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Non-deterministic Approach for Reliability Evaluation of Steel Portal Frame

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Abstract

In recent years, more researches on structural reliability theory and methods have been carried out. In this study, a portal steel frame is considered. The reliability analysis for the frame is represented by the probability of failure, P_f , and the reliability index, β , that can be predicted based on the failure of the girders and columns. The probability of failure can be estimated dependent on the probability density function of two random variables, namely Capacity R, and Demand Q. The Monte Carlo simulation approach has been employed to consider the uncertainty the parameters of R, and Q. Matlab functions have been adopted to generate pseudo-random number for considered parameters. Although the Monte Carlo method is active and is widely used in reliability research, it has a disadvantage which represented by the requirement of large sample sizes to estimate the small probabilities of failure. This is leading to computational cost and time. Therefore, an Approximated Monte Carlo simulation method has been adopted for this issue. In this study, four performances have been considered include the serviceability deflection limit state, ultimate limit state for girder, ultimate limit state for the columns, and elastic stability. As the portal frame is a statically indeterminate structure, therefore bending moments, and axial forces cannot be determined based on static alone. A finite element parametric model has been prepared using Abaqus to deal with this aspect. The statistical analysis for the results samples show that all response data have lognormal distribution except of elastic critical buckling load which has a normal distribution.

Keywords: Reliability Analysis; Monte Carlo Method; Matlab; Abaqus.

1. Introduction

The design of engineering structures is usually associated with a significant level of uncertainties due to limited information in the process of estimating the structural parameters. The impact of uncertainties needs to be quantified and propagated to obtain the reliability of a structural system Morio and Balesdent (2016) [1]. In practice, most engineering design of structures are based on deterministic parameters and often do not consider the variations in the material properties and the geometry of the structure. Ebenuwa and Tee (2019) stated that the determination of structural performance based on the deterministic model is undoubtedly a simplification because physical measurement always shows variability and randomness [2].

In many circumstances, it is impossible to describe the response of structural systems mathematically because of these uncertainties. Even after finding a mathematical model to predict the behavior of the system, there is no closed form solution for solving the equation. In such cases, simulation is one of the most applicable techniques to acquire the required information. Simulation is a special technique to approximate the quantities that are difficult to obtain

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analytically. Amongst many of simulation methods, the Monte Carlo simulation method is one of the well-known and common procedures in solving complex engineering problems Melchers and Beck (2017) [3].

The origin of Monte Carlo began in the 1940s by three scientists, John von Neumann, Stanislaw Ulam, and Nicholas Metropolis while working on a nuclear weapon project called the Manhattan Project. They conceived of a new mathematical method that would become known as the Monte Carlo method. Stanislaw Ulam coined the name after the Monte Carlo Casinos, located in Monaco south of France. Soon, applications started going up in all sorts of situations in business, engineering, science and finance [4].

Theory and methods for structural reliability have been developed substantially in the last few years and they are actually a useful tool for evaluating rationally the safety of complex structures or structures with unusual designs Gordini, et al. (2018) [5]. Recent evolution allows anticipating that their application will gradually increase, even in the case of common structures Cardoso et al. (2008) [6].

Zhange and Zhou (2013) studied the system reliability analysis of a 3D steel frame designed used AISC LRFD with respect to the collapse limit state under the dead and live loads. They evaluated the system reliability of the frame for two cases, Case 1 was ignoring the spatial variation of the live load and Case 2 was considering it. The results showed that the reliability index for Case 1 is slightly higher than Case 2. This indicated that the spatial inconsistency of the live load decreases the system reliability of the frame [7].

In 2016 Klink and Silva assessed the reliability and security of a steel I-beam profile subject to an applied bending moment. The purpose was to evaluate the suitability of the beam in handling specific project stresses. The Monte Carlo method was used, to obtain the probability of structural failure. Based on the analysis, the I-beam was oversized, thus, can be submitted to increase loading stresses without damaging the global structure [8].

The probability of failure for steel column and beam had been estimated by Manjunath and Sagar in 2017. The probability modelling adopted using Matlab and Monte Carlo method to generate pseudo-random numbers for the parameters considered in the statistical analysis. The reliability analysis show that the probability of failure for column was greater than the probability of failure for beam [9].

Zhang et al. (2018), examined the system reliabilities of a number of simple yet representative structures subjected to gravity loads, including a continuous beam, a portal frame that fails by elastic instability, and three related frames with various load redistribution capacities. The research provided an overview of the strengths and system reliabilities of these structures when designed either by the second-order inelastic method or by LRFD in AISC 360-10. Based on the system reliability analysis results for the five structures, some general observations were made. As designed by LRFD, the frames system reliability indices were quite scattered. The system-based design by inelastic analysis is well able to achieve identical system reliabilities than present member-based LRFD. This is to be predictable given that the inelastic method is explicitly based on overall system behaviors. The reason that the inelastic method leads to lower system reliabilities than LRFD was that the inelastic analysis, in contrast to LRFD, leaves little reserve strength after first yielding in the system [10].

In 2019, the system reliability analysis based limit state design criterion for 3D steel frames under wind loads had been studied by Wenyu et al. Through the Monte Carlo technique, the probabilistic characteristics of the ultimate lateral strengths of the frames are determined. It was found that despite the differences in structural configuration, system size and degree of redundancy of the frames, their ultimate lateral strengths share similar probabilistic characteristics, i.e., they can be generally described by lognormal distributions [11].

The behavior of steel frame is generally assessed based on their strength and their elastic deformations In addition to the deterministic aspects that discussed in mechanics of material, the strength and deformation of steel frames have random parts due to the scatter in the dimensions, material properties, and the applied load. These random aspects can be simulated in terms of the probability density functions that either obtained from real experimental data on the member scale level or from the simulation that based on data of sectional level [12].

This paper starts with data gathering from literature for the variation in cross-section dimensions of frame elements, the variation in the elastic modulus and yield stress of the material, and the scatter in the applied loads. Based on these data, it has been found that the variation in the sectional dimensions, elastic modulus, yield stress, and dead loads are normally distributed while the lognormal and extreme type I (Gumbel) can be adopted for the variation in the length and live loads respectively.

Monte Carlo simulation has been used to generate a sample for the parameters that effected on the frame behavior. Two samples have been generated first one is the demand sample while the second one is the capacity samples. These samples had been presented and summarized in the form of histograms. The generated sample has been statistically tested with the χ^2 test. Base on limit state function, these samples have been used to estimate the probability of failure for the portal frame. This study innovatively concerns with the randomness in structural parameters and how these

randomness effects on structure reliability by determining the probability of failure and reliability index using Monte Carlo simulation method and Approximate Monte Carlo simulation method.

2. Uncertainties in Engineering System

Every structure may contain some failed elements which lead to the whole structure failure. The probability of failure for structure can be predicated established on the failure of its elements. Hence, it is significant in reliability analysis to determine the probability of structure elements failure. First and second-order of reliability method and Monte Carlo methods can be used to analyze the reliability of elements [13]. For the statically indeterminate steel portal frame of this paper, the girder and columns failure have been used to estimate the frame probability failure.

The uncertainties included in the building engineering can be categorized according to their source into natural hazards and man-made hazards. Natural hazards may be resulted by wind, seismic, temperature differentials, snow load, or ice accretion. The natural variations of structural properties such as strength, stiffness and loads can be classified within the natural hazards. On the other hand, from a structural point of view, the man-made hazards can be sub classified into two classes: from within the building process and from outside the building process. The second one includes uncertainties due to fires, gas explosions, collisions, and similar causes, while the first one includes uncertainties due to acceptable practice and those caused by departures from acceptable practice [14]. This paper concerns with the natural hazard aspects due to change in stiffness, strength, and applied loads.

3. Performance Functions

The limit state function or performance function represents the relation between capacity, R, and demand, Q.

In this paper, the serviceability limit state deflection function and ultimate moment limit state function have been studied for the frame. The ultimate limit states can be used to determine the safety margin. The performance function can be written as follows:

$$g(R,Q) = R - Q \tag{1}$$

The structure is classified safe when $g \ge 0$ while it is unsafe when g < 0. Mathematically, the failure probability P_f is equal to the probability of g < 0 [2]:

$$P_f = P(g < 0) = P(R - Q < 0) \tag{2}$$

If R and Q have probability density functions (PDF) indicated in Figure 1, the quantity R-Q would be a random variable also with its own PDF. As shown in Figure 1, the probability of failure would correspond to the shaded area.



Figure 1. PDFs of load, resistance, and safety margin [15]

A direct determinate of P_f from Equation 2 is relatively difficult. Therefore, it would be more appropriate to express structural safety in the expression of a reliability index β , which can be described as the shortest distance from the origin to the failure limit. When R and Q are uncorrelated the β would be the inverse of the coefficient of variation of the Equation 1 [14] and the reliability index is related to the probability of failure by:

$$\beta = -\varphi^{-1} \left(P_f \right) \text{ or } P_f = \varphi(-\beta) \tag{3}$$

From a statistical point of view, the PDF of g(R, Q) and β can be determined based on a simulation process. Monte Carlo technique has been used for a simulation to determine the reliability index β numerically.

4. Monte Carlo Simulation Method

In this paper, reliability analyses have been achieved through the Monte Carlo method that has used digital computers to generate pseudo-random sampling for variables of dimensions, loads, elastic modulus, and yield stress using Matlab codes.

The method is based on running the model many times as in random sampling. For each sample, random variates are generated on each input variable; computations are run through the model yielding random outcomes on each output variable. Since each input is random, the outcomes are random [16]. The method may be described as a means of solving problems numerically in mathematics, physics, and other sciences through sampling experiments [1].

In each simulation experiment, the possible values of the input random variables $x = x_1 \cdot x_2 \cdot \dots \cdot x_n$) are generated based on predefined distribution and parameters. Then the values of the response variable, y, are determined through the performance function y = g(x) at the samples of input random variables. In this manner, a set of samples for the response variable y would be available for the subsequent statistical analyses to estimate the characteristics of the response variable y [17].

Monte Carlo simulation provides a common feasible way to determine the reliability index or the probability of failure. It is applicable to linear and nonlinear limit state function [12].

The problem to be simulated may have a probabilistic or deterministic form. In the probabilistic form, the actual random variable or function appearing in the problem is simulated, whereas in the deterministic form an artificial random variable or function is first constructed and then simulated [4]. In this paper, the frame can be classified as a deterministic form problem where the stiffness, strength response functions have been determined from the strength of the material and the design of steel structures.

5. Reliability Analysis Using Simulation

Probability failure of a structural element can be calculated based on the amount of convergence function probability distribution of two random variables, Strength (R) and Load (Q). A key point in solving reliability with Monte Carlo simulation methods is the generation of series of random variables for the probability density of each variable of limit state function and failure probability is written as below [8]:

$$P_f = \frac{Number of trials for g(x) \le 0}{N}$$
(4)

It is obvious that the approximation is more realistic with more samples [13].

Although the Monte Carlo method is seen as effective and is widely used in research for reliability, there is a problem with Monte Carlo sampling is that if the probability of failure is a small value such that of the structural design where the allowable probability of failure is in the range $(10)^{-5}$ [18], a large number of samples are needed in order to predict this accurately, causing a sharp increase in required cost and time. The other solution is to use algorithms that generate more random numbers near the tail. These kinds of algorithms mostly use a technique to change the dispersion of random numbers in order to generate more random numbers in a certain angle or a specific area such as the tail region [19]. One of these algorithms provided by Far & Wang, 2016 which is known as the approximation of the Monte Carlo sampling method for reliability analysis of structures. A simple algorithm was proposed to estimate low failure probabilities using a small number of samples in conjunction with the Monte Carlo method [20].

The proposed algorithm shown in Figure 2 approximated the failure probability to an acceptable level of accuracy equivalent to the estimation provided by the Monte Carlo method using 20000 random numbers [20]. That has been adopted to estimate the P_f for deflection and moment limit state for mid-span girder and elastic stability for the frame.



Figure 2. The proposed approximation flowchart

6. System Reliability

Most structures consist of systems of interconnected components and members. When considering the system reliability, it is important to recognize that the failure of a single component may or may not cause the failure of structure [21]. There are three idealized types of structural systems. In a series system, in a parallel system, and hybrid or combined systems. Figure 3 show examples of series and parallel systems.

• Series systems

In this system, the failure of one member leads to immediate failure of the entire system. And it is sometimes referred to as the weakest link system because the failure of the system corresponds to the failure of the weakest element in the system. Probability system failure P_f represented by the probability of element failure P_{fi} :

$$P_f = 1 - \prod_{i=1}^{n} (1 - P_{fi}) \tag{5}$$

• Parallel systems

All the members must fail before the system fails:

$$P_f = \prod_{i=1}^n P_{fi} \tag{6}$$

• Hybrid (Combined) Systems

Many structures can be considered as a combination of series and parallel systems. Such systems are referred to as hybrid or combined systems [14]. The portal frame considers as hybrid system.



Figure 3. (a) Series system; (b) Parallel system [12]

7. Random Variables with Their Statistical Parameters

The statistical characteristics for all parameters that considered in this study have been illustrated in the tables below which they gathered from the review of the literature. Kala, et al. 2009 studied the randomness in cross section dimensions for hot rolled steel sections as illustrated in Table 1 [22], the probability density function that represents the variations in this samples is Normal distribution [22] and [23]. Table 2 present the statistical characteristics for other parameters.

Random variables	Relative Mean	Relative Standard deviation
Section depth h	1.0009	0.0044233
Section width b	1.0139	0.009868
Web thick. t_1	1.0540	0.039053
Flange thick. t_2	0.9927	0.045859

Table 1. Statistical characteristics for cross-section dimensions

Random variables	Mean/ Nominal	Mean	Standard deviation	COV	Distribution type	References
Point dead load $F_D kN$	1.03			0.08	Normal	[24]
Point live load $F_L kN$	1			0.1	Gumbel	[25]
Yield strength $F_y MPa$		327.50	24.56	0.07	Lognormal	[26]
Modulus of Elasticity E MPa	0.993			0.034	Normal	[7]
Girder span m	1			0.07	Lognormal	[13]

Table 2. Statistical characteristics for parameters from different sources.

This variation can be described based on parameters and statistical distributions indicated in Table 1 and Table2. Based on these variations and statistical characteristics, Matlab random number generator has been used in this paper to generate a sample for these parameters that have been adopted in subsequent calculations.

8. Case Study of Portal Frame

8.1. Proposed Structural Sections

After check the portal frame sections which they found adequate for the requirements of [27] and assumed braced laterally, the portal frame indicated in Figure 4 has been used in this study. Four concentrated loads have been applied their values shown in Figure 4. The live loads are equal to 25, 12.5 kN and 150, 75 kN are simulate the superimposed load.



Figure 4. Portal frame with dimensions, sections details, and applied loads

8.2. Limit States Samples

Two samples group are needed for frame components, the first is demand or loads samples, and the second is the capacity. These samples have been generated with respect to uncertainties in their parameters which illustrated in Table 1 and Table 2. As Matlab software has been adopted for this issue.

• Demand samples

The demand samples are represented by deflection, the moment for the girder in addition to moment and axial force for the column. Assume these samples are obtained due to uncertainties in applied loads. The random load values have been generated for 100 values. The frame has been analysis in Abaqus software and by rerun the model for 100 times, the gathered samples have been exported to Matlab to present this data in histogram forms and calculate the statistical characteristics where M, v, s, B1, and B2 represent mean, variance, standard deviation, coefficient of skewness, and coefficient of kurtosis respectively which indicated in Figure 5 through Figure^A. The χ^2 test has been used to show that the lognormal probability density function can be adopted for these histograms with coefficients of variance equal to 0.047, 0.048, 0.052, 0.048, and 0.043 for deflection, moment girder, shear force, moment column and axial force samples.







Figure 6. Histogram and statistical characteristics for girder moment



Figure 7. Histogram and statistical characteristics for column moment values



Figure 8. Histogram and statistical characteristics for column axial force values

• Capacity samples

The capacity samples for the frame components have been randomly generated using Matlab code. These samples include the maximum allowable deflection and the moment capacity for the girder. Regarding the columns, they include moment and axial compression capacities. These samples with a size of 100 have been generated due to variation in cross-section dimensions, girder span, yield stress, and modulus of elasticity, which their statistical characteristics illustrated in Table 1 and Table 2. These samples presented in histograms in Figure 9 through Figure 12. From the histograms, a lognormal distribution null hypothesis has been adopted and verified using the χ^2 goodness of fit test by Matlab. Therefore, these samples are lognormal probability density function with coefficients of variance equal to 0.064, 0.057, 0.055, 0.053, and 0.040 for max allowable deflection, moment girder, shear force, moment column, and column axial force capacities respectively.



Figure 9. Histogram and statistical characteristics for Max. Allowable deflection





b- Statistical characteristics.





a- Histogram for randomness in column moment capacity.

b- Statistical characteristics.

Figure 11. Histogram and statistical characteristics for column moment capacity





(7)

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For the case of elastic stability, the portal frame presented in Figure 13 has been considered which subjected to proportional loads for linear perturbation analysis in Abaqus at the same position of applied loads. Other variables including girder span and columns height have been assumed constant. The frame has been analysed to determine the elastic buckling capacity sample, Eigen values, with changing the aforementioned parameters as input analysis variables using a Matlab code. These data represent the Resistance sample P_{cr} , for the stability with statistical properties mentioned in Table 3.



Figure 13. The portal frame subjected to a proportional load

The second sample is the demand or the Load sample *P*. It is represented by the required axial force for the column. The buckling limit state can be defined as [28]:

$$= P_{cr} - P$$

If g< 0 the column is unstable and represents failure case, otherwise it is stable.

Table 3. Statistical characteristics for elastic buckling capacity sample

Random variables Mean		Standard deviation	COV	Distribution type
Resistance sample P_{cr} ('kN)	14095	44.671	0.003	Normal

8.3. Reliability Analysis for Frame Components

As shown in Section 8.2 and through using the Monte Carlo simulation method, the generated samples for the demand and capacity indicate no failure at the limit state. It essential to use a larger sample size which is difficult to achieve and required long run time. Therefore, there is a need to use the Approximated Monte Carlo method which had been illustrated in Figure 2 for serviceability and ultimate limit state of the girder in addition to elastic stability limit state for the frame.

Based on the random sample 100 in size, the deflection, moment and stability limit state, LSF, has been drawn versus the standard normal variable, Z_i , as indicated in Figure 14 through Figure 16 respectively. The LSF and Z_i have been fitted Then the equation intercept has been determined to indicate the failure probability. The computed P_f have been summarized in Table 4.



Figure 14. Standard-normal variable versus LSF for deflection limit state



Figure 15. Standard-normal variable versus LSF for moment limit state



Figure 16. Standard-normal variable versus LSF for elastic stability limit state

Unfortunately, the approximated method is inapplicable for the column that is adequate when located inside the interaction diagram. To overcome this difficulty, the statistical characteristics of the generated sample have been used to generate a large sample size of 20000 using Matlab.

The axial force bending moment interaction equations specified in (AISC 360, 2010) represents the capacity of a beam-column [7]. Equation 7 has been used in the reliability analysis for the column to indicate that values greater than one represent failures cases.

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0\tag{7}$$

A large sample with a size of 20000 has been randomly generated based on the statistical properties of the input parameters. After that, the cases where ratio may be greater than 1 can be counted.

As shown above, the probability of failure for girder and column equal to zero. This may interpret in terms of the applied load magnitude that is very small compared to the failure load.

Limit state function	P _f
Deflection limit state	1.1×10^{-11}
Moment limit state	3.2×10^{-37}
Stability limit state	0
Column interaction equation	0

Table 4. Probabilities of failure for girder

8.4. Case Study for the Frame under Higher Load Value

The failure probability, P_f , and the reliability index, β , for the frame have been re-estimated with the load mean values increase to 60, and 30 kN for live load. While the superimposed load as 200, and 400 kN. As it dealt with the same frame sections and considered the same randomness for the same parameters, the statistical characteristics of Section 8.2 have been used to generate pseudo-random samples. The probability of failure and the reliability index have been presented in Table 5.

P _f	β
0.9944	-2.54
2.9987e-10	6.19
0	
0.0094	2.35
8.836e-5	3.75
	P _f 0.9944 2.9987e-10 0 0.0094 8.836e-5

Table 5.	Probabilities	of failure	for the	frame unde	r higher	load

As illustrated in Table 5, to transform from the member analysis achieved in above to a system reliability analysis for the ultimate limit state function, the frame can be represented as a hybrid system where the overall failure occurs if both columns fail or if the girder fails. Thus, the two columns behave as a parallel subsystem, while both of them are in series with the girder [14]. As the column is identical so that they have the same failure probability and it equal to 0.0094. Hence, for the combined system, the probability of failure using Equation 5 and Equation 6 is equal to $8.836 \times (10)^{-5}$ and the reliability index β is equal to 3.75. Popov (1990), explained that the β , for routine applications on order of 3 is considers appropriate [29], therefore β for ultimate limit state is appropriate while for deflection is not considers appropriate.

9. Conclusion

In this paper, the Monte Carlo based method and the Approximate Monte Carlo method have been used to estimate the reliability of a portal frame. It has been shown that these methods provide good estimates of the member reliability with a moderate computational effort. The failure probability P_f , results indicate that the deflection limit state is more sensitive than other limit state functions for randomness in the considered parameters. The elastic stability of the frame is the least sensitive one. And the variation in applied load is more significant than other random parameters, this due to higher uncertainties in the load samples especially for the live load.

It has been pointed out that the use of Monte Carlo methods for system reliability analysis has several very attractive features, the most important being that the failure criterion is usually relatively easy to check almost irrespective of the complexity of the system and the number of basic random variables.

Ultimate limit state failure probability of the whole frame is dependent on the failure probability for the girder and the columns. It is greater than the member failure probability, P_{fi} , for the girder and equal to failure probability of the columns parallel subsystem. From a statistical point of view, the probabilities of failure for elements in the connected joint have been assumed as independence events. From the theory of structure, these probabilities of failure are correlated for an indeterminate structure similar to the considered portal frame.

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11. Conflicts of Interest

The authors declare no conflict of interest.

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