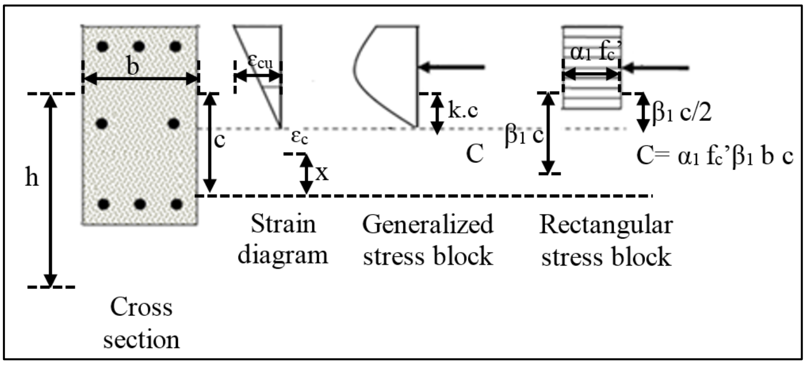

Figure 2. Equivalent rectangular stress block

4. Results and Discussion

4.1. Develop New Stress Block Parameters

In order to obtain the equivalent rectangular stress block shown in Figure , for HSC to be used to evaluate the strength capacity of the structural element, MATHEMATICA software was utilized to evaluate (α_1 and β_1) the parameters for different values of f'_c. The exact values for the stress block parameters (α_1 and β_1) are obtained from the equilibrium Equations 20 and 21 and by using the stress-strain models proposed by Hognestad, Popovics and Carreira and Chu as illustrated in Table 2. The relationships between the obtained values of the α_1 and β_1 and f'_c values are shown in Figure 1 to 8. It can be observed that a linear relationship can be used with a correlation coefficient R^2 very close to 1. It can be observed that the Hognestad model shows conservative parameters compared to the other two models. The other two models overestimate the strength capacity of HSC.

Table 2. Values of α_1 and β_1 for different f'_c or HSC

f'_c(MPa)	Hognestad		Popovics		Carreira & Chu	
	α_1	β_1	α_1	β_1	α_1	β_1
60	0.846	0.734	0.882	0.778	0.991	0.942
70	0.826	0.720	0.880	0.767	0.990	0.938
80	0.807	0.707	0.879	0.756	0.989	0.934
90	0.788	0.694	0.877	0.745	0.988	0.930
100	0.769	0.680	0.875	0.734	0.987	0.926
110	0.750	0.667	0.873	0.723	0.986	0.923
120	0.731	0.653	0.872	0.712	0.985	0.919

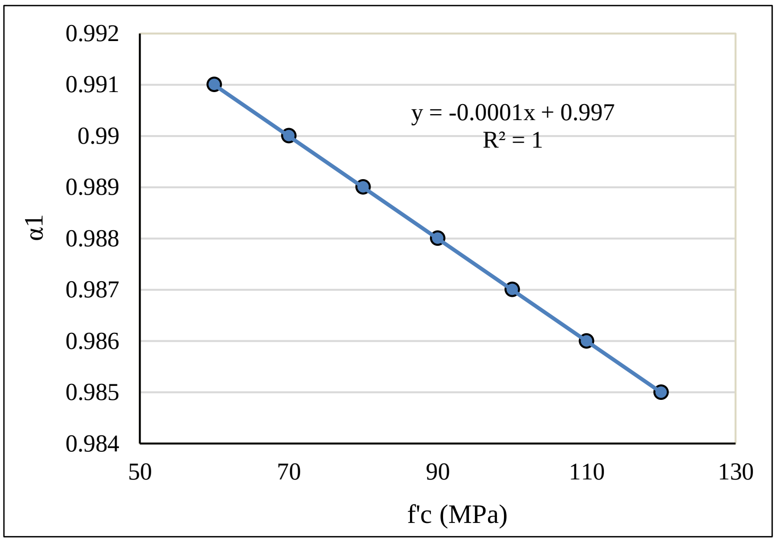

Figure 1. α_1 variation vs. concrete strength f'_c based on the stress-strain model proposed by Carreira and Chu [20]

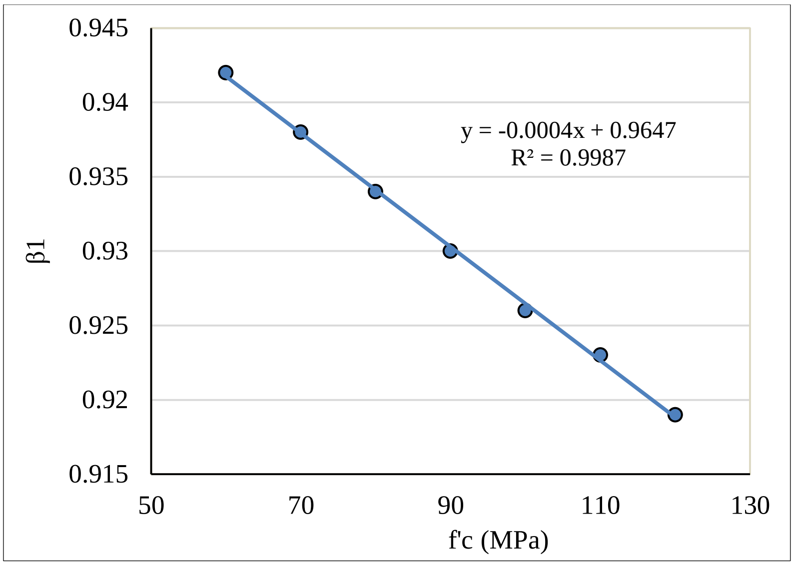

Figure 2. β_1 variation vs. concrete strength f'_c based on the stress-strain model proposed by Carreira and Chu [20]

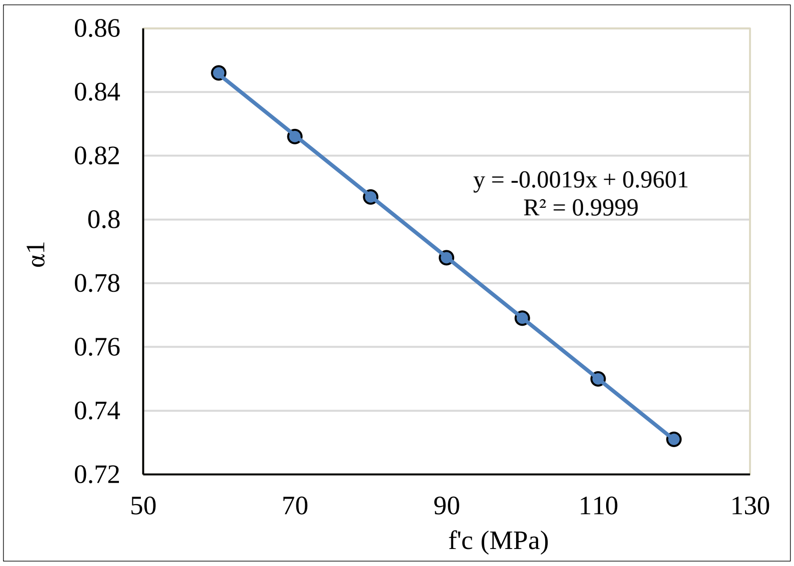

Figure 3. α_1 variation vs. concrete strength f'_c based on the stress-strain model proposed by Hognestad [18]

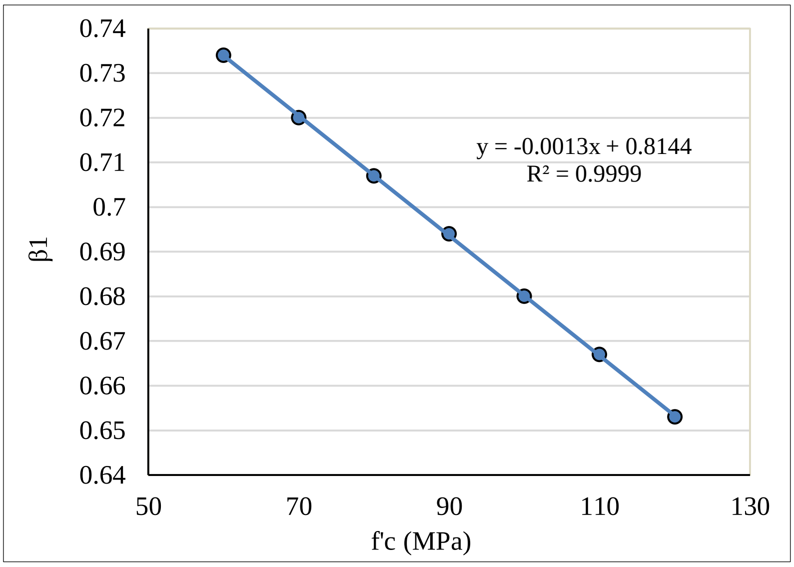

Figure 4. β_1 variation vs. concrete strength f'_c based on the stress-strain model proposed by Hognestad [18]

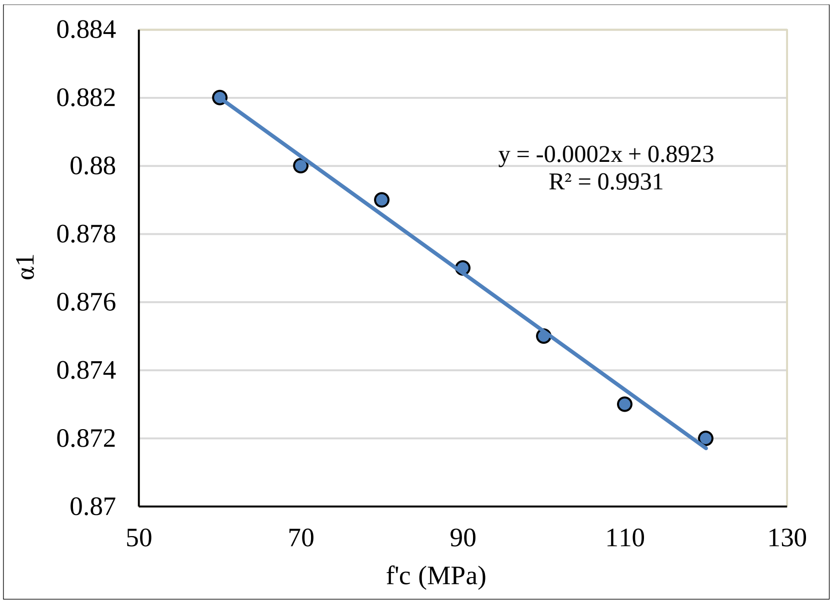

Figure 5. α_1 variation vs. concrete strength f_c' based on the stress-strain model proposed by Popovics [19]

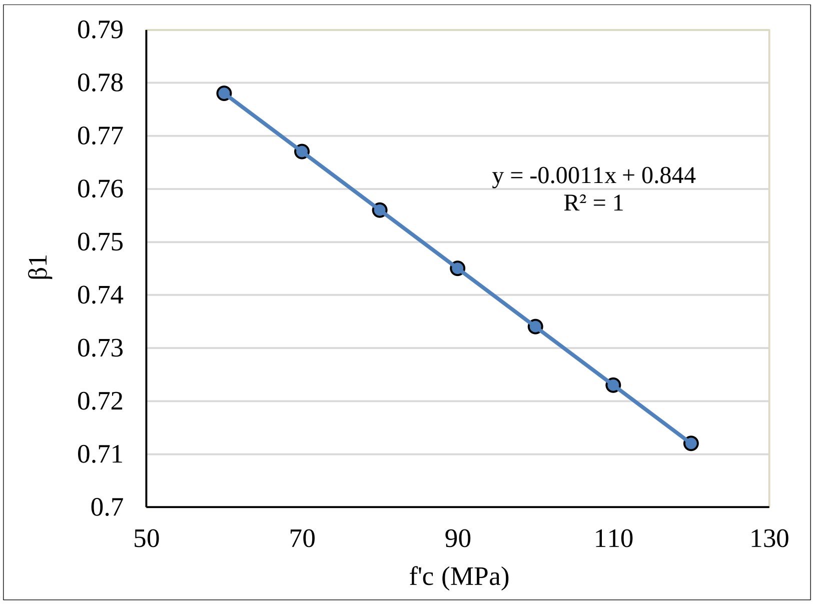

Figure 6. β_1 variation vs. concrete strength f_c' based on the stress-strain model proposed by Popovics [19]

4.2. Validation of Developed Parameters

4.2.1. Previous Experimental Works

The three different stress-strain models and their corresponding equivalent stress block parameters are verified with the experimental data for the reinforced concrete columns found in the literature. Experimental work was conducted for the four different cases shown in Table 3. Each case represents a certain value of concrete strength (f_c'). For each case, a set of experimental data for column capacity (M_n, P_n) is found. The four investigated models are performed in the developed Mathematica code to create their corresponding interaction curves. The validation of the four models with the experimental results is shown in Figures 9 to 12.

Case 1: Tested experimentally by Lloyd and Rangan [23];

Case 2: Tested experimentally by Ibrahim and MacGregor [5];

Case 3: Tested experimentally Foster and Attard [24];

Case 4: Tested experimentally by Ibrahim [25].

The material properties and cross-sectional properties for each case are shown in Table 3. The Experimental data in Table 5 of the pervious experimental works are used to verify the proposed parameters. Table 4 contains the values of the stress block parameters corresponding to the four cases and evaluated based on the developed equations of α1 and β1 in Figures 3 to 8.

Figures 9 to 12 depict the difference between the generated interaction diagrams utilizing the Mathematica code which, use the stress block parameters in Table 4 and the experimental data. It can be seen that the experimental results are much closer to the stress-strain model proposed by Hognestad [18] and this model is more conservative. New parameters for an equivalent stress block can be proposed based on this model; the proposed parameters are given in Equations 22 and 23 by:

$$\alpha_1 = 0.9601 - 0.0019f'_c(\mathrm{MPa}) \tag{22}$$

$$\beta_1 = 0.8144 - 0.00133f'_c(\mathrm{MPa}) \tag{23}$$

Table 3. Properties of the rectangular column sections

	Case 1	Case 2	Case 3	Case 4
f'_c(MPa)	97	126	90	72
f_y(MPa)	400	400	400	400
b (mm)	175	200	175	200
h (mm)	175	300	150	300
γ	0.84	0.60	0.89	0.6
ρ (Steel Percentage)	1.30	1.30%	1.30%	1.30%

γ: is the ratio of the distance between reinforcement steel on two opposite sides to the cross section length in the corresponding direction.

ρ: is the percentage of steel reinforcements to the concrete in a given column.

Table 4. Corresponding stress block parameters for each case using developed equations in Figures 3 to 8

Case	f'c(MPa)	Carreira and Chu		Popovics		Hognestad		Bae and Bayrak	
Case 1	97	0.987	0.928	0.876	0.737	0.775	0.684	0.742	0.670
Case 2	126	0.984	0.917	0.871	0.705	0.719	0.645	0.670	0.670
Case 3	90	0.988	0.930	0.877	0.745	0.788	0.694	0.770	0.670
Case 4	72	0.990	0.937	0.880	0.765	0.823	0.718	0.842	0.682

Table 5. Experimental results for the tested column corresponding to each case

Case 1		Case 2		Case 3		Case 4	
M_n (kN.m)	P_n (kN)	M_n (kN.m)	P_n (kN.m)	M_n (kN.m)	P_n (kN)	M_n (kN.m)	P_n (kN)
59.90	742.66	177.72	3746.06	20.53	1606.15	58.33	3207.77
59.11	751.47	180.44	3949.79	20.21	1659.55	108.05	2720.45
60.46	998.04	180.02	4204.89	23.01	1707.75	110.55	2729.55
58.89	965.75	202.89	4406.33	36.50	1353.12	140.34	3015.47
42.79	1969.67	226.41	4403.67	37.25	1374.52	-	-
39.69	1928.57	-	-	47.12	787.35	-	-
-	-	-	-	48.70	822.15	-	-

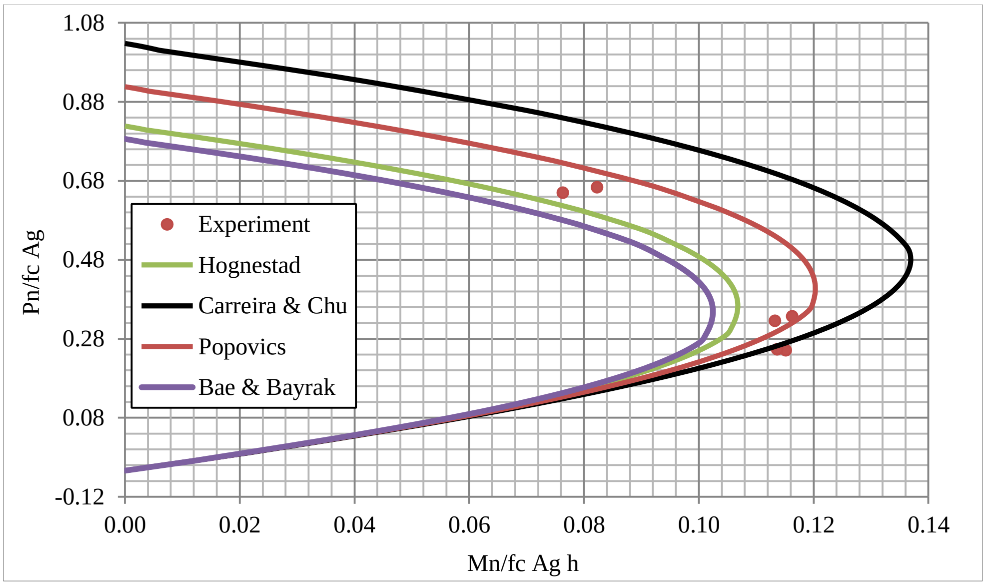

Figure 7. Interaction diagrams corresponding to case 1 using the developed equations of stress block parameters in Figures 3 to 8

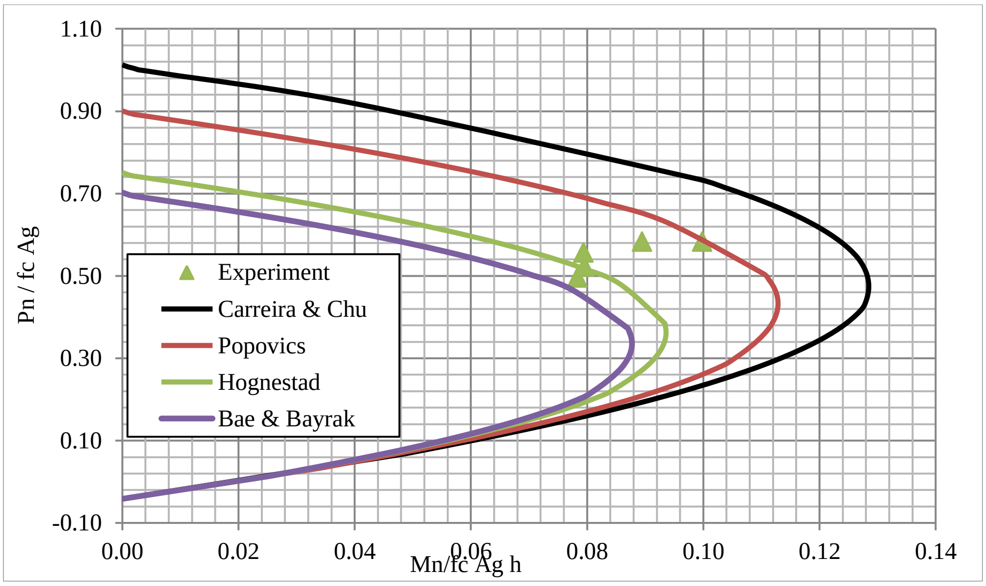

Figure 8. Interaction diagrams corresponding to case 2 using the developed equations of stress block parameters in Figures 3 to 8

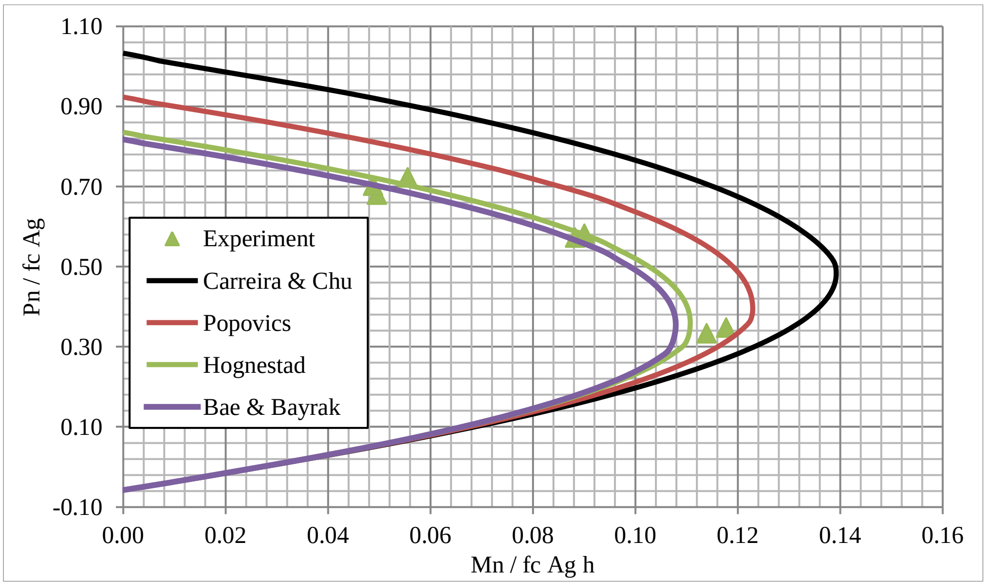

Figure 9. Interaction diagrams corresponding to case 3 using the developed equations of stress block parameters in Figures 3 to 8

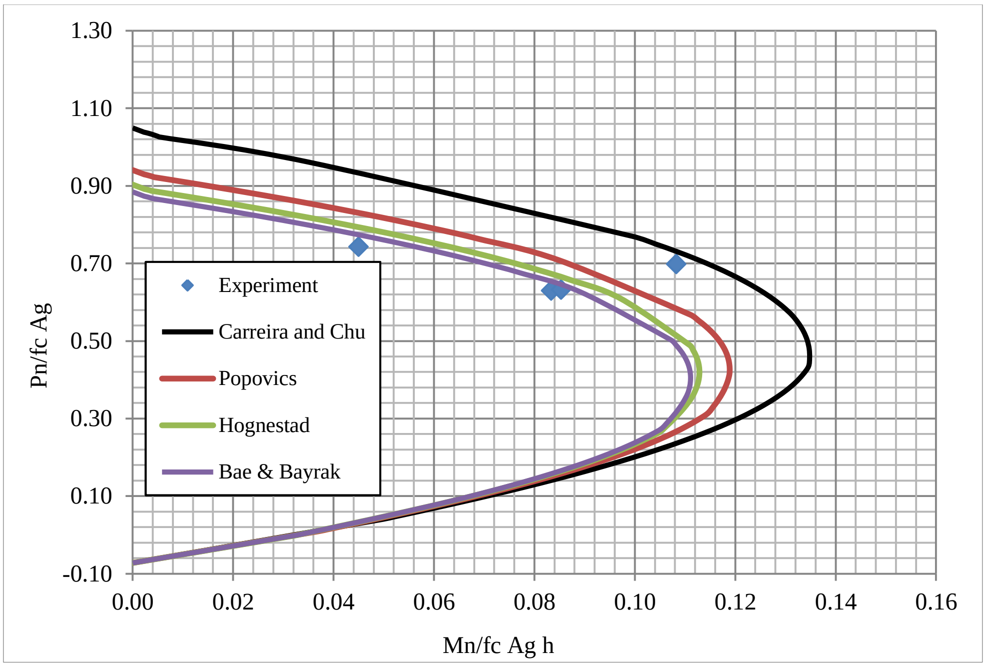

Figure 10. Interaction diagrams corresponding to case 4 using the developed equations of stress block parameters in Figures 3 to 8

4.2.2. Finite Element Model

To verify the proposed developed model, 3-D finite element modeling was developed for the columns with cases number 3 and 4. The finite element modelling was conducted using the comprehensive code ABAQUS. In this work, a dynamic explicit method was utilized to avoid the problems of convergence associated by the column concrete under eccentric loading. Figure 13(a) shows the geometry of the developed model consisting of HSC, longitudinal steel, transverse steel (ties) and steel plates to apply the load.

Finite Element Types and Interaction

A C3D8R 8-noded linear brick, reduced integration, hourglass control element is used to model both the concrete and the steel plate at the top and bottom of a column. A 2-noded linear 3D truss (T3D2) element was utilized to model both the longitudinal and transverse steel rebars. The mesh sizes were investigated in the range of 10 to 30 mm to obtain the optimum mesh size used from the model. The results showed that based on both the accuracy and a reasonable running time, the mesh size of 15 mm was used as shown in Figure 13(a).

The interaction between the different parts in the model was represented using different approaches of contact. The interaction between the concrete and steel rebars was modelled as embedded elements in which the concrete was considered as the host region. The interaction between the concrete and steel plates on the top and bottom was represented using a tie interaction (perfect bond).

Boundary Conditions and Load Application

The ends of the columns are modelled in which they are restrained in the transverse directions (U_z = 0). The loads have been applied as a displacement control at the top and bottom of the steel plates to transfer the load uniformly.

Modelling of HSC and Steel Rebars

The concrete is modelled by using the elastoplastic-damage model developed by Lubliner et al. [26] and extended by Lee and Fenves [27]. The concrete is modelled by using 8-node solid elements and the steel rebar is modelled by using truss elements. In this paper, the hardening and softening under compression of the concrete are implemented in the FE code based on the model developed by Popovics [19]. The model requires this data in the form of stress-inelastic strain. The plastic damage model parameters implemented in ABAQUS are shown in Table 6. The steel rebars were modelled as an elastic-plastic material with yield stress given in Table 3.

Table 6. Parameters Used for the Concrete in Plastic Damage Model

Young's Modulus (MPa)	Poisson's Ratio	Dilation Angle ψ (Degree)	Eccentricity ε	f_{bo}/f_{co}	K
Varied	0.2	36	0 .1	1.16	0.67

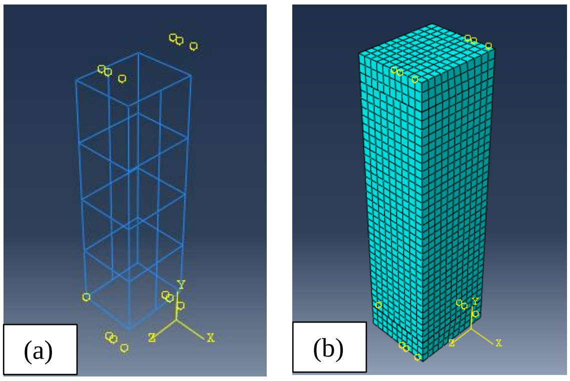

Figure 11. Finite element model: (a) Steel rebar; (b) Meshing of column

Verification of the Model

A comparison of the experimental works and the numerical results of the interaction curves was conducted for cases 3 and 4 and is shown in Figures 14 and 15, respectively. The FE results demonstrated a good agreement with the experimental ones. It can be seen that the difference between the failure loads obtained from the finite element and the experimental results of cases 3 and 4 are less than 10%.

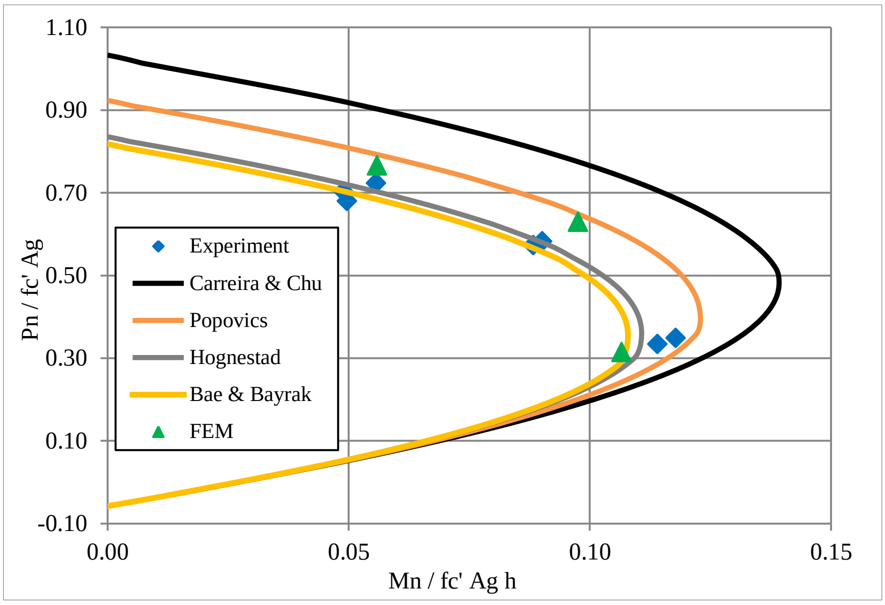

Figure 12. Experimental and numerical validation of the interaction diagrams for case 3

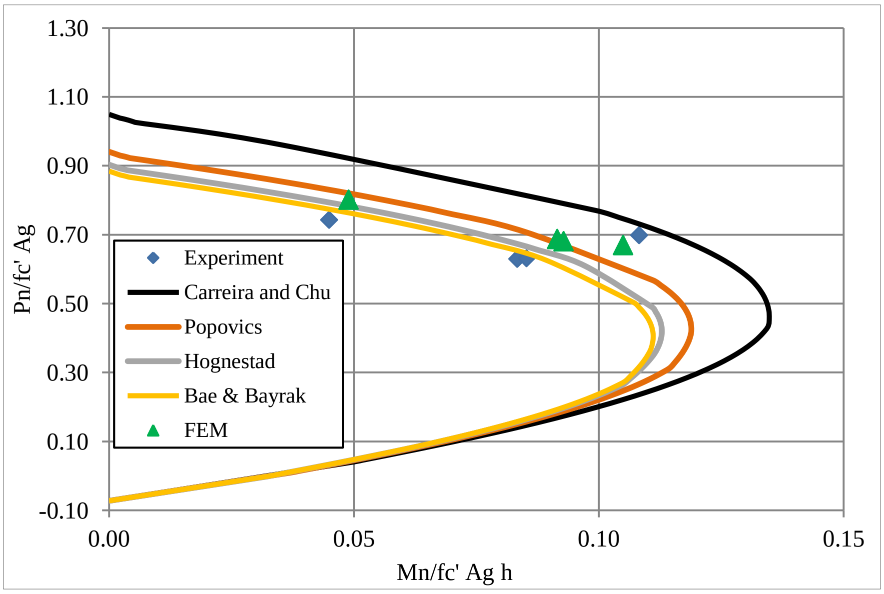

Figure 13. Experimental and numerical validation of the interaction diagrams for case 4

4.3. Stress Block Parameters Effects on Columns Interaction Charts

Stress block parameters affects the interaction diagrams of columns subjected to uniaxial bending and compression force. These effects are significant and must be considered. The concrete stress-stain diagram for HSC is not the same as that for normal concrete. The actual stress block of concrete is idealized to an equivalent rectangular stress block to simplify analysis and design of structural concrete members. The equivalent rectangular stress block of concrete has two parameters; the stress intensity parameter (α_1) and the stress block depth parameter(β_1). The MATHEMATICA code was developed to generate interaction charts for rectangular and circular columns under the effect of uniaxial bending and compression force. Using that code to draw interaction diagrams for a typical rectangular reinforced concrete column with steel on two opposite faces and subjected to uniaxial bending and compression force. The developed MATHEMATICA code was used to draw the normalized interaction charts for a rectangular column section with different values of α_1, β_1 and steel percentage (ρ) as shown in Figures 16 to 18.

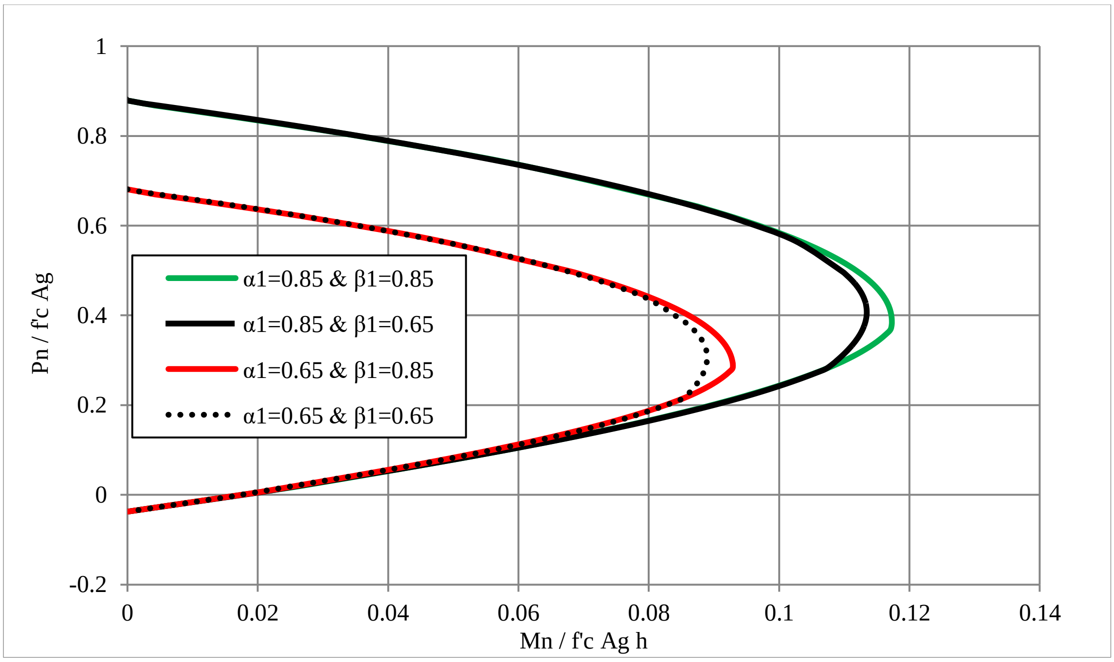

Figure 14. Stress block parameters effects on columns interaction charts for $\rho = 0.01, f_y = 414$ MPa, $f'_c = 110$ MPa and $\gamma = 0.75$

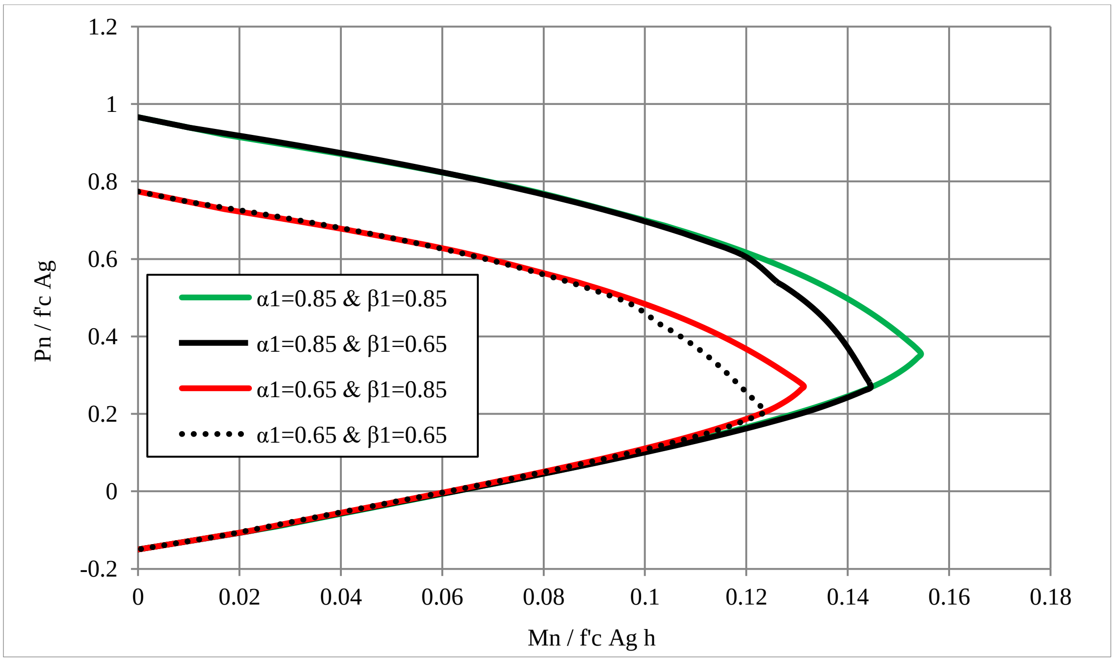

Figure 15. Stress block parameters effects on columns interaction charts for $\rho = 0.04, f_y = 414$ MPa, $f'_c = 110$ MPa and $\gamma = 0.75$

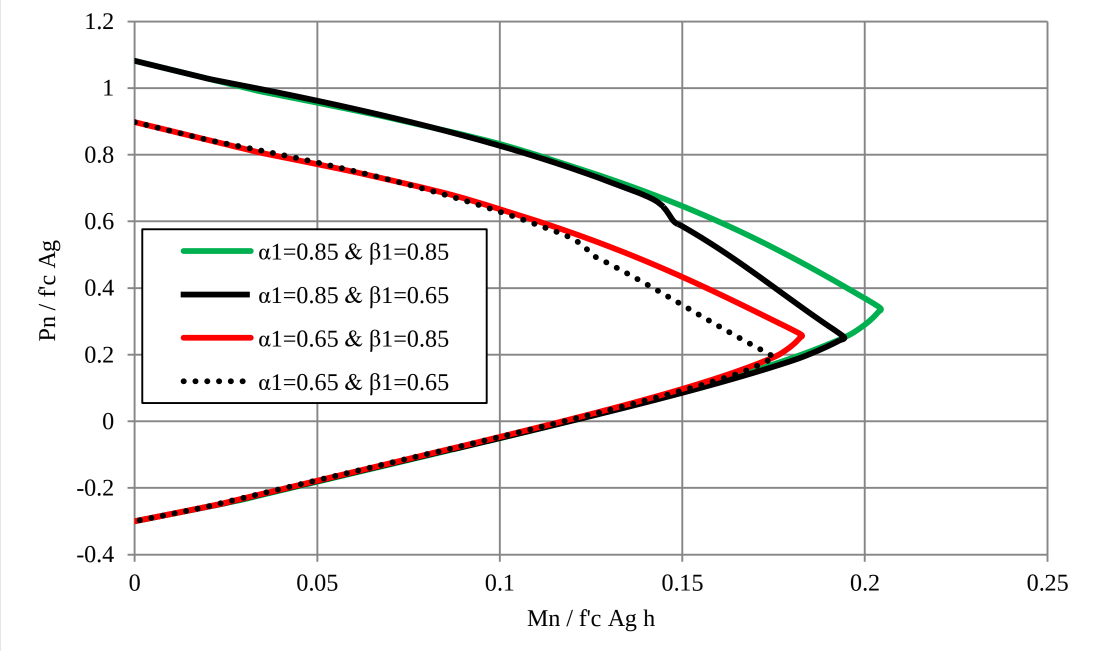

Figure 16. Stress block parameters effects on columns interaction charts for $\rho = 0.08, f_y = 414$ MPa, $f'_c = 110$ MPa and $\gamma = 0.75$

The stress intensity factor has major effects in the compression control region as shown in Figures 16 to 18. This parameter α_1 shifts the interaction diagrams upward and downward when it is equal to 0.85 and 0.65, respectively. The nominal capacity of the column is higher when the value of α_1 is high. The effect of this factor fades as it moves from the compression control region to the tension control region. Concrete in the tension control region has very slight effects and the steel controls the strength in this region. Referring to Figures 16 to 18, the effects of the stress depth parameter β_1 localized near the balanced point on the interaction diagrams. The effect of β_1 does not appear beyond the balanced point. Its effect is minor as compared to the effects of α_1.

Finally, these two parameters play an important role in evaluating the nominal capacity of concrete columns subjected to compression with uniaxial bending. Their values depend mainly on the concrete strength. The American Concrete Institute (ACI) Code provides relationships to evaluate their values for normal concrete only and the use of ACI code stress blocks for high strength concrete will overestimate the column capacity. The interaction diagram for case 3 is developed using an ACI code stress block, a finite element model and the proposed stress block parameters as shown in Figure 19. It can be seen that a good match for the failure loads is estimated by the FE, ACI code and proposed stress block parameters.

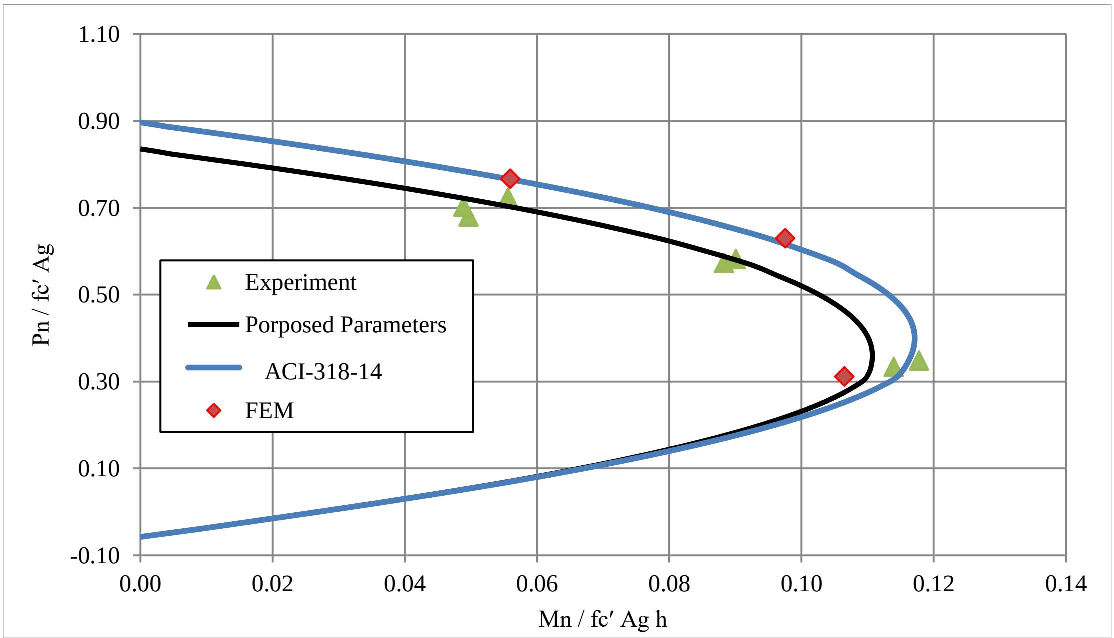

Figure 17. Interaction diagram for previous case 3 developed using ACI code and proposed stress block parameters

4.4. MATHEMATICA Code

The general MATHEMATICA code has been developed using the MATHEMATICA software. The developed code is capable of developing interaction diagrams for rectangular or circular columns subjected to compression force with uniaxial bending. The proposed stress block parameters are used as the default values in the developed code. Interaction charts for HSC columns subjected to compression with uniaxial bending can be easily found. These interaction diagrams were not previously found in codes or standards.

Code input:

Concrete strength: f_c'

Yielding strength of steel: f_y

Steel arm to whole depth ratio: γ

Evaluation of column strength

The nominal capacity of the column is evaluated based on the given neutral axis depth. The way the code evaluates the strength is iterative. Simply assume a neutral axis depth (c) and then evaluate the corresponding column capacity. Figure 20 shows the stress distribution at a certain value of c.

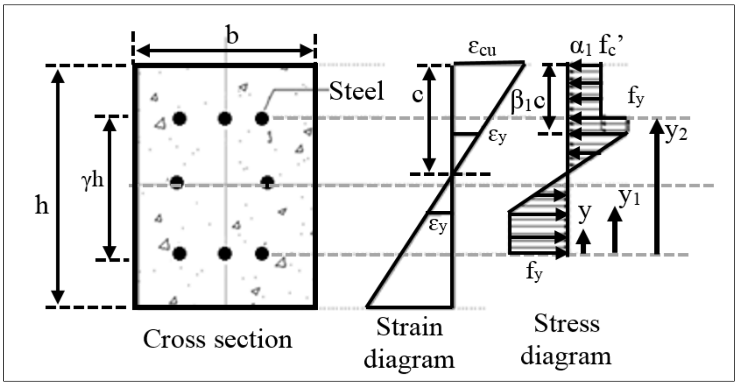

Figure 18. Typical sketch with some definitions for rectangular cross section

Where: y: Arbitrary distance from the extreme tensile steel; y_1: locates the distance to yielding under tension; y_2: locates the distance at which steel starts to yield under compression.

Code output

The output of the code is a graph with eight curves each corresponding to a certain steel percentage starting from 1% and ending with 8%. Figure 21 is a typical code output for the following data: γh; $f_c' = 80$ MPa; $f_y = 400$ MPa and $\gamma = 0.75$.

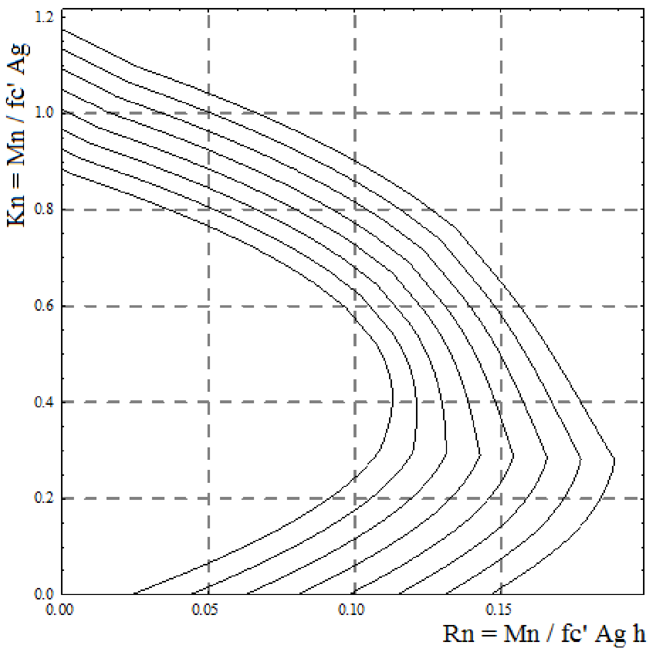

Figure 19. Main output of the developed code corresponding to the above typical example

5. Conclusions

A new stress block for high strength concrete was developed. Interaction diagrams were generated and validated with actual experimental data for the reinforced concrete columns found in the literature. The developed interaction diagrams can be used for the analysis and design of columns. At the end of this research work one can conclude the following:

- A new equivalent rectangular stress block is proposed for HSC. The equivalent rectangular stress proposed/used by ACI code is for NSC and overestimates the capacity of an HSC column. This overestimation in column capacity mainly comes from the stress intensity factor of the equivalent rectangular stress block. The stress depth factor has a minor effect near the balanced point on the column interaction diagram and its effect is considered minor compared with the intensity factor effect.
- The new proposed stress block for high strength concrete has been verified with the experimental data found in the literature and FEM. Consequently, the calculated nominal capacity of the HSC column reflects the real capacity of the column and a safe design of columns is obtained.
- Among all of the investigated models which represent the stress-strain relations of high strength concrete, Hognestad's model is considered to be best one. The stress block parameters, computed using this model, have given results that are very close to the previous experimental results and FEM, and in addition to being conservative.
- A MATHEMATICA code is proposed to develop interaction charts for HSC columns subjected to compression with uniaxial bending. The code is used for rectangular and circular sections only.

6. Acknowledgements

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7. Conflicts of Interest

The authors declare no conflict of interest.

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Appendix I: MATHEMATICA Code

Rectangular Cross Section

```
γ = 0.89;
fy = 400;
fc = 133
n1 = 15;
n2 = 15;
n3 = 15;
ρ =.; b = 1; h = 1; α1 =.; β1 =.;
ϵcu = 3/1000;  Es = 200000; ϵy = fy/Es;
If[fc ≤ 60, α1 = 85/100]
If[60 < fc ≤ 120, α1 = 0.960463 - 0.00191377 fc]
If[fc ≤ 28, β1 = 85/100]
If[28 < fc ≤ 56, β1 = 85/100 - 5/100 (fc - 28)/7]
If[56 < fc ≤  60, β1 = 65/100]
If[60 < fc ≤ 120, β1 = 0.814134 - 0.00133911 fc]
As1 = As3 = ρ b h/4;
As2 = ρ b h/2;
c =.; ϵs1 =.; ϵs3 =.; Fs1 =.; Fs2 =.; Fs3 =.; Fc =.; fs3 =.;
y1 =.; ϵ =.; P2 =.; M2 =.; P =.; M =.; ρ =.; i =.; Mc =.;
mc2 =.; mt2 =.; Fsc2 =.; Fst2 =.; Fs =.;
Do[c[i] = h, {i, 1, 1}]
Do[c[i] = (0.04 i + 1) h, {i, 2, n1}]
Do[{ϵs1[i] = ϵcu/c[i] (c[i] - h/2 (1 + γ)), ϵs3[i] = ϵcu/c[i] (c[i] - h/2 (1 - γ))}, {i, 1, n1}]
Do[ϵ[i] = ϵcu/c[i] (c[i] - x), {i, 1, n1}]
Do[If[β1 c[i] > h, Fc[i] = α1 fc b h, Fc[i] = α1 fc b β1 c[i]], {i, 1, n1}]
Do[If[β1 c[i] > h, Mc[i] = 0, Mc[i] = α1 fc b β1 c[i] ((h - β1 c[i])/2)], {i, 1, n1}]
Do[If[ϵs1[i] ≥ ϵy, fs1[i] = fy, fs1[i] = ϵs1[i] Es], {i, 1, n1}]
Do[If[ϵs3[i] ≥ ϵy, fs3[i] = fy, fs3[i] = ϵs3[i] Es], {i, 1, n1}]
Do[Fs1[i] = As1 (fs1[i] - α1 fc), {i, 1, n1}]
Do[Fs3[i] = As3 (fs3[i] - α1 fc), {i, 1, n1}]
Do[{sol = Solve[ϵ[i] == ϵy, x], x1[i] = x /. sol[[1]]}, {i, 1, n1}]
Do[If[x1[i] ≤ h/2 (1 - γ), Fsc2[i] = As2/(γ h) (NIntegrate[(ϵ[i] Es - α1 fc), {x, h/2 (1 - γ), γ h}])], {i, 1, n1}]
Do[If[x1[i] > h/2 (1 - γ) && x1[i] ≤ h/2 (1 + γ), Fsc2[i] = As2/(γ h) (NIntegrate[(ϵy Es - α1 fc), {x, h/2 (1 - γ), x1[i]}] +
      NIntegrate[(ϵ[i] Es - α1 fc), {x, x1[i], γ h}])], {i, 1, n1}]
Do[If[x1[i] > h/2 (1 + γ), Fsc2[i] = As2 fy], {i, 1, n1}]
Do[If[x1[i] ≤ h/2 (1 - γ), mc2[i] = As2/(γ h) (NIntegrate[(ϵ[i] Es - α1 fc) (h/2 - x), {x, h/2 (1 - γ), γ h}])], {i, 1, n1}]
Do[If[x1[i] > h/2 (1 - γ) && x1[i] ≤ h/2 (1 + γ),
  mc2[i] = As2/(γ h) (NIntegrate[(ϵy Es - α1 fc) (h/2 - x), {x, h/2 (1 - γ), x1[i]}] +
```

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        NIntegrate[(ϵ[i] Es - α1 fc) (h/2 - x), {x, x1[i], γ h}]]], {i, 1, n1}]
Do[If[x1[i] > h/2 (1 + γ), mc2[i] = 0], {i, 1, n1}]
Do[{mt2[i] = 0, Fst2[i] = 0}, {i, 1, n1}]

Do[{Fs2[i] = Fst2[i] + Fsc2[i], Fs[i] = Fs3[i] + Fs2[i] + Fs1[i], P[i] = Fc[i] + Fs[i]}, {i, 1, n1}]
Do[M[i] = (-Fs1[i] + Fs3[i]) (γ h/2) + Mc[i] + mc2[i] + mt2[i], {i, 1, n1}]
Do[P1[i] = FullSimplify[P[i]/(fc b h)], {i, 1, n1}]
Do[M1[i] = FullSimplify[M[i]/(fc b h^2)], {i, 1, n1}]
Do[data1[ii] = Reverse[Table[{N[M1[i]], N[P1[i]]} /. ρ → 0.01 ii, {i, 1, n1}]], {ii, 1, 8}]
c =.; ϵs1 =.; ϵs3 =.; Fs1 =.; Fs2 =.; Fs3 =.; Fc =.; fs3 =.; y1 =.; ϵ =.; P2 =.; M2 =.; P =.; M =.; ρ =.; i =.;
mc2 =.; mt2 =.; Fsc2 =.; Fst2 =.; Fs =.;
Do[c[i] = h/2 (1 + γ) - (i - 1)/n2 γ h, {i, 1, n2}]
Do[ϵ[i] = (ϵcu (x - h/2 (1 + γ)))/c[i] + ϵcu, {i, 1, n2}]
Do[{ϵs1[i] = ϵcu/c[i] (h/2 (1 + γ) - c[i]), ϵs3[i] = ϵcu/c[i] (c[i] - h/2 (1 - γ))}, {i, 1, n2}]
Do[Fc[i] = α1 fc b β1 c[i], {i, 1, n2}]
Do[If[ϵs1[i] ≥ ϵy, fs1[i] = fy, fs1[i] = ϵs1[i] Es], {i, 1, n2}]
Do[If[ϵs3[i] ≥ ϵy, fs3[i] = fy, fs3[i] = ϵs3[i] Es], {i, 1, n2}]
Do[Fs1[i] = As1 (fs1[i]), {i, 1, n2}]
Do[Fs3[i] = As3 (fs3[i] - α1 fc), {i, 1, n2}]
Do[{sol = Solve[ϵ[i] == ϵy, x], x1[i] = x /. sol[[1]]}, {i, 1, n2}]
Do[{sol = Solve[ϵ[i] == -ϵy, x], x2[i] = x /. sol[[1]]}, {i, 1, n2}]
Do[If[ϵs3[i] > ϵy, Fsc2[i] = As2/(γ h)
    (NIntegrate[(ϵy Es - α1 fc), {x, x1[i], γ h}] + NIntegrate[(ϵ[i] Es - α1 fc), {x, h/2 (1 + γ) - c[i], x1[i]}]),
  Fsc2[i] = As2/(γ h) NIntegrate[(ϵ[i] Es - α1 fc), {x, h/2 (1 + γ) - c[i], γ h}]], {i, 1, n2}]
Do[If[ϵs1[i] > ϵy, Fst2[i] =
  As2/(γ h) (NIntegrate[(-ϵy Es ), {x, 0, x2[i]}] + NIntegrate[(ϵ[i] Es ), {x, x2[i], h/2 (1 + γ) - c[i]}]),
  Fst2[i] = As2/(γ h) (NIntegrate[(ϵ[i] Es ), {x, 0, h/2 (1 + γ) - c[i]}])], {i, 1, n2}]
Do[{Fs2[i] = Fst2[i] + Fsc2[i], Fs[i] = Fs3[i] + Fs2[i] - Fs1[i], P[i] = Fc[i] + Fs[i]}, {i, 1, n2}]

Do[If[ϵs3[i] > ϵy, mc2[i] = As2/(γ h) (NIntegrate[(ϵy Es - α1 fc) (x - γ h/2), {x, x1[i], γ h}] +
        NIntegrate[(ϵ[i] Es - α1 fc) (x - γ h/2), {x, h/2 (1 + γ) - c[i], x1[i]}]),
  mc2[i] = As2/(γ h) NIntegrate[(ϵ[i] Es - α1 fc) (x - γ h/2), {x, h/2 (1 + γ) - c[i], γ h}]], {i, 1, n2}]
Do[If[ϵs1[i] > ϵy, mt2[i] = As2/(γ h) (NIntegrate[(-ϵy Es ) (x - γ h/2), {x, 0, x2[i]}] +
        NIntegrate[(ϵ[i] Es ) (x - γ h/2), {x, x2[i], h/2 (1 + γ) - c[i]}]),
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mt2[i] = As2/(γ h) (NIntegrate[(ϵ[i] Es) (x - (γ h)/2), {x, 0, h/2 (1 + γ) - c[i]}])], {i, 1, n2}]

Do[M[i] = (Fs1[i] + Fs3[i]) ((γ h)/2) + Fc[i] ((h - β1 c[i])/2) + mc2[i] + mt2[i], {i, 1, n2}]
Do[P2[i] = FullSimplify[P[i]/(fc b h)], {i, 1, n2}]
Do[M2[i] = FullSimplify[M[i]/(fc b h^2)], {i, 1, n2}]
Do[data2[ii] = Table[{N[M2[i]], N[P2[i]]} /. ρ → 0.01 ii, {i, 1, n2}], {ii, 1, 8}]
c =.; ϵs1 =.; ϵs3 =.; Fs1 =.; Fs2 =.; Fs3 =.; Fc =.; fs3 =.; y1 =.; ϵ =.; P3 =.; M3 =.; P =.; M =.; ρ =.; i =.;
mc2 =.; mt2 =.; Fsc2 =.; Fst2 =.; Fs =.;
Do[c[i] = h/2 (1 - γ) - (i - 1)/n3 * 5/12 * (h/2 (1 - γ)), {i, 1, n3}]
Do[{ϵs1[i] = ϵcu/c[i] (h/2 (1 + γ) - c[i]), ϵs3[i] = ϵcu/c[i] (h/2 (1 - γ) - c[i])}, {i, 1, n3}]
Do[ϵ[i] = (ϵcu (h/2 (1 + γ) - x - c[i]))/c[i], {i, 1, n3}]
Do[Fc[i] = α1 fc b β1 c[i], {i, 1, n3}]
Do[If[ϵs1[i] ≥ ϵy, fs1[i] = fy, fs1[i] = ϵs1[i] Es], {i, 1, n3}]
Do[If[ϵs3[i] ≥ ϵy, fs3[i] = fy, fs3[i] = ϵs3[i] Es], {i, 1, n3}]
Do[Fs1[i] = As1 (fs1[i]), {i, 1, n3}]
Do[Fs3[i] = As3 (fs3[i]), {i, 1, n3}]
Do[{sol = Solve[ϵ[i] == ϵy, x], x2[i] = x /. sol[[1]]}, {i, 1, n3}]
Do[Fsc2[i] = 0, {i, 1, n3}]
Do[If[x2[i] < γ h, Fst2[i] = As2/(γ h) (NIntegrate[(ϵy Es), {x, 0, x2[i]}] + NIntegrate[(ϵ[i] Es), {x, x2[i], γ h}]),
  Fst2[i] = As2/(γ h) (NIntegrate[(ϵ[i] Es), {x, 0, γ h}])], {i, 1, n3}]
Do[{Fs2[i] = -Fst2[i] + Fsc2[i], Fs[i] = -Fs3[i] + Fs2[i] - Fs1[i], P[i] = Fc[i] + Fs[i]}, {i, 1, n3}]
Do[mc2[i] = 0, {i, 1, n3}]
Do[If[x2[i] < γ h, mt2[i] =
   As2/(γ h) (NIntegrate[(-ϵy Es) (x - (γ h)/2), {x, 0, x2[i]}] + NIntegrate[-(ϵ[i] Es) (x - (γ h)/2), {x, x2[i], γ h}]),
  mt2[i] = As2/(γ h) (NIntegrate[-(ϵ[i] Es) (x - (γ h)/2), {x, 0, γ h}])], {i, 1, n3}]
Do[M[i] = (Fs1[i] - Fs3[i]) ((γ h)/2) + Fc[i] ((h - β1 c[i])/2) + mc2[i] + mt2[i], {i, 1, n3}]
Do[P3[i] = FullSimplify[P[i]/(fc b h)], {i, 1, n3}]
Do[M3[i] = FullSimplify[M[i]/(fc b h^2)], {i, 1, n3}]
Do[data3[ii] = Table[{N[M3[i]], N[P3[i]]} /. ρ → 0.01 ii, {i, 1, n3}], {ii, 1, 8}]
P0 =.; t =.; d =.; Mr =.;
Mr = Max[Table[N[M2[i]] /. ρ → 0.08, {i, 1, n2}]];
P0 = ((Ag - ρ Ag) α1 fc + ρ Ag fy)/(fc Ag);
t = -(ρ Ag fy)/(fc Ag);
Do[d[i] = ListPlot[Join[{{0, P0}} /. ρ → 0.01 i, data1[i], data2[i], data3[i], {{0, t}} /. ρ → 0.01 i],
   Joined → True, PlotStyle → {Thickness[0.003], Black}], {i, 1, 8}]
Show[{d[1], d[2], d[3], d[4], d[5], d[6], d[7], d[8]}, AspectRatio → 1, GridLines → Automatic,
 GridLinesStyle → Directive[Gray, Dashed], Frame → True, PlotRangePadding → 0,
 PlotRange → {{Mr + 0.01, 0}, {0, P0 /. ρ → 0.08 + .01}}, FrameLabel → {"Rn=Mn/fc Ag h", "Kn=Pn/fc Ag"}]
```

Circular Cross Section

```
ClearAll["Global`*"]
γ = 0.7;
fy = 400;
fc = 55;
n1 = 20;
n2 = 30;
n3 = 30;
ρ =.; h = 1; α1 =.; β1 =.;
ϵcu = 3/1000;  Es = 200 000; ϵy = fy/Es;
If[fc ≤ 60, α1 = 85/100]
If[60 < fc ≤ 120, α1 = 0.960463 - 0.00191377 fc]
If[fc ≤ 28, β1 = 85/100]
If[28 < fc ≤ 56, β1 = 85/100 - 5/100 (fc - 28)/7]
If[56 < fc ≤  60, β1 = 65/100]
If[60 < fc ≤ 120, β1 = 0.814134 - 0.00133911 fc]
Ag = π/4 h^2;
As = ρ Ag;
c =.; θ =.; θ1 =.; θ2 =.; θ3 =.; θ4 =.; θ5 =.; fs =.; P =.; M =.; Ff =.; Mf =.; Fc =.; Mc =.;
Fs =.; i =.; M1 =.; P1 =.; ρ =.; ϵs1 =.; ϵs3 =.; Fsc =.; Fst =.; Msc =.; Mst =.; x =.; θθ3 =.;
Do[c[i] = (0.02 i + 1) h/2 (1 + γ), {i, 1, n1}]
Do[{ϵs1[i] = ϵcu/c[i] (c[i] - h/2 (1 + γ)), ϵs3[i] = ϵcu/c[i] (c[i] - h/2 (1 - γ))}, {i, 1, n1}]
Do[{θ1[i] = N[ArcCos[1/γ (1 - 2 c[i]/h)]], θ2[i] = N[ArcCos[1 - (2 β1 c[i])/h]],
  θ3[i] = N[ArcCos[1/γ (1 - (2 c[i])/h (1 - ϵy/ϵcu))]], θ5[i] = N[ArcCos[1/γ - (2 β1 c[i])/(γ h)]]}, {i, 1, n1}]
Do[If[β1 c[i] > h, Fc[i] = α1 fc Ag, Fc[i] = FullSimplify[-α1 fc/2 h^2 NIntegrate[(Sin[θ])^2, {θ, θ2[i], 0}]]],
 {i, 1, n1}]
Do[If[β1 c[i] > h, Mc[i] = 0, Mc[i] = FullSimplify[-α1 fc/4 h^3 NIntegrate[(Cos[θ]) (Sin[θ])^2, {θ, θ2[i], 0}]]],
 {i, 1, n1}]
Do[If[β1 c[i] ≥ h/2 (1 + γ), Ff[i] = -α1 fc ρ Ag, Ff[i] = (-α1 fc ρ Ag)/π θ5[i]], {i, 1, n1}]
Do[If[β1 c[i] ≥ h/2 (1 + γ), Mf[i] = 0, Mf[i] = -(α1 fc ρ Ag γ h Sin[θ5[i]])/(2 π)], {i, 1, n1}]
Do[fs[i] = N[(Es ϵcu)/c[i] (c[i] - h/2 (1 - γ Cos[θ]))], {i, 1, n1}]
Do[x[i] = ((ϵcu - ϵy)/ϵcu) c[i], {i, 1, n1}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ), θθ3[i] = ArcCos[1/γ - (2 c[i])/(γ h) (1 - ϵy/ϵcu)]], {i, 1, n1}]
Do[If[x[i] ≤ h/2 (1 - γ), Fsc[i] = As/(2 π) (NIntegrate[fs[i], {θ, 0, π}])], {i, 1, n1}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ),
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    Fsc[i] = As/(2 π) (NIntegrate[fy, {θ, 0, θθ3[i]}] + NIntegrate[fs[i], {θ, θθ3[i], π}])], {i, 1, n1}]
Do[If[x[i] > h/2 (1 + γ), Fsc[i] = As/(2 π) (NIntegrate[fy, {θ, 0, π}])], {i, 1, n1}]
Do[If[x[i] ≤ h/2 (1 - γ), Msc[i] = (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fs[i], {θ, 0, π}])], {i, 1, n1}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ), Msc[i] =
    (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fy, {θ, 0, θθ3[i]}] + NIntegrate[(Cos[θ]) fs[i], {θ, θθ3[i], π}])], {i, 1, n1}]
Do[If[x[i] > h/2 (1 + γ), Msc[i] = 0], {i, 1, n1}]
Do[{Mst[i] = 0, Fst[i] = 0}, {i, 1, n1}]
Do[{P[i] = 2 (Fsc[i] + Fst[i]) + Fc[i] + Ff[i], M[i] = (Mst[i] + Msc[i]) + Mc[i] + Mf[i]}, {i, 1, n1}]
Do[{P1[i] = FullSimplify[P[i]/(fc Ag)], M1[i] = FullSimplify[M[i]/(fc Ag h)]}, {i, 1, n1}]
Do[data1[ii] = Reverse[Table[{M1[i], P1[i]} /. ρ → 0.01 ii, {i, 1, n1}]], {ii, 1, 8}]
c =.; θ1 =.; θ2 =.; θ3 =.; θ4 =.; θ5 =.; fs =.; P =.; M =.; P2 =.; M2 =.; Ff =.; Mf =.; Fc =.;
Mc =.; Fs =.; ϵs1 =.; i =.; M =.; ρ =.; ϵs1 =.; ϵs3 =.; Fsc =.; Fst =.; Msc =.; Mst =.;
Do[c[i] = h/2 (1 + γ) - (0.9 i)/n2 (γ h), {i, 1, n2}]
Do[{ϵs1[i] = ϵcu/c[i] (h/2 (1 + γ) - c[i]), ϵs3[i] = ϵcu/c[i] (c[i] - h/2 (1 - γ))}, {i, 1, n2}]
Do[{θ1[i] = N[ArcCos[1/γ (1 - 2 c[i]/h)]],
    θ2[i] = N[ArcCos[1 - (2 β1 c[i])/h]], θ3[i] = N[ArcCos[1/γ (1 - (2 c[i])/h (1 - ϵy/ϵcu))]],
    θ4[i] = N[ArcCos[1/γ (1 - (2 c[i])/h (1 + ϵy/ϵcu))]], θ5[i] = N[ArcCos[1/γ - (2 β1 c[i])/(γ h)]]}, {i, 1, n2}]
Do[If[β1 c[i] > h/2 (1 - γ), Ff[i] = -(α1 fc ρ Ag)/π θ5[i], Ff[i] = 0], {i, 1, n2}]
Do[If[β1 c[i] > h/2 (1 - γ), Mf[i] = -1/(2 π) α1 fc ρ Ag γ h Sin[θ5[i]], Mf[i] = 0], {i, 1, n2}]
Do[{fs[i] = N[(fy γ)/(2 (c[i]/h) (ϵy/ϵcu)) (Cos[θ] - Cos[θ1[i]])],
    Fc[i] = FullSimplify[-α1 fc/2 h^2 NIntegrate[(Sin[θ])^2, {θ, θ2[i], 0}]],
    Mc[i] = FullSimplify[-α1 fc/4 h^3 NIntegrate[(Cos[θ]) (Sin[θ])^2, {θ, θ2[i], 0}]]}, {i, 1, n2}]
Do[If[ϵs1[i] > ϵy, Fst[i] = As/(2 π) (NIntegrate[fs[i], {θ, θ1[i], θ4[i]}] + NIntegrate[-fy, {θ, θ4[i], π}]),
    Fst[i] = As/(2 π) (NIntegrate[fs[i], {θ, θ1[i], π}])], {i, 1, n2}]
Do[If[ϵs3[i] > ϵy, Fsc[i] = As/(2 π) (NIntegrate[fs[i], {θ, θ3[i], θ1[i]}] + fy θ3[i]),
    Fsc[i] = As/(2 π) (NIntegrate[fs[i], {θ, 0, θ1[i]}])], {i, 1, n2}]
Do[If[ϵs1[i] > ϵy, Mst[i] =
    (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fs[i], {θ, θ1[i], θ4[i]}] + NIntegrate[-fy (Cos[θ]), {θ, θ4[i], π}]),
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  Mst[i] = (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fs[i], {θ, θ1[i], π}])], {i, 1, n2}]
Do[If[εs3[i] > εy, Msc[i] =
    (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fs[i], {θ, θ3[i], θ1[i]}] + NIntegrate[(Cos[θ]) fy, {θ, 0, θ3[i]}]),
  Msc[i] = (γ h As)/(2 π) (NIntegrate[(Cos[θ]) fs[i], {θ, 0, θ1[i]}])], {i, 1, n2}]
Do[{P[i] = 2 (Fsc[i] + Fst[i]) + Fc[i] + Ff[i], M[i] = (Mst[i] + Msc[i]) + Mc[i] + Mf[i]}, {i, 1, n2}]
Do[{P2[i] = FullSimplify[P[i]/(fc Ag)], M2[i] = FullSimplify[M[i]/(fc Ag h)]}, {i, 1, n2}]
Do[data2[ii] = Table[{M2[i], P2[i]} /. ρ → 0.01 ii, {i, 1, n2}], {ii, 1, 8}]
c =.; θ =.; θ1 =.; θ2 =.; θ3 =.; θ4 =.; θ5 =.; fs =.; P =.; M =.; Ff =.; Mf =.; Fc =.; Mc =.;
Fs =.; i =.; M3 =.; P3 =.; ρ =.; εs1 =.; εs3 =.; Fsc =.; Fst =.; Msc =.; Mst =.; x =.; θθ4 =.;
Do[c[i] = (i/n3) h/2 (1 - γ), {i, 1, n3}]
Do[{εs1[i] = εcu/c[i] (c[i] - h/2 (1 + γ)), εs3[i] = εcu/c[i] (c[i] - h/2 (1 - γ))}, {i, 1, n3}]
Do[{θ1[i] = N[ArcCos[1/γ (1 - 2 c[i]/h)]], θ2[i] = N[ArcCos[1 - (2 β1 c[i])/h]],
  θ3[i] = N[ArcCos[1/γ (1 - (2 c[i])/h (1 - εy/εcu))]], θ5[i] = N[ArcCos[1/γ - (2 β1 c[i])/(γ h)]]}, {i, 1, n3}]
Do[Fc[i] = FullSimplify[-α1 fc/2 h^2 NIntegrate[(Sin[θ])^2, {θ, θ2[i], 0}]], {i, 1, n3}]
Do[Mc[i] = FullSimplify[-α1 fc/4 h^3 NIntegrate[(Cos[θ]) (Sin[θ])^2, {θ, θ2[i], 0}]], {i, 1, n3}]
Do[{Mf[i] = 0, Ff[i] = 0}, {i, 1, n3}]
Do[fs[i] = N[(Es εcu)/c[i] (-c[i] + h/2 (1 - γ Cos[θ]))], {i, 1, n3}]
Do[x[i] = ((εcu + εy)/εcu) c[i], {i, 1, n3}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ), θθ4[i] = ArcCos[1/γ - (2 c[i])/(γ h) (1 + εy/εcu)]], {i, 1, n3}]
Do[If[x[i] ≤ h/2 (1 - γ), Fst[i] = As/(2 π) (NIntegrate[-fy, {θ, 0, π}])], {i, 1, n3}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ),
  Fst[i] = As/(2 π) (NIntegrate[-fs[i], {θ, 0, θθ4[i]}] + NIntegrate[-fy, {θ, θθ4[i], π}])], {i, 1, n3}]
Do[If[x[i] > h/2 (1 + γ), Fst[i] = As/(2 π) (NIntegrate[-fs[i], {θ, 0, π}])], {i, 1, n3}]
Do[If[x[i] ≤ h/2 (1 - γ), Mst[i] = 0], {i, 1, n3}]
Do[If[x[i] > h/2 (1 - γ) && x[i] < h/2 (1 + γ),
  Mst[i] = (γ h As)/(2 π) (NIntegrate[-(Cos[θ]) fs[i], {θ, 0, θθ4[i]}] + NIntegrate[-(Cos[θ]) fy, {θ, θθ4[i], π}])], {i,
  1, n3}]
Do[If[x[i] > h/2 (1 + γ), Mst[i] = (γ h As)/(2 π) (NIntegrate[-(Cos[θ]) fs[i], {θ, 0, π}])], {i, 1, n3}]
Do[{Msc[i] = 0, Fsc[i] = 0}, {i, 1, n3}]
Do[{P[i] = 2 (Fsc[i] + Fst[i]) + Fc[i] + Ff[i], M[i] = (Mst[i] + Msc[i]) + Mc[i] + Mf[i]}, {i, 1, n3}]
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Do[{P3[i] = FullSimplify[P[i]/(fc Ag)], M3[i] = FullSimplify[M[i]/(fc Ag h)]}, {i, 1, n3}]
Do[data3[ii] = Reverse[Table[{M3[i], P3[i]} /. ρ → 0.01 ii, {i, 1, n3}]], {ii, 1, 8}]
P0 =.; t =.; d =.; Mr =.;
Mr = Max[Table[N[M2[i]] /. ρ → 0.08, {i, 1, n2}]];
P0 = ((Ag - ρ Ag) α1 fc + ρ Ag fy)/(fc Ag);
t = -(ρ Ag fy)/(fc Ag);
Do[d[i] = ListPlot[Join[{{0, P0}} /. ρ → 0.01 i, data1[i], data2[i], data3[i], {{0, t}} /. ρ → 0.01 i],
   PlotStyle → {Thickness[0.003], Black}, Joined → True], {i, 1, 8}]
Show[{d[1], d[2], d[3], d[4], d[5], d[6], d[7], d[8]}, AspectRatio → 1, GridLines → Automatic,
 GridLinesStyle → Directive[Gray, Dashed], Frame → True, PlotRangePadding → 0,
 PlotRange → {{Mr + 0.01, 0}, {0, P0 /. ρ → 0.08 + .01}}, FrameLabel → {"Rn=Mn/fc Ag h", "Kn=Pn/fc Ag"}]
```