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Investigation of Soil Structure Interaction and Wall Flexibility Effects on Natural Sloshing Frequency of Vessels

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Abstract

The main purpose of this study is to establish the effects of vessel walls flexibility on its natural sloshing frequency considering soil-structure-fluid interaction theory. Furthermore, two new efficiently relations to find both of wall flexibility and soil-structure interaction effects on natural frequency are developed. Regarding the aim of current study three different conditions of elevated tanks are applied. Fixed base condition with an emphasis on recommendations of international code ACI-350, analytical FSSI regarding equivalent mass spring method, and the numerical direct method regarding theory of finite element are taken into consideration. Results indicate that there is no significant effect of walls flexibility on natural sloshing frequency regarding fixed base assumptions of vessels. On the contrary, significant effects of wall flexibility are achieved considering SSI theory. Results of international code ACI-350 show that, the international codes assumptions have imprecise estimations of natural sloshing frequency in the range of hard to very soft soil categories. On the other hand, it is observed that the wall flexibility has a more highlighted effect on natural frequency in soft soils rather than soil-structure interaction. The significance of wall flexibility effect on natural frequency is more than that of SSI considering soil softening.

Keywords: Natural Sloshing Frequency; Finite Element; Flexibility; Vessel; Soil-Structure Interaction.

1. Introduction

Regarding recent reports, there are many vessels that were collapsed or experienced remarkable damage during earthquakes that have occurred all around the world. A large proportion of such loss have been observed during recent earthquakes, which have occurred from reasons like unsuitable design of construction of tanks on loose soil and ignoring the effects of the soil structure interaction (SSI) [1, 2]. Because of a complete recognition of hydrodynamic liquid effects on vessels of elevated tanks, lots of researchers recommended inquiries to fluid-structure interaction (FSI) method [3-5]. The most accurate method for liquid storage /elevated tanks modelling is Fluid-Structure-Soil Interaction (FSSI) [6]. To simplify the issue all international codes and regular relations assume that walls of vessels are rigid and there is no deformation in the body of vessel considering fluid fluctuation.

As a good method to find the soil effects on superstructures, pervious researcher recommended to use the analytical equivalent mass spring method [7, 8]. The soil stiffness matrix of the surrounding soil is represented by a 2 × 2 matrix where K_x, K_Θ and $K_{x\Theta}$ are the sway, rocking and sway-rocking coupling terms of the corresponding static stiffness matrix, respectively [7, 9].

$$\begin{bmatrix} K_x & K_{x\Theta} \\ K_{x\Theta} & K_\Theta \end{bmatrix}$$

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Soil stiffness is attached to the central point of the rigid circular foundation [7, 10]. The practical formula to find explained soil stiffness for circular rigid foundations supported is presented in Table 1. [9, 11, 12]

Table 1: Regular recommendation formula to find soil stiffness [12]

Soil Stiffness mode	Equations
Horizontal Stiffness (kN/m)	$K_x = \frac{8GR}{2-\upsilon}$
Rocking stiffness ($kN.m$)	$K_\theta = \frac{8GR^3}{3(1-\upsilon)}$

Where, $G, R,$ and υ are shear modulus, Poisson's ratio of soil and radius of equivalent circular foundation. All of spring mass methods to find SSI effects of soil on structures response assume that the ground mass participation would be negligible [12-14]. To find fluid structure interaction effects on vessel response, many different analytical and semi-analytical methods have developed, which can be used to verify and develop more advanced numerical methods [5]. Many useful and applicable analytical methods were developed by Westergaard (1933) [15]. Several regular approximate methods also developed by Housner (1963), Haroun and Ellaithy (1985) are used in international codes such as Eurocode 8 (2006) and ACI 350 [16, 17, 30]. All mentioned codes and regular relations assume that the walls of vessel would be rigid. However, in real conditions there are many vessels with no rigid walls.

Various research methods have been applied to study vessels in recent decades. Haroun and Ellaithy (1985); Reshidat and Sunna (1986); Haroun and Temraz (1992); Livaoğlu and Doğangün (2006); Livaoglu and Dogangun (2007); Dutta et al. (2004); Marashi and Shakib (2008); Livaoglu et al. (2011); Ghahramani et al. (2010); Ghanbari and Abbasi Maedeh (2015) have evaluated dynamic behaviours of vessels [4, 6, 17-25]. There is no direct study to compare results of flexible and rigid walls results and the SSI effects on flexible walls sloshing results. The majority of studies were dedicated to appraise the behaviour of the fluid and the supporting structure by using the fixed base assumptions and they have neglected soil effect and wall flexibility on tanks behaviour.

In this study, the changes of natural sloshing frequency for vessels by using the numerical finite element software, international code ACI-350 and the analytical method theory would be evaluated. The natural frequency of vessel by using both of rigid and flexible walls considering SSI would be investigated. To find SSI and wall flexibility effects, the numerical finite element theory by use of direct method technic for vessel is chosen. Different category of soil stiffness is evaluated in explained analysis. The equivalent mass spring used for analytical FSSI analysis. Concluded results of natural frequency are compared with methods. The innovations of this study are classified as follows:

- Find the wall flexibility effects on natural sloshing period of vessels.
- Find the SSI effects on flexible and rigid walls vessel.
- Find the rate of wall flexibility and SSI effects on natural frequency of sloshing model.

Developing two new efficiently relations to find the natural sloshing frequency considering FSSI and wall flexibility effects.

2. Basic Concept and Assumptions

To find natural periods of a MDOF system, the following equations would be defined and solved. In general, for a system of n degree of freedom, the equation of motion is a set of second linear differential equation as given in Equation 1. [26].

$$[K]\{U\} + [C]\{\dot{U}\} + [M]\{\ddot{U}\} = \{F\} \tag{1}$$

The vector $\{U\}$ function of time denotes the displacement response at all degrees of freedom. The matrices $[K], [C]$ and $[M]$ represent stiffness matrix, damping matrix and mass matrix respectively, which are constant for a linear system. The vector of $\{F\}$ denotes the prescribed loads at the corresponding degree of freedom as a function of time. In order to estimate the fundamental frequency of the system under harmonically varying load, Equation 2. can be written as:

$$\big[[K] - \omega^2[M]\big] = \{0\} \tag{2}$$

The n degree of freedom system Eigen value equation considering arrays of soil and super structure matrix which is substituted into Equation 2. is developed as Equation 3. [7, 27]:

$$\begin{bmatrix} [K_s] & [K_{sf}] \\ [K_{fs}] & [K_{soil}] \end{bmatrix} - \{\omega^2\} \begin{bmatrix} [M_s] & [M_{sf}] \\ [M_{fs}] & [M_{soil}] \end{bmatrix} \tag{3}$$

Solving Equation 3. n vibration frequencies corresponding to n degrees of freedom are driven. The radian frequency ω_n, can be found. The natural frequency equation for the multi mass assumption of water tank with an emphasis on FSSI system is written as Equation 4:

$$\begin{bmatrix} K_c & -K_c & 0 & 0 \\ -K_c & K_c + K_{str} & -K_{str} & 0 \\ 0 & -K_{str} & K_{str} + K_x & K_{x\theta} \\ 0 & 0 & K_{x\theta} & K_\theta \end{bmatrix} - \{\omega^2\} \begin{bmatrix} m_c & 0 & 0 & mc(hc) \\ 0 & m_{str} & 0 & mstr(hi) \\ 0 & 0 & m_f & 0 \\ m_c(h_c) & m_{str}(h_i) & 0 & I_f \end{bmatrix} = 0 \tag{4}$$

The recommendations of ACI-350 international code are used for fixed base analytical system modeling. Numerical model of fixed and flexible (direct method) for vessel are made by using following assumptions. The general-purpose computer code ANSYS is chosen to implement FEM analyses [28]. A filled elevated tank is selected and FEM analyses are conducted using displacement-based (D-Fluid) Elements. The extracted results through free vibration analysis, including natural periods of vibration are compared with those obtained from analytical recommended and international code approximation. D-Fluid elements use displacements as the variables in the liquid domain. These elements are considered as an extracted of structural solid elements in which the element shear modulus are set to zero and the liquid bulk modulus; K is used to establish the elastic stress–strain relations. D-Fluid elements are particularly useful in modeling fluids contained within vessels having no net flow rate [28]. It can be employed for transient as well as free vibration analysis [5, 29]. The stress–strain relationship of the D-Fluid element is established using the stiffness matrix as follows:

$$\begin{bmatrix} \varepsilon_{bulk} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \\ R_x \\ R_y \\ R_z \end{bmatrix} = \begin{bmatrix} 1/k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/S & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/B & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/B \end{bmatrix} \times \begin{bmatrix} P \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \\ M_x \\ M_y \\ M_z \end{bmatrix} \tag{5}$$

Where $\varepsilon_{bulk} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = bulk\ strain$

K= Fluid elastic (bulk) modulus

P= Pressure

γ= Shear strain

S=$K \times 10^{-9}$ arbitrary small number to give element some shear stability

τ= Shear stress

R= Rotation about axis i

B= $K \times 10^{-9}$ arbitary small number to give element some rotation stability

M_i= Twisting force about axis i

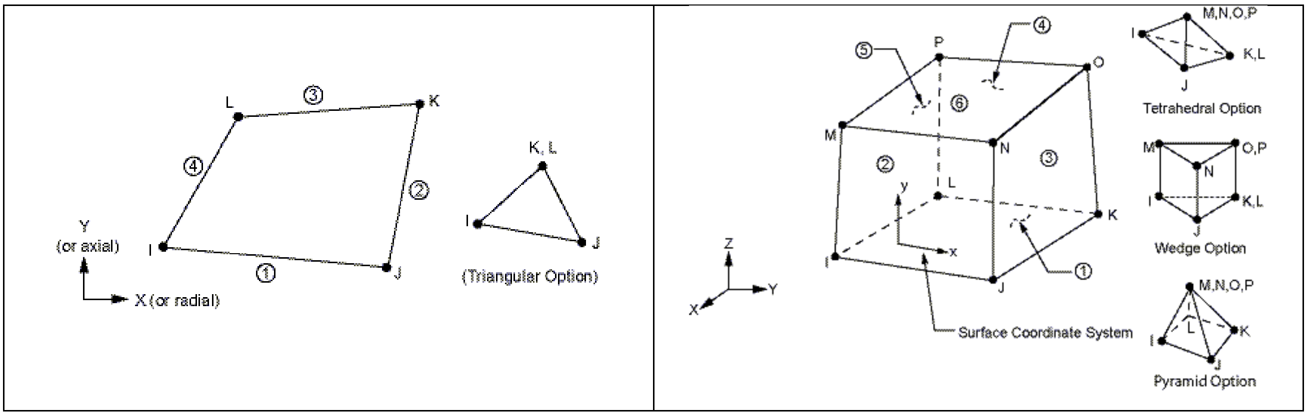

Figure 1. D-fluid element picture left 2D fluid element; right 3D fluid element [28]

The equation of motion of the system can be solved by the direct integration method. However, when the response of the structure is linear, more efficient analysis using the concept of modal superposition [24]. According to ACI 371R-08 for each model, a sufficient number of modes (both impulsive and convective) to obtain a combined modal mass participation of at least 90% of the actual mass of the structure in the direction under consideration were employed [5].

In this study, firstly it is assumed that the 10×10 vessel by height of 5 meters and the liquid height of 4 meters is placed on ground. The tank is established on dry and non-saturated soil with different mechanical properties. The stiffness of the equivalent springs for various varieties of soil has been obtained from values of shear modulus G of soil. Different categories of current studies soil properties show in Table 2.

Table 2. Soil classification and general soil properties [6]

Soil category	ν	γ (kN/m^2)	E (kN/m^2)	G (kN/m^2)	Vs (m/s)
Very hard	0.2	19	4.90E+06	2041667	1026.71
Hard	0.3	18	7.63E+05	293461.5	399.92
Soft	0.35	17	9.63E+04	35666.67	143.46
Very soft	0.4	13	3.20E+04	11428.57	92.86

To find a regular range of wall thickness for vessel, regarding literature and the property of real vessels, which constructed on different place, it is observed that the regular wall thickness for general vessels is filled between 0.1 to 0.25 meters. Some of real vessels wall thickness which collected to find true estimation of limit of wall thickness is reported in in Table 3.

Table 3. regular wall thickness of vessels, which collected in current study

Vessel type	Concrete wall thickness (m)	Reference or location
Clio vessel	0.3	Iran,Sari
Underground vessel	0.2	Yousefi samangani et al.,2013
Elevated tank	0.2	Omidnasab and shakib, 2011
Elevated tank	0.3	Kalani Sarkolae et al., 2013
Elevated tank	0.2	Uddin, 2013
Elevated tank	0.25	Livaoglu and dugnagun, 2008
Elevated tank	0.2	EQ8 guide line
Ground supported tank	0.25	EQ8 guide line

3. Results and Discussion

To simplify the numerical models and also saving the time firstly current study regarding results of numerical modeling verified that there is no difference between results of 2D and 3D natural sloshing period in the rigid wall vessels. For this reason, the 20×15 meter vessel is made by use of FEM software. D-Fluid elements were used for liquid and also plain elements for body of the tank. Fluid and plain domain should be meshed in such a way that the location of each node of the fluid domain on the interface coincides exactly with that of the corresponding shell element.

Fluid nodes should be coupled at all interfaces with a containing structure; as a result, all fluid nodes located at the interface with the tank's floor should be restrained in vertical direction. In the finite element idealization, the fluid domain is modeled using eight-node brick fluid elements as described previously and the tank's wall is modeled using four-node quadrilateral shell elements. The shell element has six degrees of freedom (translations and rotations) at each node and both bending and membrane behaviors are permitted. The concluded frequency which reported from 2D and 3D sample models are 0.1957 Hz (Figure 2).

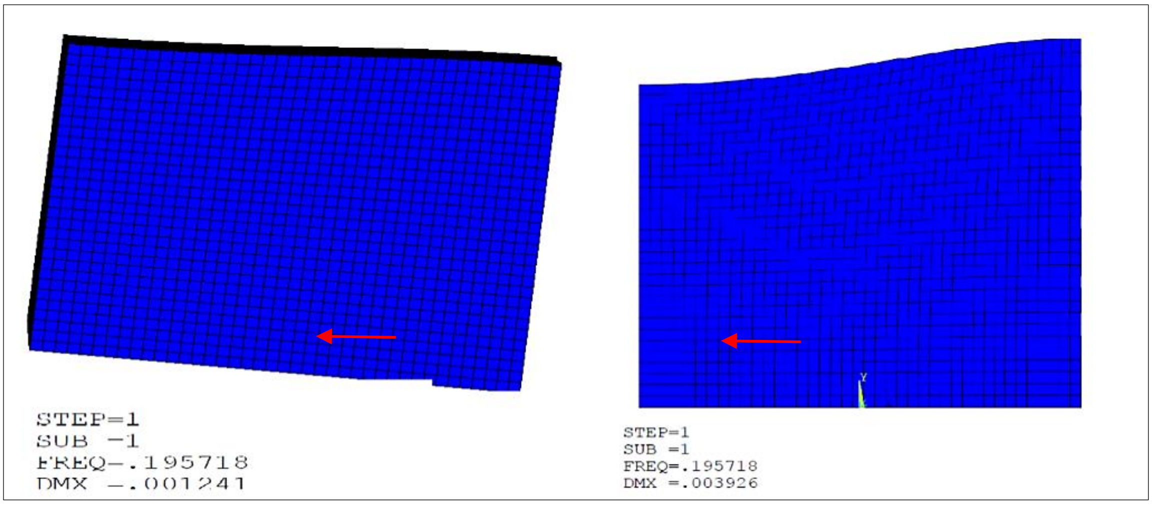

Figure 2. 2D (right) and 3D (left) natural sloshing frequency (1/sec) of a rigid wall vessel

The basic model of current study was a 10×10 meter vessel by using real wall thickness and 4 meters of liquid height. It made in FEM software to find the natural sloshing frequency. The elastic modulus of wall concrete is selected $2.24e10\ Pa$, the poison ratio of concrete is assumed 0.25 and the density of concrete is assumed 2500 kg/m^3. Results of this case of vessel analyzing by using regular assumption of wall thickness (20 cm) and fixed base are reported as following. The maximum horizontal displacement of liquid and body of vessel occurred on corner of the vessel wall. Furthermore, it is observed that by using flexible wall maximum settlements of liquid occurred on top of the liquid surface. In Figure 3. the maximum horizontal and vertical displacement of liquid and vessels body is shown.

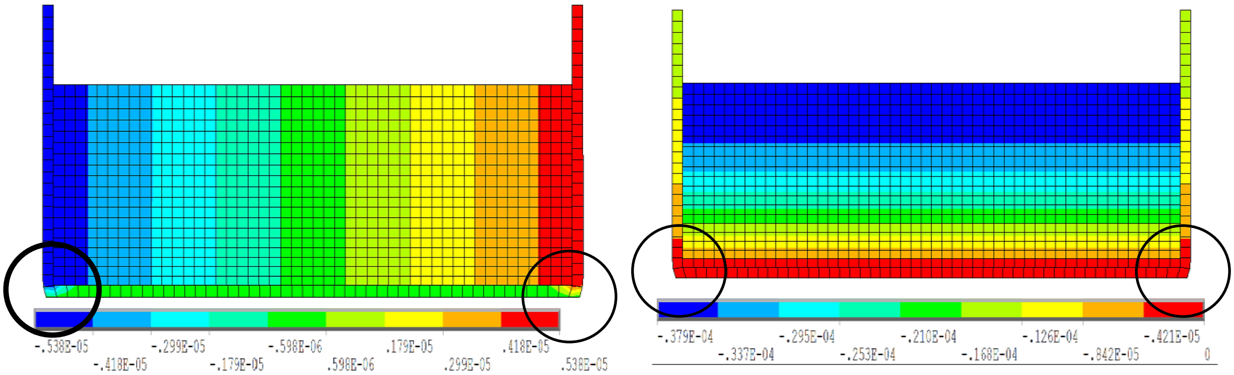

Figure 3. Flexible wall total displacement right: vertical displacement (m) of liquid and wall; left: horizontal displacement (m) of liquid and wall

Fixed base vessels with rigid walls are also made by using numerical finite element software. It is observed that there are no horizontal and vertical displacement for rigid walls and its inlet liquid. The vertical and horizontal displacement of fixed base rigid wall vessel condition shows in Figure 4.

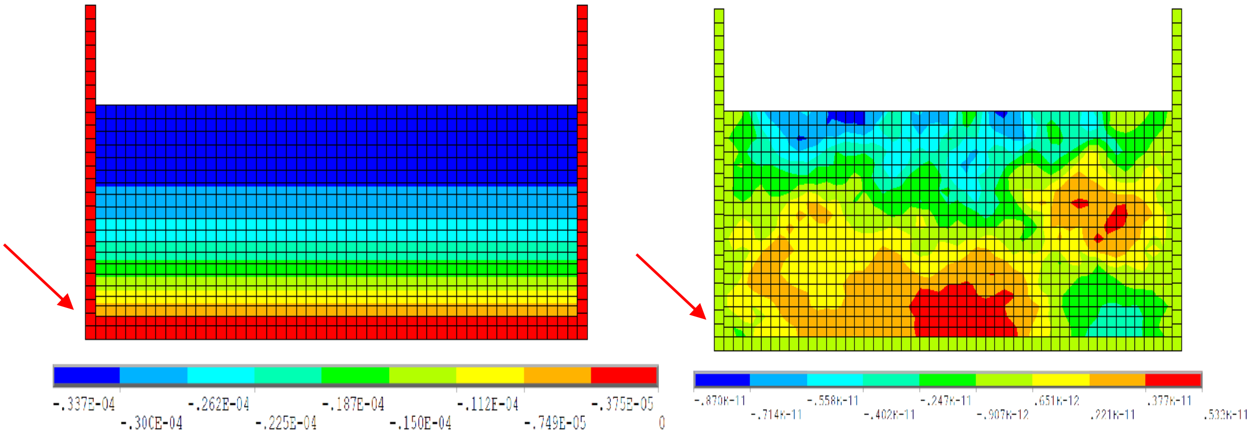

Figure 4. Rigid wall displacement right: horizontal displacement (m) of liquid and wall; left: vertical displacement (m) of liquid and wall

Regarding flexible vessel walls and fixed base condition; the values of liquid settlement considering gravity, use of different martial mechanical property for wall body which has been shown in Figure 5. Results show that there are no important settlements of liquid by using rigid wall vessel. In addition, there are significant differences between the 10 to 25 cm walls thickness. Current study results show that the maximum deformation which affected from this settlement is occurred in near the base of the vessels and the end of vertical walls.

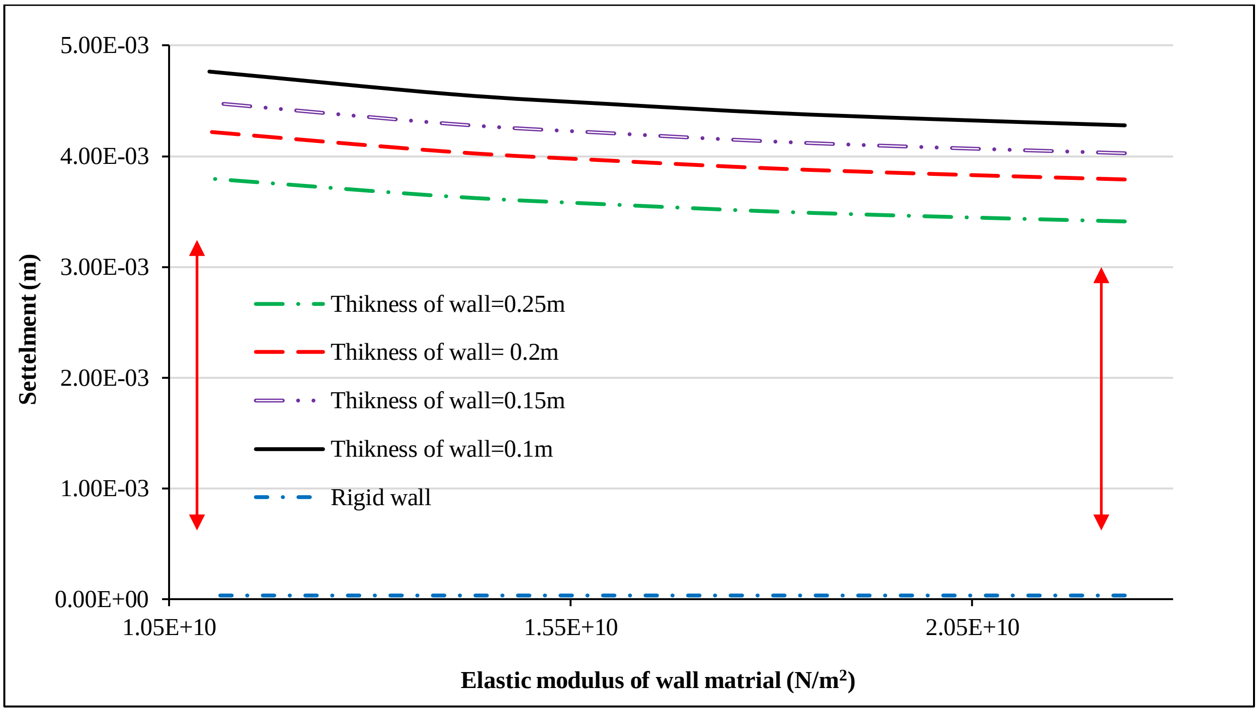

Figure 5. Liquid settlement values considering wall thickness and wall martial mechanical properties

To find the natural frequency of the vessel the block lanczos analysis by using finite element model is done. It is observed that the natural sloshing frequency has no dependency to vessels wall flexibility in the fixed base condition. The finite element analysis (Figure 6.) used for both of flexible and rigid wall considering fixed base condition reported that, the extracted natural sloshing frequency is 0.263 (1/sec) for all assumed. While by using the international code ACI-350 recommended technical method as a result, it is concluded that natural sloshing frequency is 0.26 (1/sec).

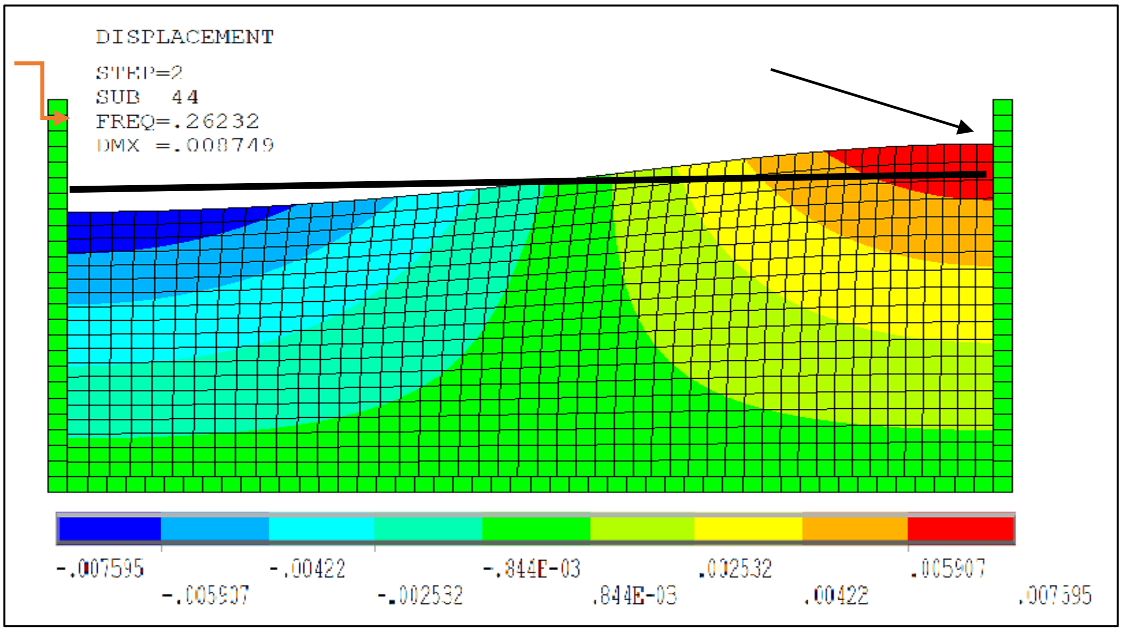

Figure 6. Natural sloshing frequency (1/sec) mode of flexible wall vessel considering fixed base condition

The SSI analysis through the combination of kinematic and inertial interactions has been evaluated using a direct method configuration in this paper (Figure 7.) [31-33]. The direct method estimation allows the experimenter to analyze the entire soil foundation- structure system as a complete system in a single step, in which the free field input motions are specified along the base and sides of the model [4, 34]. By using the direct method of soil structure interaction theory, the FSSI model of vessels, by considering the rigidity and flexibility of wall, are made. In numerical finite element model, the interface elements are defined between the bottom of vessel walls and the surface of soil domain. Current study assumed that the soil has elastic and linear behavior. For each of introduced soil category in Table 2. the natural sloshing frequencies are evaluated. The concept of finite element model which uses to find natural sloshing mode shown in Figure 7.

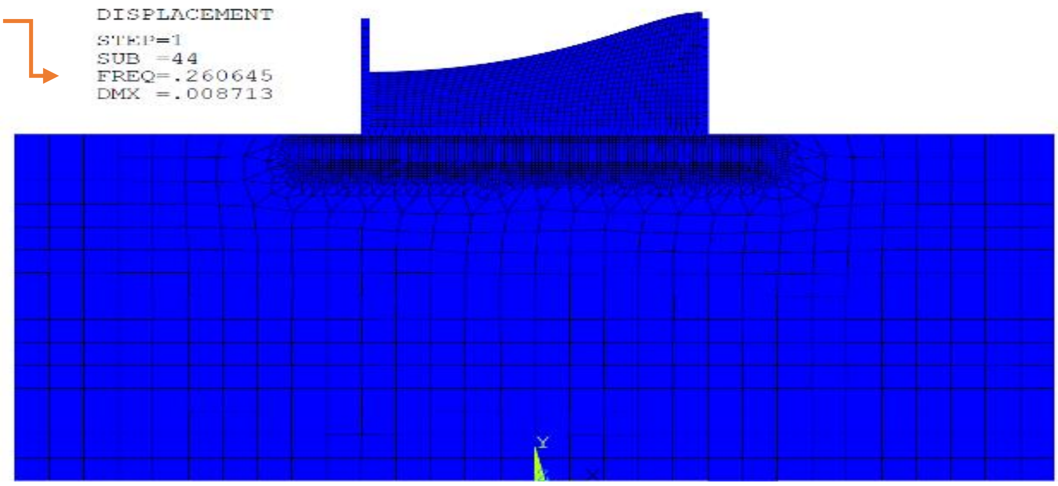

Figure 7. The numerical direct method of Fluid- structure- soil interaction model of vessel

Regarding numerical finite element theory the natural sloshing frequency of flexible vessels wall with reference to soil category condition is shown in Figure 8. It is observed that, by softening the soil condition, the natural sloshing frequency will be reduced. Furthermore, achieved results show that by softening the soil condition the maximum vertical displacement of liquid will be reduced. Pervious researcher (Goudarzi and Sabbagh yazdi 2008) regarding analytical and numerical method using showed that, the increasing of the maximum height of sloshing wave by decreasing the natural sloshing period will be occurred.

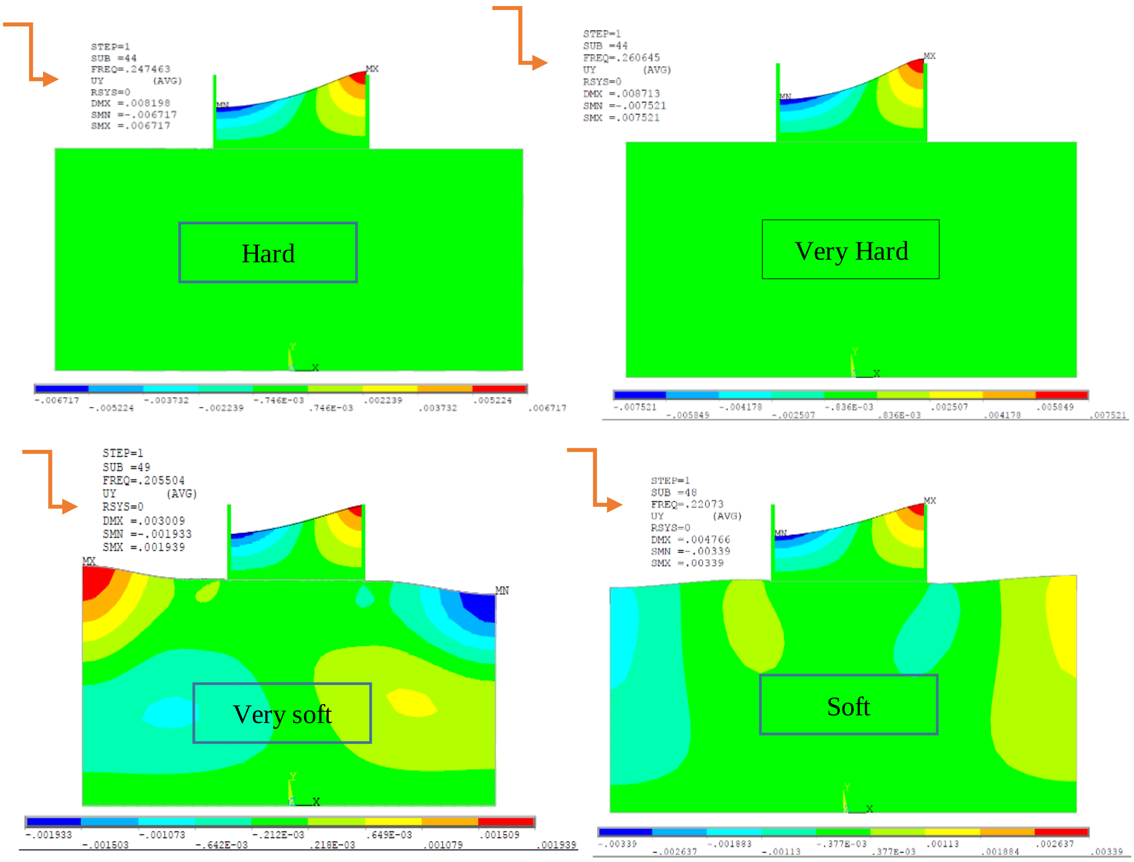

Figure 8. Sloshing natural frequency (1/sec) modes extracted from numerical analysis (direct method) considering FSSI by using flexible wall (0.2 m) condition

The rigid wall vessel uses numerical model theory SSI and shown in Figure 9. Regarding soil category changes, the natural sloshing frequency of this kind of vessel is achieved.

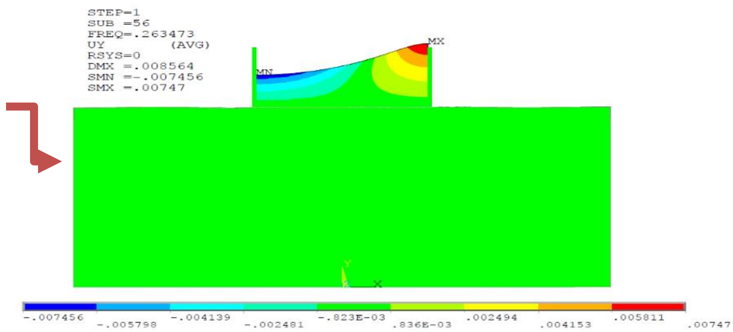

Figure 9. Sloshing natural frequency (1/sec) modes extracted from numerical analysis (direct method) considering FSSI by rigid wall condition

In conclusion, results of flexible and rigid wall vessel considering fixed base and SSI theory are shown in Figure 10. It is observed that, there are negligible differences between results of international ACI-350 code and numerical FSSI considering rigid wall and fixed base condition. The numerical FSSI analysis results show that there are no significant differences between results of FSSI considering rigid walls, international code (ACI-350), and numerical FSI. Results of FSSI (rigid wall) show that there are significant differences among this method analysis, FSI and international code which extracted from soft and very soft soil. By changing the soil stiffness from hard to soft soil, the natural frequency of FSSI (rigid wall) 7% reduced. It is observed that the natural frequency would be decreased by reduction the soil stiffness.

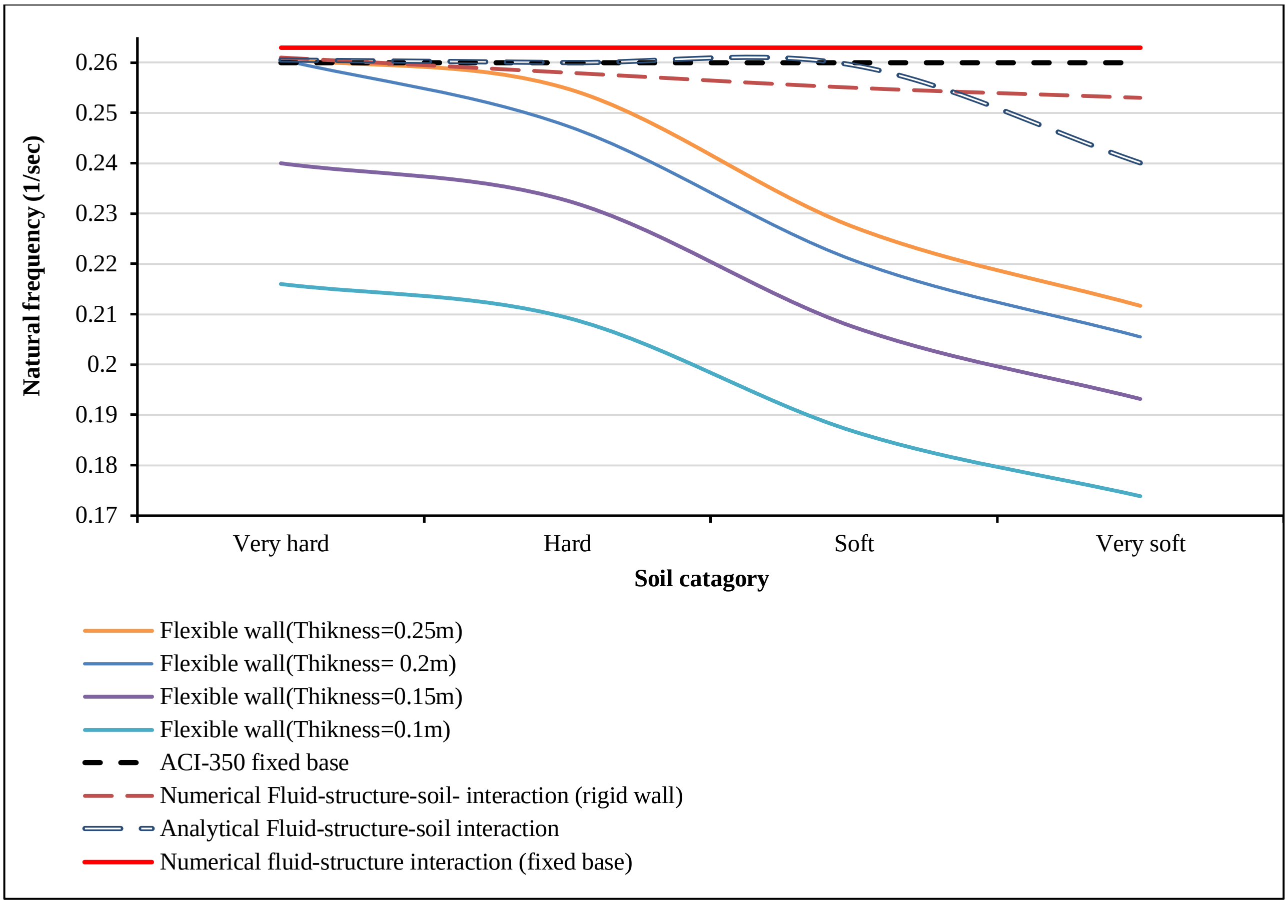

Figure 10. The natural frequency values of vessel considering SSI effects and wall condition

In conclusion this study shows in hard and very hard soil category use of 0.2 and 0.25 meters wall thickness, the extracted natural sloshing frequency are approximately same. However, by changing the walls thickness to 0.15 and 0.1 meters it is concluded that, hard and very hard soil reduced compare with thicker walls.

Extracted results of Figure 10. show that natural sloshing frequency of inlet liquid considering SSI and use of the rigid walls analysis are concluded by below developed relation:

$$f_{SSI} = f_{Fixed\ base}\left(1 - G(k)\right) \tag{6}$$

Where $f_{Fixed\ base}$ and f_{SSI} are natural frequency of fixed base condition and natural frequency considering soil-structure interaction. In addition G (k) is a semi linear mathematical function which dependent to ground stiffness (k). The values of this function would be zero for very hard soils and would be changed to 0.1 in flexible soil.

In the event of flexible walls vessel, the developed relation is changed to following equation:

$$f_{SSI(Flexible\ Wall)} = f_{SSI}(1 - S(k)) \tag{7}$$

Where $S(k)$ is a nonlinear mathematical function which shows effects of vessel wall flexibility considering the natural sloshing frequency. The $S(k)$ value depends to the wall thickness, wall stiffness, and soil stiffness. For a concrete wall by thickness of 0.1 to 0.3 m, results show that the values of $S(k)$ are filled among 0.1 to 0.5 In Figure 11, participation values of SSI and wall flexibility effects on natural sloshing frequency shown.

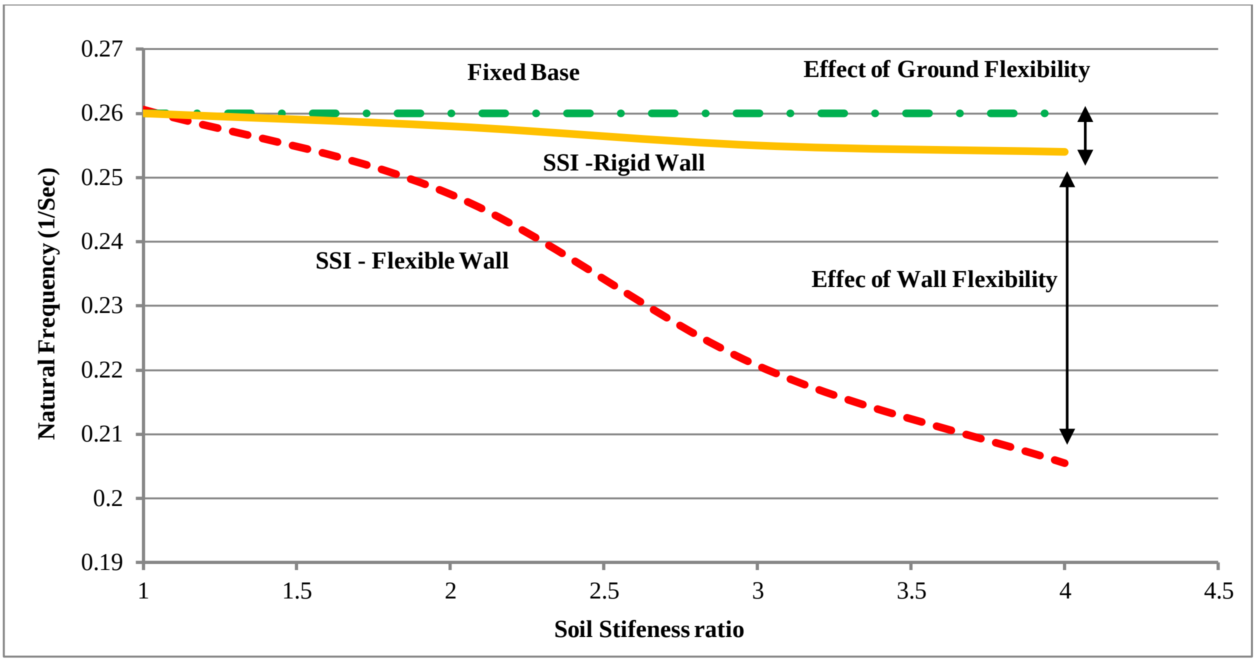

Figure 11. Soil structure interaction and wall flexibility, participation effects on natural sloshing frequency of vessels

4. Conclusion

The effects of rigid and flexible walls considering SSI by using numerical, analytical, and international codes recommendation are evaluated. With compare the concluded values of natural sloshing frequency which extracted from different analysis types, two newly efficient related to estimate the natural sloshing frequency considering both of wall flexibility and FSSI effects are developed. The following results are concluded from current study evaluation:

- It is concluded that the results of natural sloshing frequency considering 2D and 3D numerical modeling have no significant difference. To simplify the vessels modeling, the 2D models have good estimation of natural sloshing frequency.
- Regarding the fixed base condition for vessels, results of this study show that there are no significant effects of flexible or rigid walls on the natural sloshing frequency values.
- Results of liquid settlement considering gravity show that, by increasing the wall stiffness maximum vertical displacement of liquid reduced. It is noted that there are no settlement of liquid in rigid walls vessel. The numerical software analysis reported that non compressibility of water liquid assumption has a good scientific credibility.
- Regarding numerical results, it is observed that the SSI has a significant effect on natural sloshing frequency. More precise results show that wall flexibility considering SSI effect is effective on natural sloshing frequency. Results of flexible walls vessel show the natural sloshing frequency reduced considering soil softening.
- The results of rigid and flexible walls with considering FSSI show that the rate of SSI effects is lower than wall flexibility in case of soft soil. In case of hard and very hard soil, results show that both of wall flexibly and SSI effect have no significant effects on natural sloshing frequency.
- The ACI-350 analysis results show that the natural sloshing frequency value, which extracted from international code, are in the same as the rigid walls vessel, which extracted numerical FSSI.
- Current study results show that, the international codes have no good estimation of natural sloshing frequency considering soft soil and flexible walls effects. To find the real dynamic behavior of inlet liquid to modify in relation to find to find the wall flexibility and SSI effects on natural sloshing frequency.
- To find FSSI effects considering wall flexibility, two new relations are developed. First relation use of the G (K) only shows effects of soil structure interaction on sloshing natural frequency. To find both effects of wall flexibility and SSI on natural sloshing frequency the secondly new relation use of S (K) is developed.

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