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Investigating the Local Buckling of Rectangular Corrugated Plates

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Abstract

With advances in technology in recent years, the use of orthotropic materials to exclude the mechanical deficiencies of homogeneous plates has increased. Sinusoidal corrugated plates are known as orthotropic plates, as a result of changes in their mechanical properties in two orthogonal directions. Since use of corrugated plates, in particular steel shear walls instead of flat steel plates, has increased, the present study investigated local buckling of sinusoidal corrugated plates under uniform uniaxial loading on the transverse edges of the plate (vertical loading on the sinusoidal wave of corrugated plates), using the Galerkin method. This method is very powerful with regard to solving differential equations, and directly uses these equations in the process of problem-solving. Finally, the results obtained for the critical buckling load of sinusoidal corrugated metal plates and the results relating to the metal homogeneous flat plates were compared using the same supporting conditions and loading.

Keywords: Sinusoidal Corrugated Plates; Local Buckling; Rectangular Plate; Uniform Loading; Orthotropic.

1. Introduction

With the worldwide development of the construction industry, the quality and speed of construction is highly regarded and continues to evolve. Structures with cold-rolled steel walls have emerged as one of the modern manufacturing innovations of this industry, and this type of structure has become a serious competitor to those constructed from low-level traditional steel and concrete. In the United States of America, shear walls with corrugated plates have been used in the industry since 1946 [1]. Leo and Edlund [2] examined buckling of trapezoidal corrugated plates by using the finite-strip method under thin-walled structures, while Qiang et al. [3] discussed local buckling of the compression flange of H-beams with corrugated webs. Liew et al. [4] investigated buckling of metal sinusoidal and trapezoidal corrugated rectangular plates using a mesh-free Galerkin method based on first-order shear deformation theory under uniform loading, perpendicular to the edge of the plate in the sinusoidal waves of metal corrugated plates. Kabir and Karbasi [5] examined the shear buckling of thin-steel shear walls with smooth and corrugated plates, while Babaqassabha and Shaghaghi Moghaddam [6] studied the numerical analysis of composite flat and corrugated sandwich buckling plates with a soft core. In addition, Babaqassabha and Abkenari [7] investigated free vibration of a corrugated composite sandwich plate with an aluminum flat coating. Garivani and Moqimi [8] evaluated the performance of a steel plate beam with corrugated web, while Esmaeilzadeh and Shabanzadeh [9] studied the use of corrugated plate instead of simple plate in steel shear walls. Rostami et al [10] examined the elastic shear buckling resistance of zigzag corrugated steel plates, and Nakhaei et al. [11] assessed the system behavior of non-prismatic plate beam and trapezoidal corrugated plates using ABAQUS software. Moodi and Ghasemi [12] examined shear buckling of steel corrugated webs (corrugated cardboard), while Valadi and Aghajari [13] examined the impact of different widths of corrugated zigzag plates on the seismic behavior of corrugated steel shear walls under monotonic and cyclic loading. However, no previous study has examined buckling of sinusoidal corrugated plates under uniform loading perpendicular to sinusoidal waves, so we investigated local buckling of sinusoidal corrugated plates under uniaxial uniform loading on the transverse edges of the plates (loading perpendicular to the sinusoidal wave of corrugated

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plates), using the Galerkin method. We also compared the local buckling coefficient of corrugated metal plates with flat metal plates under different supporting conditions, and studied the impact of ratio of length to width of plate, as well as the effect of ratio of sinusoidal half-wave plate height on buckling coefficient, on the basis of the classical theory for thin plates.

2. Flexural stiffness of sinusoidal corrugated plate

Corrugated plates possessing geometric characteristics in two orthogonal directions that are dissimilar (Figure 2.) are called orthotropic plates [14]. The final differential equation of an orthotropic plate was calculated on the basis of applied loads along the x axis perpendicular to the transverse edge (Figure 1.), according to Equation 1 [15].

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} - N_{xx} \frac{\partial^2 w_0}{\partial x^2} = 0 \tag{1}$$

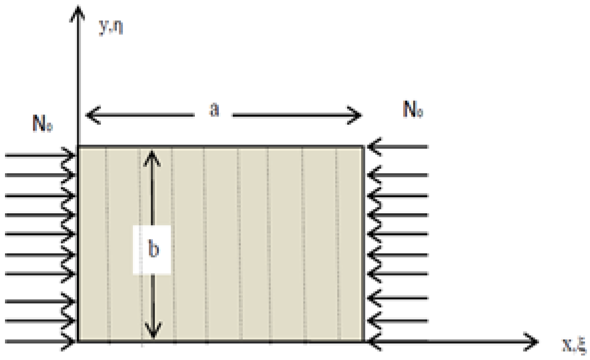

Figure 1. Loading perpendicular to the transverse edge of the sinusoidal corrugated plate (direction of loading perpendicular to the sinusoidal waves)

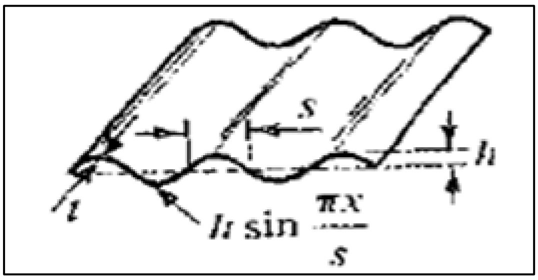

Figure 2. Cross section of sinusoidal corrugated plate [15].

In equation 1:

D_x, D_y, D_{xy} are the flexural stiffness degrees of the plate in the directions of x, y, xy, and H average torsional stiffness of the plate was calculated on the basis of Equations 2. to 5. [15, 19]:

$$D_x = \frac{s}{\lambda} \frac{Et^3}{12(1-\nu^2)} \tag{2}$$

$$D_y = EI \tag{3}$$

$$D_{xy} = 0 \tag{4}$$

$$H = \frac{\lambda}{s} \frac{Et^3}{12(1+\nu\ )} \tag{5}$$

In Equations 2. to 5., s is the length of half of the sinusoidal wave of corrugated plate, as shown in Figure 2., E is the plate's elasticity modulus, which has been assumed equal to the line of x and y. ν is the Poisson ratio, t is the plate thickness and parameters λ, I, which are the moment of inertia and the wave lengths of the sinusoidal corrugated plate, respectively, were calculated according to equations 6. and 7. [19, 15];

$$\lambda = s\left(1+\frac{\pi^2 h^2}{4s^2}\right) \tag{6}$$

$$I = 0.5h^2 t\left[1-\frac{0.81}{1+2.5\left(\frac{h}{2s}\right)^2}\right] \tag{7}$$

3. Use of the Galerkin method with different boundary conditions

The desired problem, which is a rectangular plate with a sinusoidal corrugated plate under uniform loading, perpendicular to the transverse edge (perpendicular to sinusoidal half-waves of the plate), is shown in Figure 1.; b is plate width, a is plate length, and t is the thickness of the plate. The problem was considered with regard to different supporting conditions of the plate, and its local buckling coefficient was finally drawn in terms of length to width ratio of the plate under different supporting conditions of the plate.

In order to analyze the behavior of the plate, trigonometric functions in the longitudinal direction, which satisfied the boundary conditions, were used, and to satisfy the boundary conditions in the transverse direction of the plate, the use of a polynomial function appeared to be appropriate, since the use of trigonometric functions that will meet their necessary conditions is difficult. Finally, to determine the edge conditions of the plate, the letters S (simple support), c (fixed support), f (free support), and G (guided support) were used.

4. Formulation of the problem

The final differential equation, which described the behavior of a thin orthotropic plate with loading in one direction in an elastic buckling state, was expressed according to Equation 1. In order to accelerate the analysis of the problem, we must transfer Equation 1. to the dimensionless device [16]. In order to do so, the following equations were used:

$$\zeta = \frac{x}{a},\ \ \eta = \frac{2y-b}{b},\ \ \phi = \frac{a}{b},\ \ W = \frac{w}{h} \tag{8}$$

Using Equation 8. and letting them in (1) we obtained:

$$\left(\frac{D_x}{\phi^2}\frac{\partial^4 w}{\partial \xi^4} + 2H\frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + D_y \phi^2 \frac{\partial^4 w}{\partial \eta^4}\right) - N_x b^2 \frac{\partial^2 w}{\partial \xi^2} = 0 \tag{9}$$

The solving of the equation would, of course, be analytically extremely difficult, if (closed solution) not impossible, given that a partial differential equation has variable coefficients. Here, as previously stated, the Galerkin method was used for problem-solving.

Solving differential Equation 9. or the interpolation function of plate deformation was considered as follows:

$$W = \sum_m \sum_n q_{mn} f_m(\xi) g_n(\eta) \tag{10}$$

We considered the boundary condition for the transverse edges of the plate (the loaded edges) as (joint-joint), such that the following sine function in the direction of the length of the plate was used for consideration of the boundary condition [18]:

$$f_m(\xi) = \sin(m\pi\zeta) \qquad\qquad \beta = \left(\frac{m\pi}{\phi}\right)^2 \tag{11}$$

In the above equation, m=1, 2, 3…

With regard to the transverse direction of the plate, the following polynomial function was used:

$$g_n(\eta) = \eta^{n+4} + A_n\eta^{n+3} + B_n\eta^{n+2} + C_n\eta^{n+1} + D_n\eta^{n} \tag{12}$$

Function $g_n(\eta)$ also had four unknown parameters relating to the existence of four boundary conditions. The parameters were determined on the basis of the conditions of the plate's longitudinal edges. These constants are evaluated by applying the equations in Tables 1. and the unloaded edges of the plate are assumed to be simply supported or clamped supported. Functions satisfy the boundary conditions at the edges $(\eta = 0, \quad \eta = 1)$, as shown in Table 1. Whether simply supported or clamped, one category of the following equations must be satisfied.

Table 1: Boundary condition for calculating $g_n(\eta)$ at unloaded edges

Longitudinal edges is Simply supported(S)	$[\frac{\partial^2 w}{\partial \eta^2}]_{\eta=-1,1} = 0$	$[w]_{\eta=-1,1} = 0$
Longitudinal edges is Clamped(C)	$[\frac{\partial w}{\partial \eta}]_{\eta=-1,1} = 0$	$[w]_{\eta=-1,1} = 0$

And function $f_m(\zeta)$ exactly stated the behavior of the plate in the loaded direction.

By using the parameters of Equation 12., Equation 10. can be written as follows:

$$w = \sum_{n=1,2,3} q_n f_m(\xi)[\eta^{n+4} + A_n\eta^{n+3} + B_n\eta^{n+2} + C_n\eta^{n+1} + D_n\eta^{n}] \tag{13}$$

In short, Equation 13. can be considered in the form of Equation 14.:

$$w = Y.\sin(m\pi\xi) \tag{14}$$

In the above equation, m is the number of sinusoidal half-waves that formed after uniaxial loading in the longitudinal direction of the plate, and in which:

$$Y = \sum_n q_n[g_n(\eta)] \tag{15}$$

By putting Equation 15. in a differential Equation 9. and simplifying it, the following fourth degree differential equation was obtained:

$$\bar{S} = D_y \frac{d^4Y}{d\eta^4} f(\xi) + \frac{2H}{\emptyset^2}\frac{d^2Y}{d\eta^2}\frac{d^2f(\xi)}{d\xi^2} + D_x \frac{1}{\emptyset^4} Y \frac{d^4f(\xi)}{d\xi^4} - \frac{b^2}{\emptyset^2} N_x Y \frac{d^2f(\xi)}{d\xi^2} = 0 \tag{16}$$

Using the Galerkin method, and the use of functions $\frac{\partial Y}{\partial q_n}$, we obtained:

$$\int_0^1\int_{-1}^1 [\bar{S}] \frac{\partial[Y \times f(\xi)]}{\partial q_n} d\eta d\xi = 0 \tag{17}$$

Equation (17) can be written as equation (18):

$$\sum_{n=0,1,..}^{n=j} q_n \int_{-1}^{1} [D_y \frac{d^4Y_n}{d\eta^4} - 2H\beta \frac{d^2Y_n}{d\eta^2} + D_x\beta^2 Y_n - \beta N_0 b^2 Y_n] Y_j d\eta = 0 \qquad k = N_0 b^2 \tag{18}$$

In this equation, j is the number of sentences required for convergence of result solution that n and j in Equation 18. change from zero to j.

Investigation of Equation 18. led to the formation of a square matrix, as follows:

$$\begin{bmatrix} (L_{00}-KM_{00}) & (L_{01}-KM_{01}) & ... & (L_{0J}-KM_{01}) \\ (L_{10}-KM_{10}) & . & ... & . \\ . & . & ... & . \\ . & . & ... & . \\ . & . & ... & . \\ (L_{J0}-KM_{J0}) & . & ... & (L_{JJ}-KM_{JJ}) \end{bmatrix} \begin{bmatrix} q_0 \\ . \\ . \\ . \\ . \\ q_J \end{bmatrix} = 0 \tag{19}$$

In which:

$$L_{ij} = \int_{-1}^{1} [D_y \frac{d^4 Y_i}{d\eta^4} - 2H\beta \frac{d^2 Y_i}{d\eta^2} + \beta^2 D_x Y_i] Y_j \, d\eta \tag{20}$$

And

$$M_{ij} = \beta \int_{-1}^{1} [Y_i] Y_j \, d\eta \tag{21}$$

In this relationship, matrix $[L]$ represents the plate stiffness matrix, and matrix $[M]$ is called the geometric matrix, or plate stability. Finally, through solving a specific value problem, buckling coefficients were obtained by solving the following problem:

$$|[L] - K[M]| = 0 \tag{22}$$

The values of m were chosen in such a way that the minimum buckling coefficients were obtained.

5. Validation of results with previous studies

Table 2. shows the results relating to the critical buckling load of the sinusoidal corrugated rectangular plate with supporting conditions of four simple articular sides (SSSS) under uniform loading on the transverse edge, controlled by Equation 23., taken from Ventsell [19]. It was found that the percentage error of the results was less than or equal to 0.3%, which indicates acceptability. In order to verify the data, the assumptions of section 6 were considered for the sinusoidal corrugated plate:

$$q_{x,cr} = \frac{2\pi^2 \sqrt{D_x D_y}}{b^2} (1 + \frac{H}{\sqrt{D_x D_y}}) \qquad if \quad a \geq\geq b \tag{23}$$

Table 2. Validation of the critical buckling load (N) of sinusoidal corrugated rectangular plate with supporting conditions SSSS, loading on transverse edge [19]

Amount of critical uniform load	
Present study	11755.2
Relationship of 23 from reference [19]	11722.94
Percentage error	0.3%

6. Results

On the basis of the relationships obtained in previous sections, a computer program was developed in a Matlab environment, which is capable of analyzing the question of eigenvalues and of calculating buckling coefficient under different boundary conditions after calculating stiffness and stability matrices. It should be noted that the critical buckling load f_{cr} was initially in the computer program, and the local buckling critical coefficient of plate k_{cr} was achieved by using Equation 24.;

$$k_{cr} = f_{cr}\frac{b^2}{\pi^2 D_x} \tag{24}$$

Figure 1. shows the local buckling coefficient of the sinusoidal corrugated plate under loading, perpendicular to the transverse edge (perpendicular to the sinusoidal wavelengths of the plate) in terms of length-to-width changes of the plate, under different supporting conditions at the longitudinal and transverse edges, in which the plate width was equal to $b = 1800\,mm$, the dimensions of the sinusoidal waves (based on Figure 2.) were$s = 100\,mm, h = 18\,mm, \nu = 0.3$, the plate thickness was equivalent to 0.01 of its length, and the elasticity modulus was considered as E=30 GPa. In naming supporting conditions (e.g. SSCS), the first two English letters indicate the boundary conditions of the loaded transverse edges (SS), and the second two English letters represent the boundary conditions of the longitudinal edges of the plate and without loading (CS).

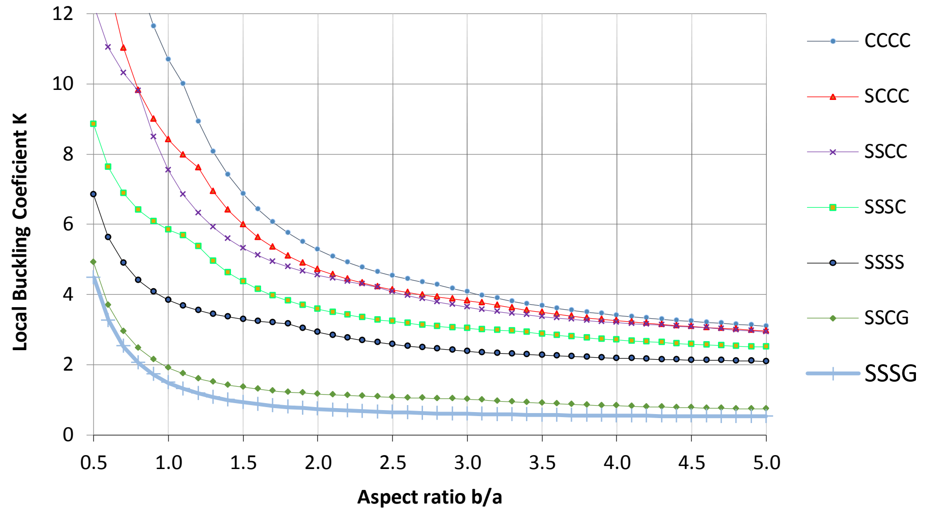

Figure 3. Local buckling coefficient of a corrugated rectangular plate with different supporting conditions, loading on a transverse edge (perpendicular to the sinusoidal waves).

Figure 1. shows that the local buckling coefficient reduced (this tended to be by a constant number) if the length-to-width ratio of the corrugated plate was increased. By changing the supporting conditions of the plate's longitudinal and transverse edges, the supporting fixed conditions of the plate's edges were reduced, so the local buckling coefficient of the plate was reduced. In addition, as shown in Diagram 1, the corrugated plate of four sides of the joint had the greatest critical load, and the SSSG plate (three sides were articular and one side was fixed-guided) had the lowest critical load. The critical buckling coefficient of the flat rectangular plate under uniform loading, perpendicular to the transverse edge in terms of length-to-width ratio is plotted in Figure 2.

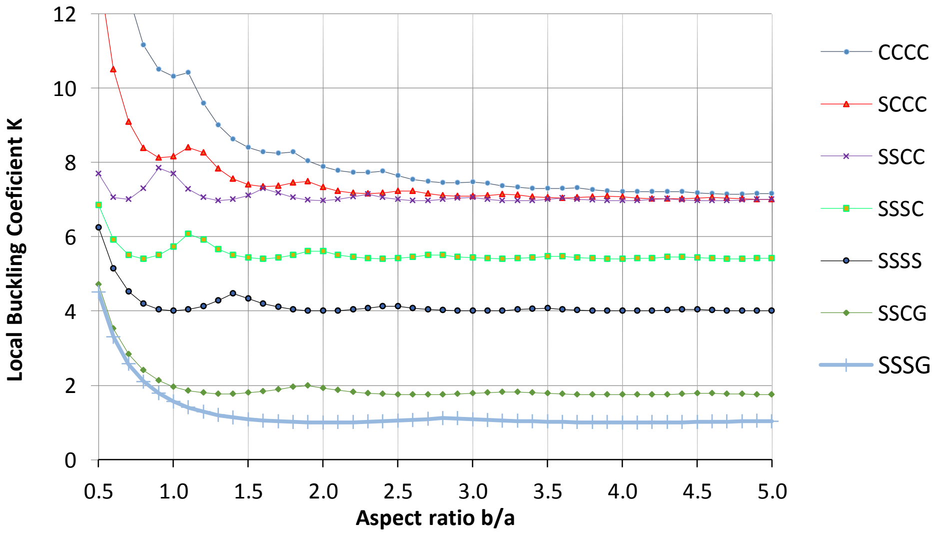

Figure 4. Local buckling coefficient of a steel flat rectangular plate with different supporting conditions

A comparison of Figure 1. (Buckling coefficient of the corrugated plate) and Figure 2. (Buckling coefficient of the flat plate) shows that the local buckling coefficient reduced more than the flat plate if the length-to-width ratio of the corrugated plate was increased, or if the corrugated plates tolerated lower loads than the flat plates in larger sizes. Figure 3 was plotted to compare the buckling coefficient of a sinusoidal corrugated plate and a flat plate with fixed supporting conditions (four sides fixed). It shows that the length-to-width ratio, (the square plate) of the buckling coefficient of the corrugated plate was greater than the buckling coefficient of the flat plate, but that the buckling coefficient of the flat plate was greater than the coefficient buckling of the corrugated plate if the length-to-width ratio of the plate was increased from 1. Reduction of the buckling coefficient of the corrugated plate for larger length-to - width ratios was too high.

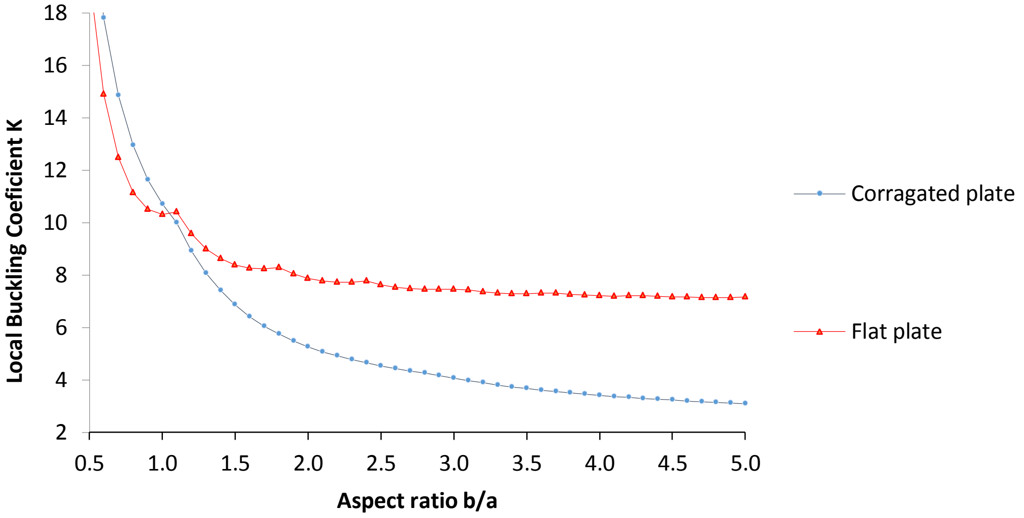

Figure 5. Local buckling coefficient comparison of a corrugated rectangular plate and flat plate with supporting conditions CCCC, loading on the transverse edge

Figure 4. shows the local buckling coefficient of the sinusoidal corrugated plate under loading, perpendicular to the transverse edge, in terms of sine wave height changes to width of plate sine wave for fixed supporting conditions.

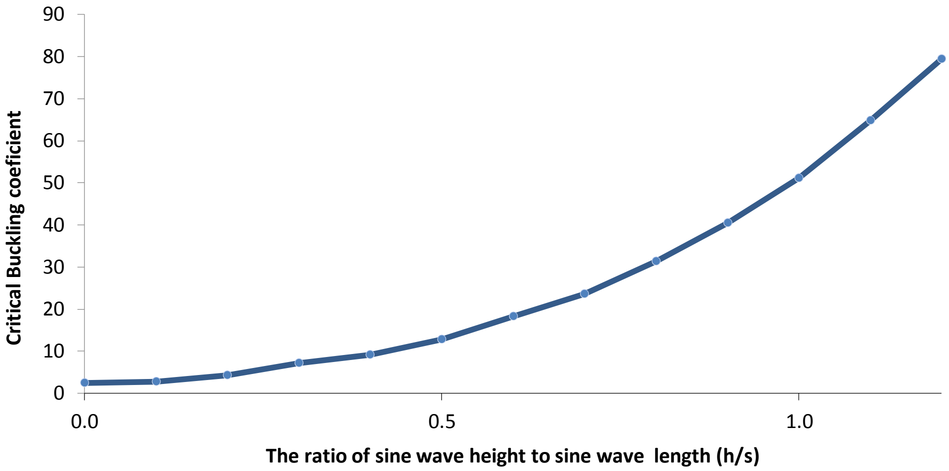

Figure 6. Local buckling coefficient of a corrugated rectangular plate with fixed supporting conditions, loading on the transverse edge, in terms of the ratio of sine wave height to sine wave length of the corrugated plate

Figure 4. shows that the local buckling coefficient increased when the height-to-width of the sine wave of the corrugated plate under uniform loading, perpendicular to the edge of the plate, was increased.

7. Conclusions

- If the length-to-width ratio of the corrugated plate was increased, the local buckling coefficient was reduced. If the supporting conditions of the transverse and longitudinal edges of the plate were changed, the fixedness of the plate reduced, and the local buckling coefficient was reduced.
- If the length-to-width ratio of the corrugated plate was increased, its local buckling coefficient was reduced more than that of the flat plate.
- The diagrams show that the length-to-width ratio (square plate) of the buckling coefficient of the corrugated plate was greater than the buckling coefficient of the flat plate, but that the buckling coefficient of the flat plate was greater than the coefficient buckling of the corrugated plate if the length-to-width ratio of the plate was increased from 1. Reduction of the buckling coefficient of the corrugated plate for larger length-to-width ratios was too high.
- The local buckling coefficient was increased if the height-to-width ratio of the sine wave of the corrugated plate under uniform loading, perpendicular to the edge of the plate was increased.

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