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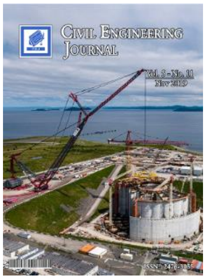

Effectiveness of Connections Type on Vibration Response of Steel Beam

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Abstract

In a steel structure, choosing the connections type are one of the most important parameters in design consideration. How a connection type affects the vibration of steel beams has been investigated in this paper. The most effective connection type in reducing beam vibration has been highlighted. The study was conducted using different finite element models to simulate each connection type. Firstly, the model was validated by comparing its results with the results obtained by the analytical approach. In the numerical model, a linear frequency analysis was performed to determine beam natural frequency, then it has been compared with the corresponding value obtained by the Euler-Bernoulli approximations for simply supported beams. After that, two analysis procedures have been executed, steady-state analysis and transient analysis. In the steady-state analysis, a harmonic load with different frequencies was applied to the beam mid-span, while an impulsive load has been applied in the transient analysis. The results indicate that the deflection could be reduced by 72%, furthermore steady vibration of the beam can be reduced by 81% with using one of the moment connections instead of the traditional shear connection.

Keywords: Vibration Analysis; Steel Beam; Finite Element Modeling; Steady State Analysis; Transient Analysis.

1. Introduction

The full potential of structure composing material is the main goal of structural engineers for a long time. The modern construction techniques led to use as the highest strength to weight performance as possible. A direct consequence of this new design trend is a considerable increase in problems related to unwanted floor vibrations. For this reason, the structural floors systems become vulnerable to excessive vibrations produced by impacts or other sources [1]. The main objective of this research is to present the effectiveness of connection type in reducing beam vibration. Where the incorrect choice of connection type in steel structure could be led to serviceability problem. Therefore, the most effective connection in reducing beam response to vibration has been highlighted in this work. The investigation was done through an extensive finite element modeling using ABAQUS software. Steady-state and transient analyses have been performed to measure the connection effect on beam response to vibration. The model was validated by comparing the beam natural frequency obtained from finite element simulation with the corresponding value obtained from the traditional analytical approach after that multi connections were simulated and its response to vibration was pointed out.

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2. Literature Review

2.1. Vibration

The vibration problem is one of the most serious serviceability issues caused by design optimization where lightweight members and high strength materials are required. Even though the vibration issues does not refer to strength problems, it can cause human discomfort and rise serviceability problems that make the structure unusable, The dynamic aspects must be considered in detail [2]. Therefore, in many times vibration controls over strength and deflection [3].

Reducing vibration effects has interested many researchers such as Zainulabidin and Jaini (2013) [4] where they tried to reduce the vibration by attaching a dynamic vibration absorber (DVA) to a fixed end beam. The study highlighted the exited beam response at resonance and how the absorber can reduce the vibration successfully at resonance and other frequencies.

Damping system can be used to minimize vibration problems as mentioned earlier by Zainulabidin and Jaini (2013) [4]. Since the damping system can be expensive, therefore it is recommended to avoid them by change design methodologies to minimized vibration [3], and that is the main goal of this research.

Following some related researches that concern about the improvement and behavior of steel structure response excited due to vibration;

- Málaga-Chuquitaype and Ilkanaev (2018) presented the improvement of incorporate resonators in new novel beam configuration and how it can enhance the vibration comfort response of timber flooring systems while retaining the original environmental benefits of wood in construction by using finite element simulation, the results pointed out the dynamic advantages of using the new novel beam; decreasing the peak displacement has been noticed when the steady-state analysis was conducted. Furthermore, when performing a hammer impact test, the acceleration at mid span has been decreased significantly [5].
- Varsha et al. (2017) studied the effect of including damage/cracking of the materials in the dynamic analysis for steel structure by using finite element simulation, where a cantilever beam with I section profile was tested. The results indicated that the damage/cracking could alter the dynamic properties of the steel beam like its natural frequency, mode shapes, and stiffness especially when the crack is located near the support [6]. Vader and Raikar (2017) concerned with that conclusion, where they studied the behavior of fixed ends composite beam, by using numerical analysis [7].
- Ghodge et al. (2018) investigated the dynamic behavior of cantilever and simply supported beams with different materials (gray cast iron, structural steel, copper alloy, and aluminum alloy), by using finite element simulation to obtained the beams natural frequencies. The results pointed out that; the structural steel has the highest natural frequencies [8].

To improve dynamic connection behavior a new beam to box column connection has been proposed by Rezaifar and Younesi (2016) [9]. The connection consists of eight stiffeners in line with beam flanges and five horizontal bar mats in concrete-filled tube columns, the results pointed out an increase in strength and rigidity by 8.08% and 3.01% respectively, furthermore, an improvement in ductility and energy dissipation capacity has been noticed.

In a multi-story building, the load position could affect the dynamic properties as presented in Chandravanshi and Mukhopadhyay (2013) studies [10], where they noticed that the natural frequency of the building is higher when the load is applied on first floor than if it is applied in the second floor. Furthermore, if the load is applied at the second floor the natural frequency is lower than the unloaded structure.

The impact of mechanical equipment (such as electrical generators and compressors) on steel structure production platform has been investigated in Guilherme et al. (2016) [11], where a numerical analysis was performed to study the dynamic behavior; a steady-state analysis was conducted, where the effect of vibration on human comfort has been presented.

The effect of vibration on material properties has been investigated in Jadhav et al. (2016) studies [12], where they performed a harmonic analysis to steel plates, the material properties such as elastic modulus and stiffness have been determined in terms of natural frequency, where they experimentally tested a free ball impact on steel plates. The tests results indicated that modulus of elasticity, stiffness, and fundamental frequency remains approximately the same under various impact energy.

Aggogeri et al. (2017) investigated the use of hybrid material to reduce vibration. The selected material has high dynamic characteristics and capacity to damp mechanical vibrations. They studied the dynamic properties of Al Foam sandwiches (AFS), Al Corrugated sandwiches (ACS), and materials reinforced by carbon fibers (CFRP). The properties were evaluated using experimental tests and numerical analysis. The results indicated that; the AFS could give damping coefficient 20 times greater than the conventional steel, the CFRP could reduce weight by 48.5% and satisfy the requirements, and finally the ACS presents a good trade-off between damping and stiffness [13].

Soltani et al. (2016) evaluated the natural frequency of non-prismatic beam sitting on an elastic foundation by using the finite difference method. The work objective was to simplify the fourth-order differential equation of motion, thus new coefficients have been presented within a relative error of 0.1%–0.3% [14].

2.2. Connections

The structural members rarely fail; most failures are caused by poorly designed connections. Therefore, the connections of steel structures are one of the most important parameters in design consideration [15]. One way to choose the connection is to determine the force direction acting on fastener [16]. Following are some of the main connection types.

a. Shear-Resisting Connections

When the connections between members transmit shear only with almost no moment it is referred to as shear connection and can be treated as simple support [15]. As shown in the shear connection of Figure 1; the beam flanges are not connected to the column, and the web connection is designed to be flexible enough to allow some relative rotation at the joint. Only a very small rotation is necessary for a connection to be treated as pinned [15].

The failure modes for such connection was investigated by Liang (2018) [17] where they tested the connection up to failure. They recommended to consider the following failure modes; shear fracture in the connecting plate, holes bearing failure, net section fracture, and weld fracture if it is used.

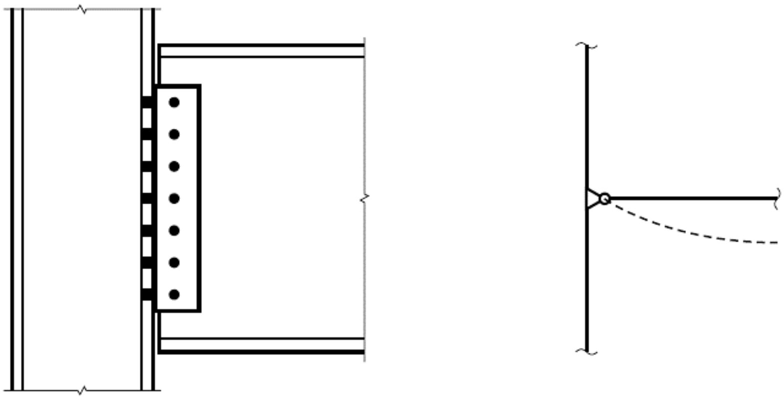

Figure 1. Small rotation in the shear connection [15]

b. Moment-Resisting Connections (Welded Flange)

If the beam flanges are connected to the column via welding as shown in Figure 2; moment can be transmitted in addition to the shear. This kind of connections is referred to as moment connection [15].

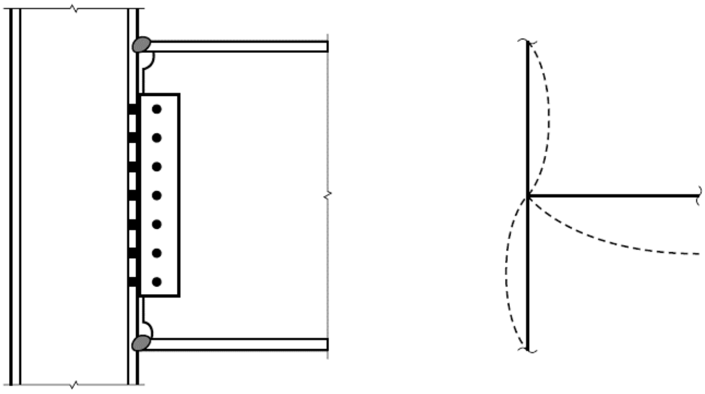

Figure 2. Moment resisting connection [15]

c. Moment Resisting Connection (Bolted Flange)

Another kind of moment connection is shown in Figure 3, where the beam flanges are attached to the plates via bolts [15].

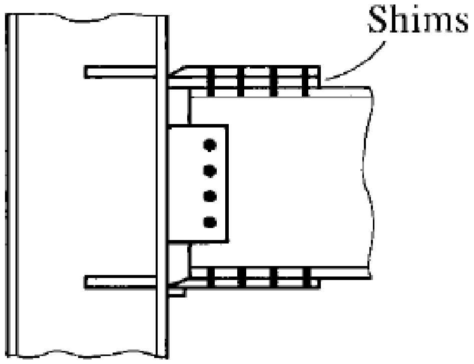

Figure 3. Moment resisting connection (bolted flange)

d. End Plate Connections

In some cases, a plate is welded to the beam ends and bolted to the columns or another beam as shown in Figure 4, this kind of connections is referred to as end plate connection. However, if the web is only connected it can be treated as a shear connection and if the flanges are welded to the plate, a moment will be transmitted through this connection [15].

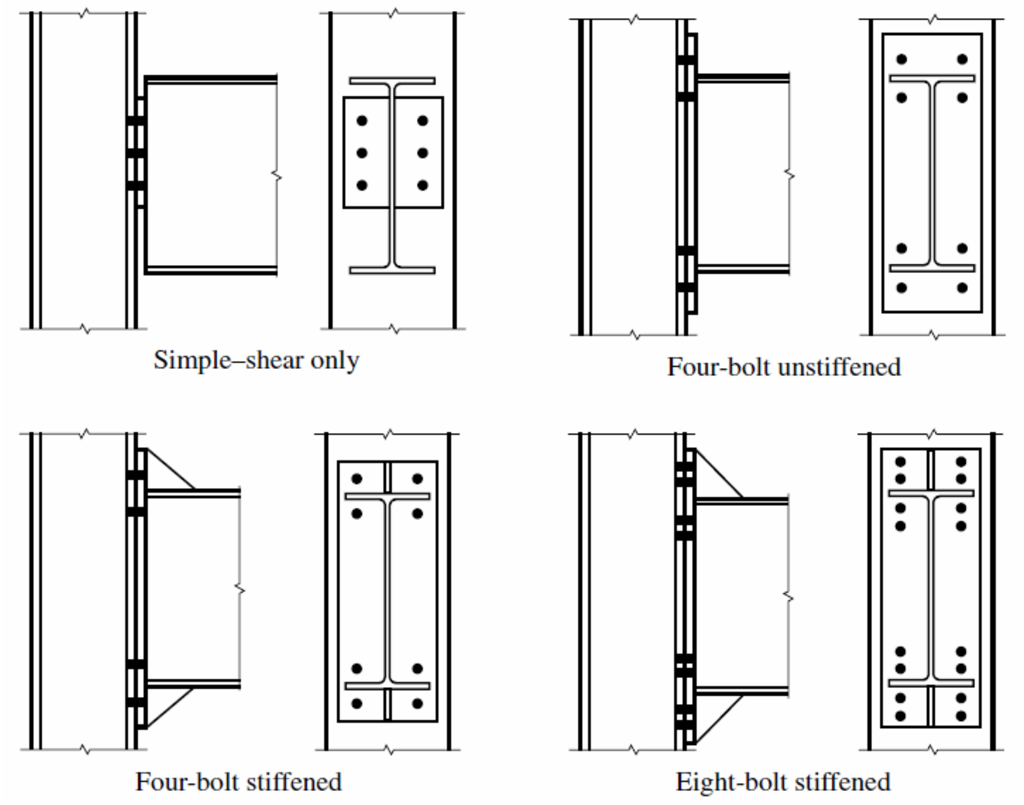

Figure 4. End Plate connections [15]

3. Finite Element Modeling

For the numerical analysis, the steel beam was simulated in a 3D space. The shell elements were adopted to simulate the beam web and flanges as well as all connections of interest. The vibration study was conducted with two analysis procedures as follow.

3.1. Analysis Procedures

e. Steady state analysis

The Steady-state dynamic analysis can predict the linear response of a structure subjected to a continuous harmonic excitation. Steady-state linear dynamic analysis in Abaqus uses a set of eigenmodes to measure the solution progress as a function of the frequency for certain excitation. Abaqus offers two procedures; "direct" and a "subspace" steady-state linear dynamic analysis, wherein the direct procedure the equations of motion are solved directly without eigenmodes usage, while in the subspace procedure, the equations of motion are dependent in the preselected eigenmodes. These multiple options are projected for; frequency-dependent system, when the damping is included, or when the governing equations are not symmetric [18].

The projection of the equations of motion of the system onto the αth mode gives

$$\ddot{q}_\alpha + c_\alpha \dot{q}_\alpha + \omega_\alpha^2 q_\alpha = \frac{1}{m_\alpha}(f_{1\alpha} + if_{2\alpha}) \exp(i\Omega t) \quad (1)$$

Where q_α is the response amplitude at mode α, c_α is the damping, ω_α is the undamped frequency, m_α is the mass, and $(f_{1\alpha} + if_{2\alpha}) \exp(i\Omega t)$ is the force associated with the mode (α) and the frequency Ω [18].

$$q_\alpha = H_{0\alpha} f_{0\alpha} \exp i(\Omega t + \Psi_\alpha) \quad (2)$$

Where $f_{0\alpha} = \sqrt{(f_{1\alpha})^2 + (f_{2\alpha})^2}$ is the projected load vector amplitude, $H_{0\alpha}$ is the transfer function amplitude for the associated mode, and Ψ_α is the response phase angle [18].

The steady-state dynamic analysis provides the steady-state amplitude and phase of the response of a system due to harmonic excitation at a given frequency. Usually, such analysis is done as a frequency sweep by applying the loading at a series of different frequencies and recording the response. The mode-based steady-state dynamic analysis has been adopted in this simulation so that the response is based on modal superposition techniques; the modes of the system must first be extracted using the eigenfrequency extraction procedure [19].

Damping is almost always specified for a steady-state analysis [19]. In this work the damping ratio was 2% of the critical damping as it can be adopted for steel structures [20].

f. Transient analysis

ABAQUS conducts the transient response by performing a model dynamic procedure, where the time history analysis is provided for the linear system. The system excitation is specified as a function of time: it is assumed that the amplitude curve is identified so that the excitation magnitude varies linearly within each increment [18].

The following equations are obtained when the model is projected onto the eigenmodes used for its dynamic representation:

$$\ddot{q}_\beta + C_{\beta\alpha}\dot{q}_\alpha + \omega^2 q_\beta = (f_t)_\beta = f_t - \Delta t + \frac{\Delta f}{\Delta t}\Delta t \quad (3)$$

Where q_β is the response amplitude at mode β, α and β indices span the eigenspace, $C_{\beta\alpha}$is the viscous damping, $\omega_\beta = \sqrt{k_\beta / m_\beta}$ is the undamped natural frequncy, $(f_t)_\beta$ is the loading magnitude, and Δf is the change in the f over the time (Δt).

The general solution of the uncoupled system can be written as a particular integral for the loading as follow;

$$\begin{pmatrix} q_t + \Delta t \\ \dot{q}_t + \Delta t \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{pmatrix} q_t \\ \dot{q}_t \end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{pmatrix} f_t \\ f_t + \Delta t \end{pmatrix} \quad (4)$$

Where a_{ij} and b_{ij} are constents depending on the system damping and should be found accordingly.

Equation 4 can be generalized to include the full coupling system, by introducing the splite of matrix C that have digonal and off-diagonal parts as follow

$$C = C_{diag} + C_{off} \quad (5)$$

With that addition, the equation of the uncoupled system can be rewritten as:

$$\begin{pmatrix} q_t + \Delta t \\ \acute{C}\dot{q}_t + \Delta t \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} - b_{11}C_{off} \\ a_{21} & a_{22} - b_{21}C_{off} \end{bmatrix} \begin{pmatrix} q_t \\ \dot{q}_t \end{pmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{pmatrix} f_t \\ f_t + \Delta t \end{pmatrix} + \begin{bmatrix} 0 & -b_{12}C_{off} \\ 0 & 0 \end{bmatrix} \begin{pmatrix} q_t + \Delta t \\ \dot{q}_t + \Delta t \end{pmatrix} \quad (6)$$

Where Ć is given by

$$\acute{C} = I + b_{22}C_{off} \quad (7)$$

The modal dynamic analysis is used in this study to analyze dynamic behavior. Where this procedure uses modal superposition, in which it can be performed only after a frequency extraction. The transient modal dynamic analysis gives the response of the model as a function of time-based on a given time-dependent loading. The structure response is based on a subset of the modes of the system, which must first be extracted using an eigenfrequency extraction procedure. The method is very accurate because the integration operator used is exact whenever the forcing functions vary piecewise linearly with time [19].

The damping coefficient can be defined for all or some of the modes used in the response calculation. As in the steady-state analysis, a damping ratio of 2% of the critical damping was assumed.

g. Geometric properties

An IPE300 steel beam has been chosen to record the beam response due to vibration, the adopted geometric properties are indicated in Table 1 and Figure 5 below.

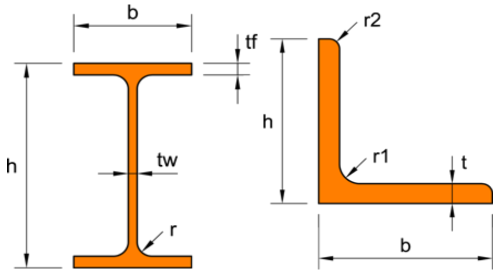

Figure 5. IPE steel beam and angles cross sections

Table 1. Model geometric properties

Part	h	b	t_f	t_w	Length
IPE 300 Steel beam	300	150	10.7	7.1	6000
Angle	100	100	5	5	200

After creating the required parts, the model has been assembled in the assembly module as shown in Figure 6.

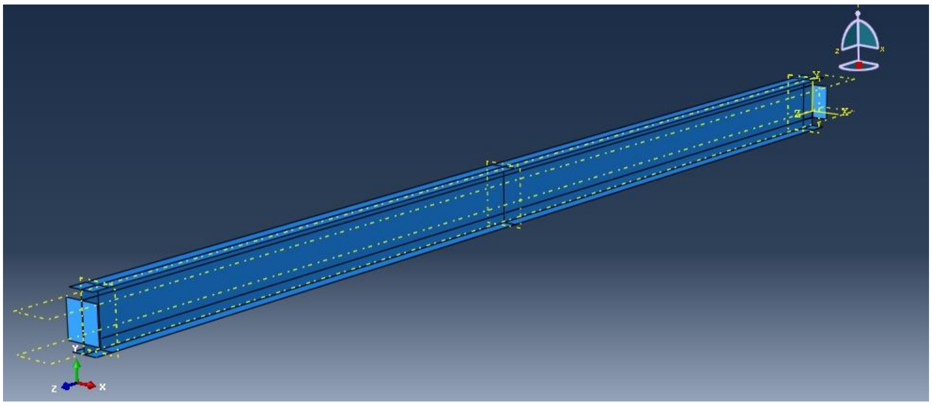

Figure 6. Model assembles

To simulate all connections types of interest, the top and bottom plates were created for the moment connection, as well as the end plate with and without stiffeners have been created in the part module then it was gathered in the assembly module as shown in Figure 7 and Figure 8.

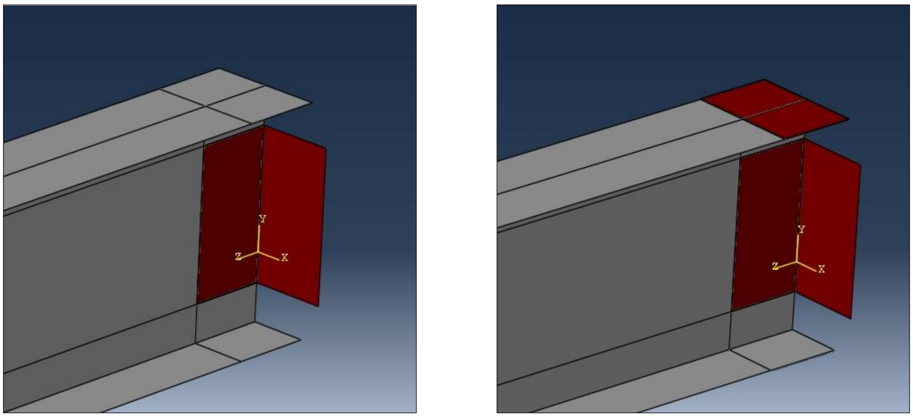

Figure 7. Modeling shear and bolted moment connection

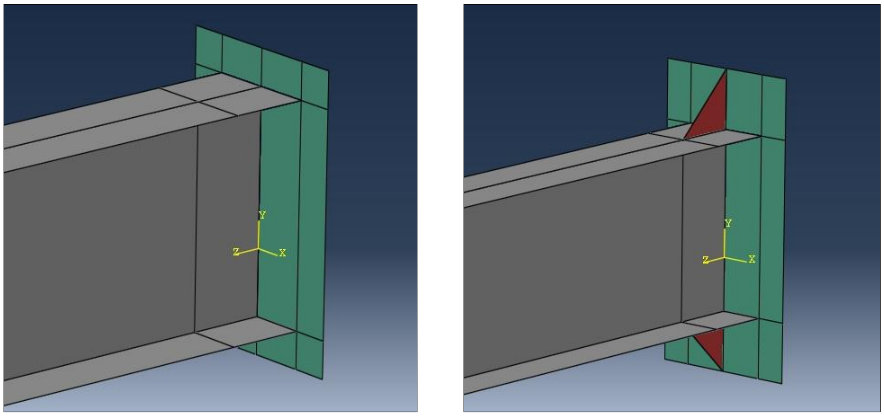

Figure 8. Modeling end plate connection with and without stiffeners

3.2. Material Property

As stated earlier this work aim to study the dynamic response of the steel with varies connections types, therefore, the material behavior was assumed to be limited in the elastic zone with a modulus of elasticity and Poisson ratio equal to 200,000 MPa and 0.3 respectively.

3.3. Loading and Boundary Conditions

At beginning the first model was a beam with simply supported ends where the boundary conditions have been a signed directly i.e. the beams both ends were constrained against the movement in X and Y directions and the movement in span direction (Z-axis) were constrained for one end only. It should be noticed that the simplification of boundary conditions are considered, due to the possibility of high natural frequencies may arise if the model is over-constrained [21]. After assigning the required boundary conditions, a concentrated load was applied at mid-span to measure the dynamic response as shown in Figure 9, the intent of this model is to validate the simulation of the beam when the shear connection end are included in the model.

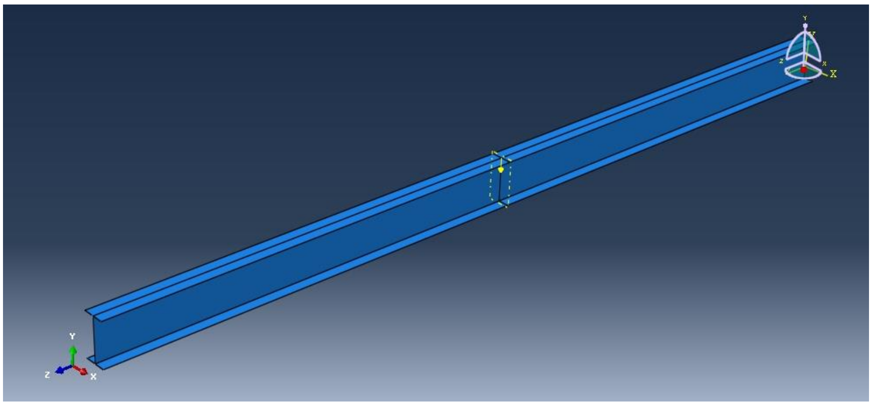

Figure 9. Appling load and boundary conditions

As stated earlier the adopted beam has 6m span length. Therefore, a lateral bracing should be assumed to prevent the unwonted out of plane failure.

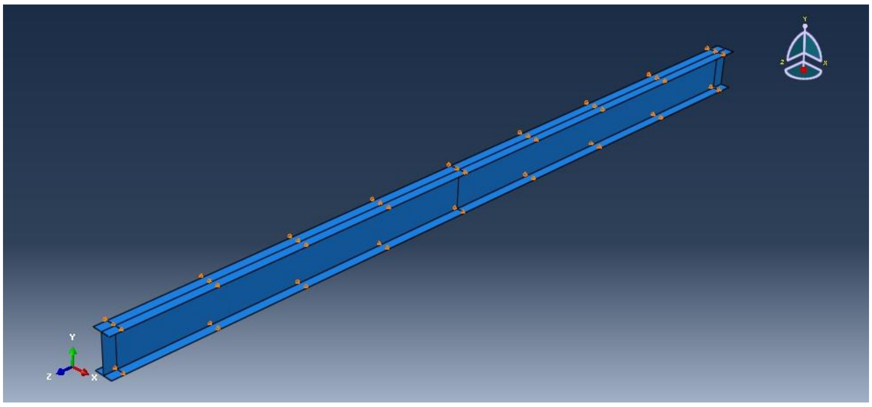

Figure 10. Appling lateral bracing

After that, all models with different connection types were simulated i.e. shear connection, moment connection, and end plate connection. The boundary conditions were assigned accordingly as shown in Figure 11 through Figure 13.

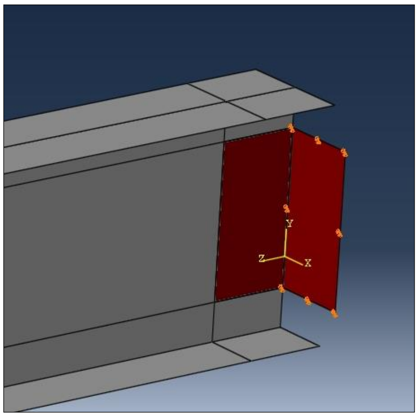

Figure 11. Appling Boundary condition to the shear connection beam

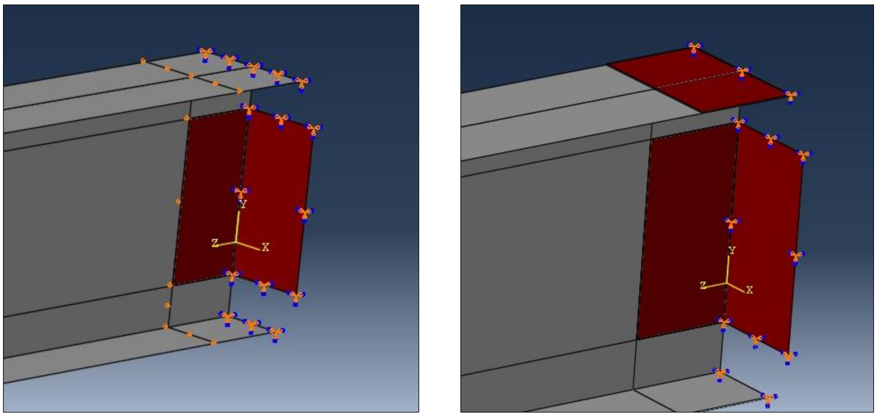

Figure 12. Appling Boundary condition to the welded and bolted moment connection beams

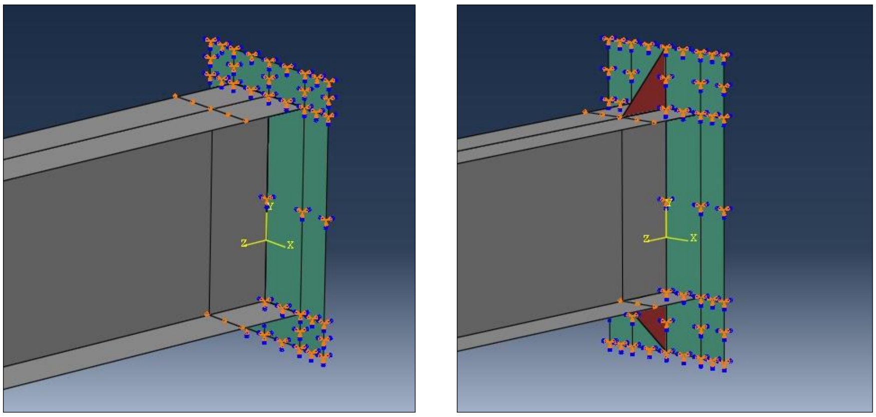

Figure 13. Appling Boundary condition to the unstiffened and stiffened end plate connection beams

3.4. Choosing the Mesh Type and Size

ABAQUS S8R6 quadrilateral shell element with eight nodes and six degrees of freedom per node was selected for the steel beam finite element models as it is capable to provide sufficient degrees of freedom to explicitly model deformations. A mesh size of 75×100 mm is adopted for flanges and web as shown in Figure 14.

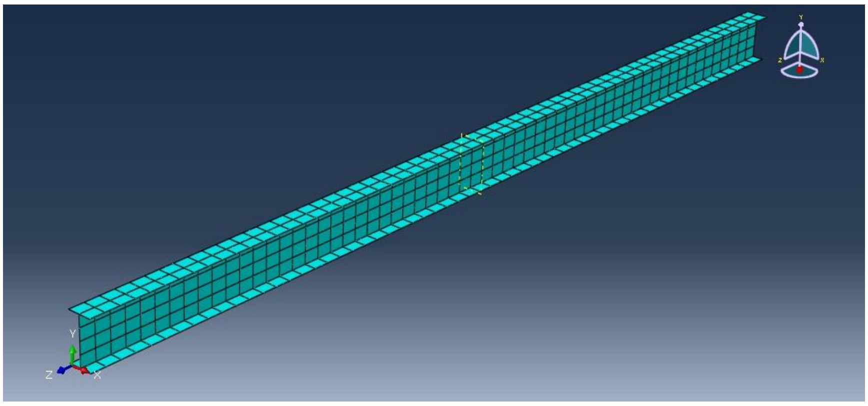

Figure 14. Beam Meshing

4. Model Validation

The proposed finite element model was validated by comparing the beam natural frequency obtained from the finite element analysis and its corresponding value obtained from traditional analytical approach.

In the traditional approach, initial estimates of the vibration frequencies of beams were obtained using Euler-Bernoulli approximations for simply supported beams [22]:

$$\omega = (n\pi)^2 \sqrt{\frac{EI}{\rho L^4}} \tag{1}$$

Where E is the elastic modulus, ρ is the mass per unit length, L is the beam span, n is the mode number, and I is the section moment of inertia. Figure 15 presents the results of the comparison between the two approaches, were the proposed finite element model shows a good agreement with the analytical approach.

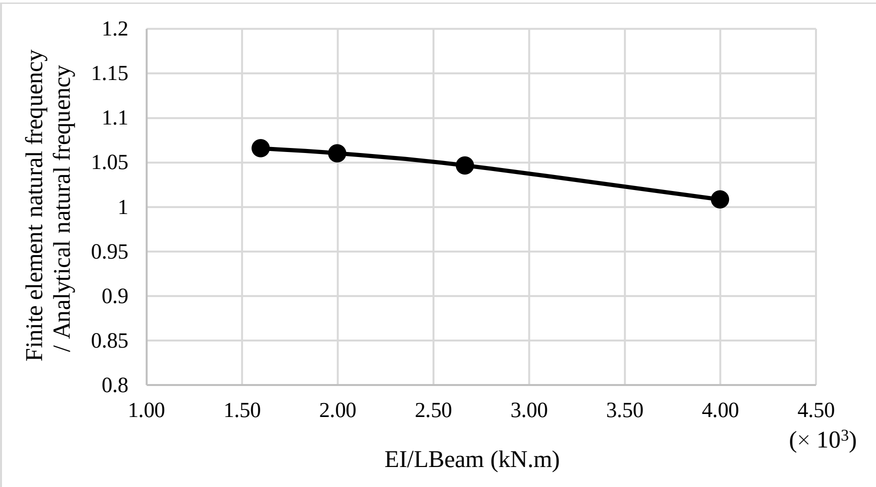

Figure 15. Validation results

5. Results and Discussion

First, linear frequency analysis has been performed for all connection types, a summary of the first mode shape are presented in Table 2 below. The results of frequency analysis concord with Varsha et al. (2017) [6] and Vader and Raikar (2017) [7] finding, where the beam natural frequency has beam changed due to support status. As the connections are contributing more to beam constraint; the natural frequency increases.

Table 2. Frequency analysis results.

#	Connection type	Natural frequency (first mode) cycle/sec
1	Shear connection	28.99
2	Bolted Moment connection	33.58
3	Welded moment connection	61.499
4	End plate connection	58.38
5	Stiffened end plate connection	58.97

All mode shapes and natural frequencies regarding connection types are presented from Figure 16 through Figure 20.

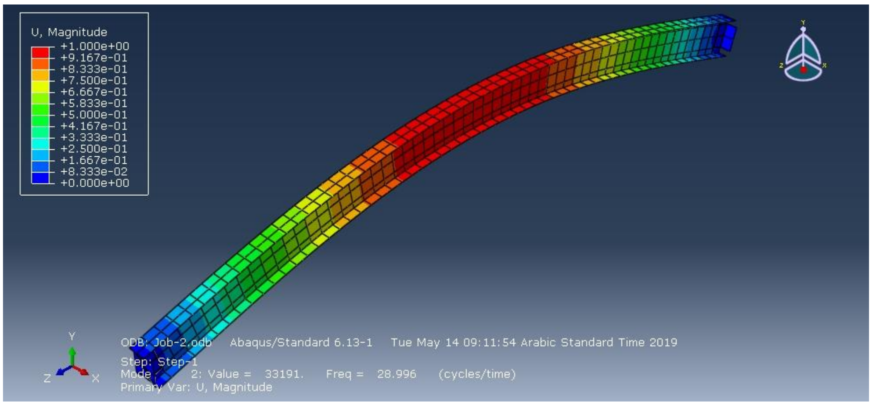

Figure 16. Shear connection

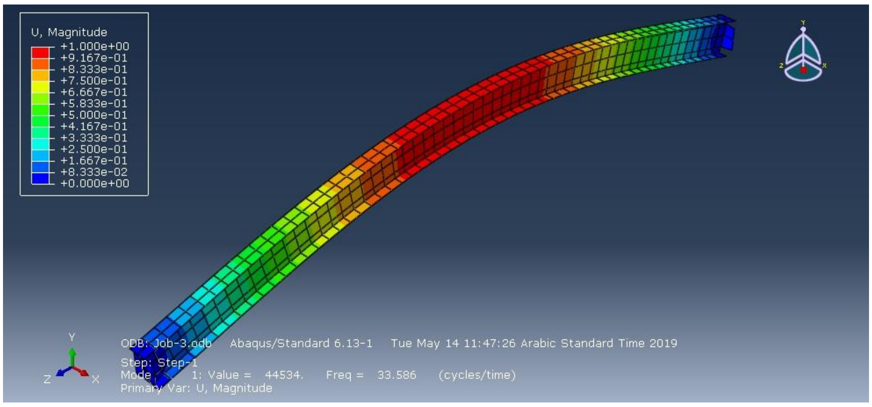

Figure 17. Bolted moment connection

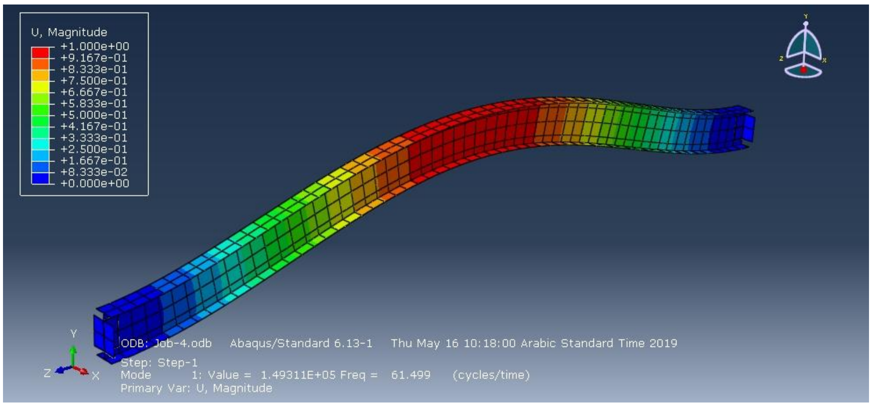

Figure 18. Welded moment connection

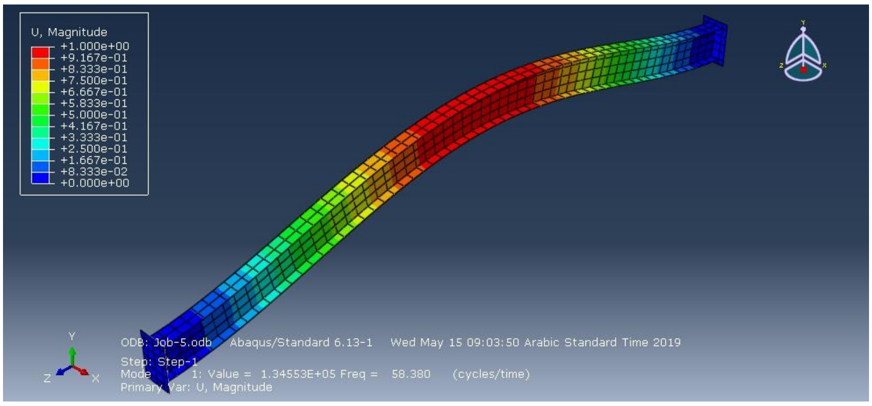

Figure 19. End plate connection

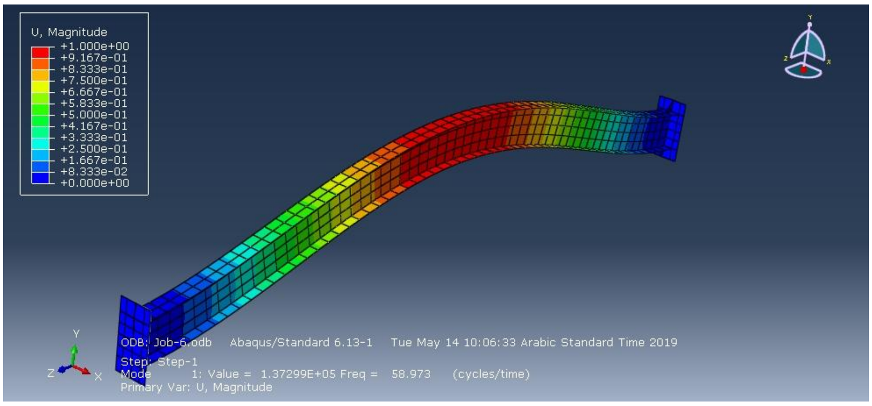

Figure 20. Stiffened end plate

After that and as stated earlier the beam response to vibration will be compared with different connection type and different analysis procedures as follow.

5.1. Steady State Analysis

The response to the steady-state analysis is presented in Table 3 below, which indicates that the beam with moment connection ends and the end plate showed better resistance to vibration when comparing it with the beam that is connected by the traditional shear connection. However, the peak deflection reduced in the beams with bolted and welded moment connection by 22.3% 72.2% respectively by comparing it with shear connection beam, such response is due to simulation differences; in the bolted moment connection, two plates were modeled in the top and bottom beam flanges to simulate this connection, the rotation of beam ends is depending on the plates rigidity, unlike the beam with welded flanges where both flanges have been braced against movement and rotation to simulate welding, therefore this connection will prevent the beam end from rotation and that will give better vibration resisters.

On the other hand, the beams with unstiffened and stiffened end plate connection showed the approximate same response to vibration. However, in both of those connections, the peak deflection also reduced by 70% when comparing it with the shear connection beam.

Table 3. Beam response to the Steady state analysis

#	Connection type	Peak Displacement (mm)
1	Shear connection	300
2	Bolted Moment connection	233
3	Welded moment connection	83
4	End plate connection	90
5	Stiffened end plate connection	88.6

The displacement versus frequency graphs is presented below from Figure 21 through Figure 23. In which it will compare the beam response with the traditional shear connection beams.

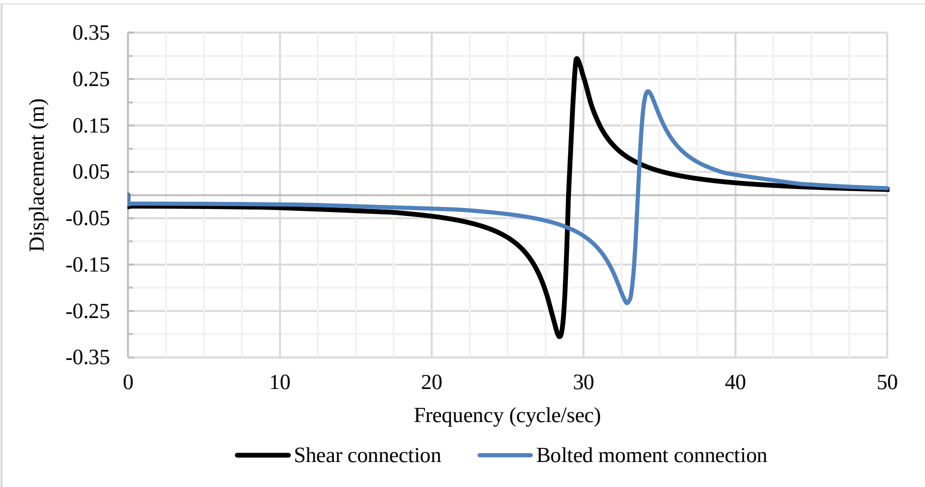

Figure 21. Steady state response for the beams with shear and Bolted moment connection

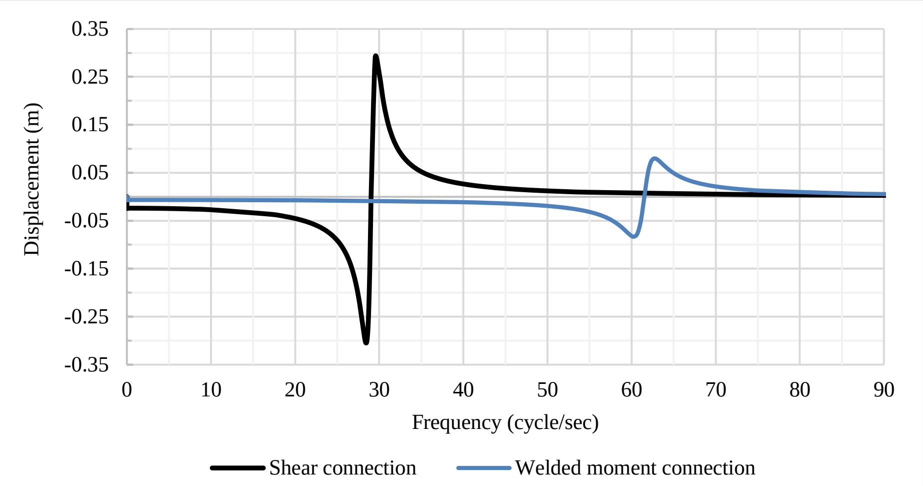

Figure 22. Steady state response for the beams with shear and Bolted moment connection

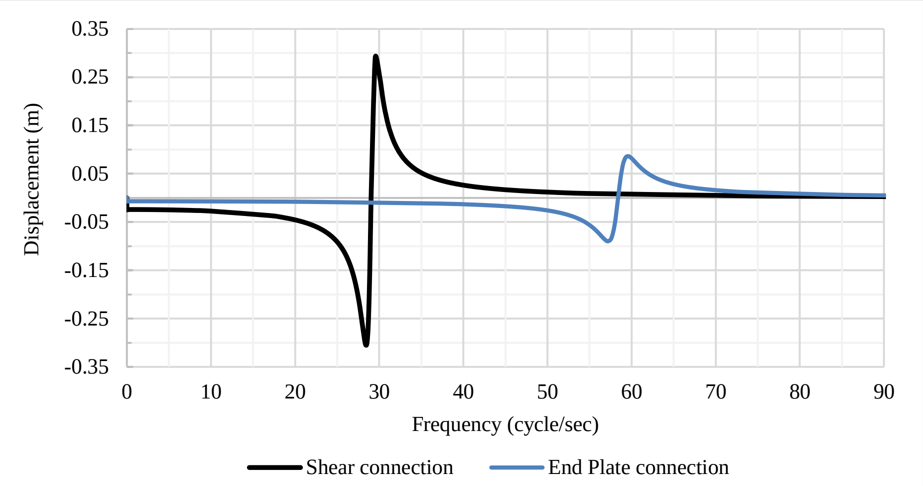

Figure 23. Steady state response for the beams with shear and Bolted end plate connection

5.2. Transient analysis

The responses for all connections types are summarized in Table 4 below, which indicated that the moment connection and end plate connection show better resistance to vibration by comparing it with the traditional shear connection. However, the beams with bolted and welded moment connection approximately stopped vibrating by 25% and 80.9% respectively before the beam with shear connection, and in the other hand the beams with stiffened and unstiffened end plate showed approximate same response. However, both of those stopped vibrating by 81.7% before the beam with the shear connection.

Table 4. Transient analysis response

#	Connection type	Peak Displacement (mm)	Peak Displacement at 0.25 sec (mm)	Peak Displacement at 0.5 sec (mm)	Approximate time to 1mm deflection (sec)
1	Shear connection	36.7	17.5	6.8	1.04
2	Bolted Moment connection	32.8	9.8	3.4	0.782
3	Welded moment connection	9.6	0.7	0.1	0.198
4	End plate connection	15.2	0.665	0.11	0.19
5	Stiffened end plate connection	13.2	0.65	0.1	0.187

The displacement versus time is presented below from Figure 24 through Figure 26. In which it will compare the beams response with the traditional shear connection beams.

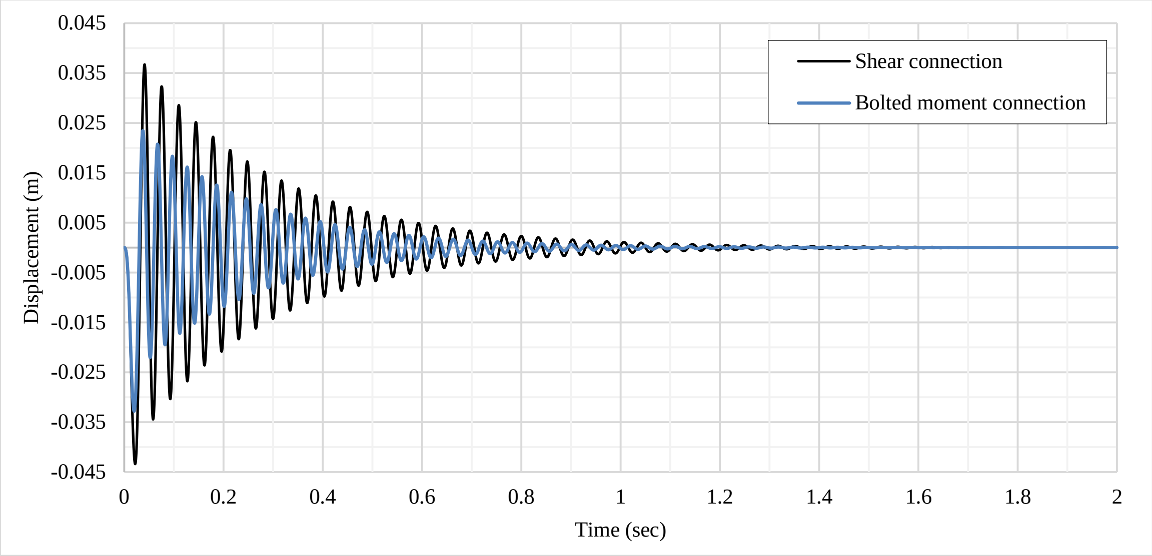

Figure 24. Transient analysis response for the beams with shear connection and bolted moment connection

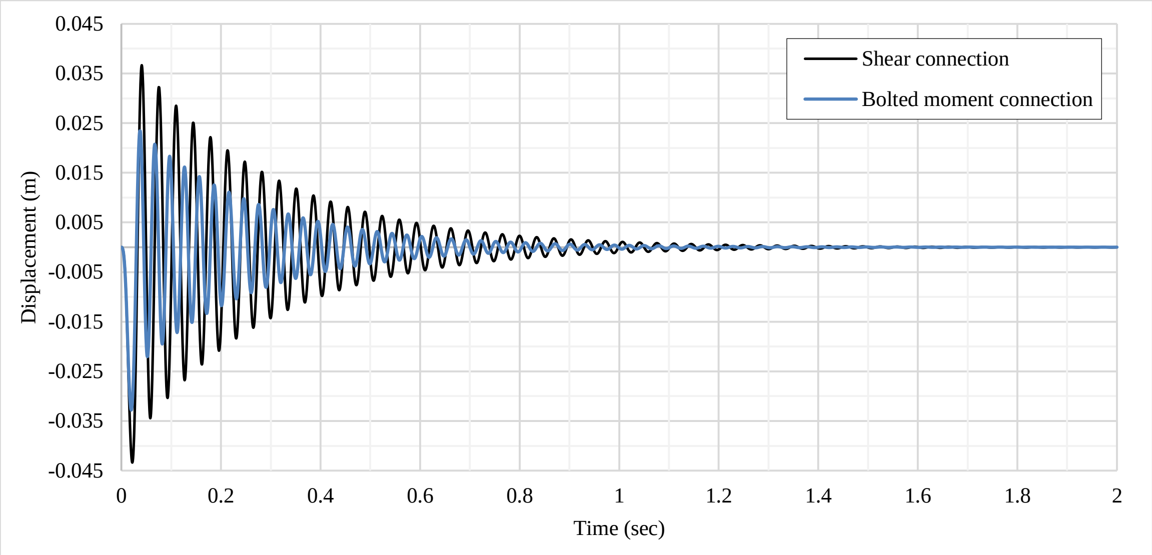

Figure 25. Transient analysis response for the beams with shear connection and welded moment connection

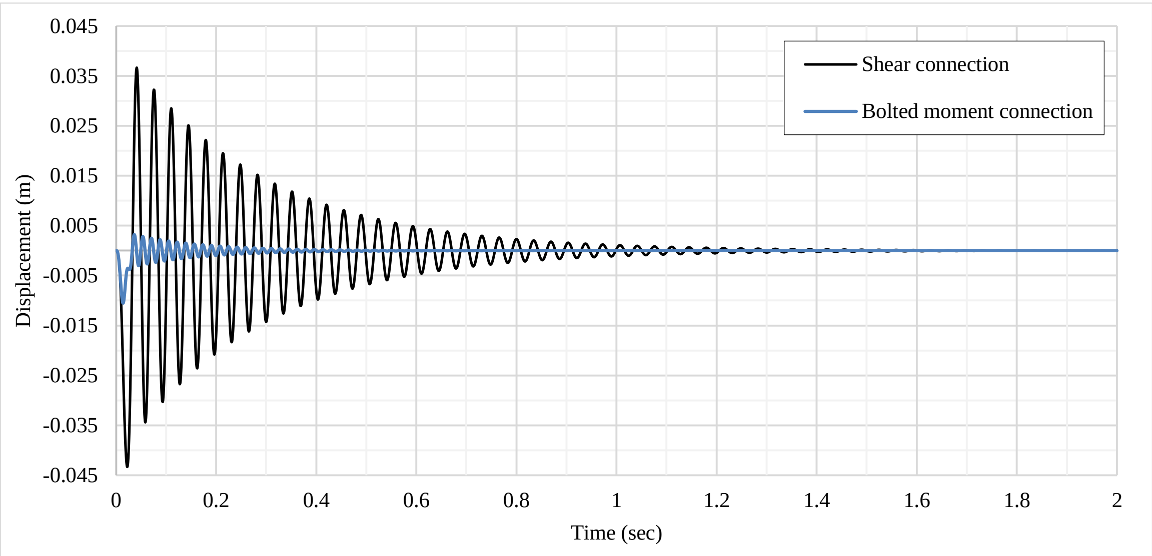

Figure 26. Transient analysis response for the beams with shear connection and end plate connection

The beams behavior can be explained by; in the shear connection the beam is attached to the supports via web only, so that the beam vibrate more freely comparing with the moment connection, where in that connection the supports are attached to beam web and flanges as mentioned earlier thus it can give the beam more stiffness to resist vibration. The welded moment and end plate connections are more effective to resist vibration than the bolted moment connection, since in the first two connections, the beam flanges are restrained against movement completely unlike when the bolted moment connection is used where the beam end is connected to supports via plates, so that the vibration resistance is depending on plate stiffness.

6. Conclusions

The connection types of steel beams subjected to vibration have been investigated in this study by using finite element simulation. The proposed finite element models use shell element to simulate beam flanges and web and use the linear Eigen solver frequency analysis to estimate the beam natural frequency. Two analysis procedures have been performed to record beam response to vibration i.e. steady-state analysis and transient analysis. The model has been verified through comparison its results with the corresponding value from the analytical approach, which has presented a good agreement.

From different case studies and by comparing the response with the traditional beam having shear connection ends, it has been pointed out that; the beam response to vibration is highly affected by connections type. The following conclusions have been drawn:

- The peak deflection in the bolted moment connection has reduced by 22.3% when performing the steady-state analysis.
- When the transient analysis is performed, the beam with the bolted connection ends approximately stopped vibrating by 25% before the beam with the shear connection ends.
- The peak deflection in the welded moment connection has reduced by 72.2% when performing the steady-state analysis.
- When the transient analysis is performed, the beam with the welded connection ends approximately stopped vibrating by 80.9% before the beam with the shear connection ends.
- Different responses have been noticed in the beams with moment connection. The beam with welded flanges presents more stiffness to resist vibration. However, in the bolted moment connection, the rotation of beam ends is depending on rigidity of the plates, unlike the beam with welded flanges where both flanges have been braced against movement, therefore, this connection will prevent the beam end from rotation and that will give better vibration resisters.
- The peak deflection in the unstiffened and stiffened end plate connection approximately have the same response. However, the stiffened end plate reduced the steady-state response by 70%.
- When the transient analysis is performed, the beam with the unstiffened and stiffened end plate connections also approximately presents the same response. However, the stiffened beams approximately stopped vibrating by 81.7% before the beam with the shear connection ends.

The main conclusion is that; connections that differ in their main behavior from simple to a rigid support, have a significant effect on the steel beam vibration response.

7. Conflicts of Interest

The authors declare no conflict of interest.

8. References

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