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Comparison of Coupled and Uncoupled Consolidation Equations Using Finite Element Method in Plane-Strain Condition

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Abstract

In the current paper, the consolidation settlement of a strip footing over a finite layer of saturated soil has been studied using the finite element method. In Biot's coupled consolidation equations, the soil deformation and excess pore pressure are determined simultaneously in every time step which refers to the hydro-mechanical coupling. By considering a constant total stress throughout the time and by assuming that volume strain is a function of isotropic effective stress, uncoupled consolidation equations can be obtained using coupled consolidation equations. In these uncoupled equations, excess pore pressure and deformation are determined separately. In this approach, the excess pore pressure can be identified in the first stage. Using the calculated excess pore pressure, the soil deformation is determined through effective stress-strain analyses. A computer code was developed based on coupled and uncoupled equations that are capable of performing consolidation analyses. To verify the accuracy of these analyses, the obtained results have been compared with the precise solution of Terzaghi's one-dimensional consolidation theory. The capability of these two approaches in estimation of pore water pressure and settlement and to show Mandel-Crayer's effect in soil consolidation is discussed. Then, the necessity of utilizing coupled analyses for evaluating soil consolidation analysis was investigated by comparing the coupled and uncoupled analyses results.

Keywords: Coupled Consolidation Analysis; Uncoupled Consolidation Analysis; Finite Element Method; Strip Footing; Excess Pore Pressure.

1. Introduction

Consolidation of a saturated porous medium is a critical problem in geotechnical engineering. Due to the time-dependent nature of consolidation, the settlement of structures placed upon compressible soils is regarded as a significant design consideration. In practice, the consolidation theories can be used to evaluate the amount and rate of consolidation settlements. In most cases, the results of laboratory Oedometer consolidation test, based on one-dimensional consolidation theory, are used to determine the compressibility indexes and consolidation coefficient of consolidating soils. This theory was presented by Terzaghi in 1925 by using simple assumptions [1]. The major statements in this theory include:

(i) Incompressibility of solid particles and pore fluid,

(ii) One-dimensional seepage

(iii) Validity of Darcy's law to model the fluid flow through the porous medium.

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In 1936, Rendulic [2] expanded the Terzaghi's theory to use it for the three-dimensional conditions (i.e. deformation in one direction while seepage flows in three directions). At a later time, Biot [3-4] expanded the basic equations of consolidation theory by considering deformations and seepage flows in three directions. This theory, known as coupled consolidation theory, calculates excess pore pressure and deformations simultaneously by considering compressibility of soil particles and pore fluid.

Because solving coupled consolidation equations is complicated, Booker, Small [5] and other researchers calculated excess pore pressure and deformations separately by using uncoupled equations. To achieve such types of uncoupled equations, some assumptions need to be considered in coupled theory; these assumptions may affect the accuracy of the results.

In Biot's consolidation analysis, in contrast to Terzaghi's solution and uncoupled consolidation theory, the excess pore pressure may increase to its peak in the early stages of the consolidation process, depending on boundary conditions. This issue was first mentioned theoretically in the consolidation study of a rectangular prism of soil mass in the plane-strain condition by Mandel in 1953 and then in the study of a spherical clay model by Crayer in 1963 [2]. Such non-uniform response (temporary increase) of excess pore pressure in early stages of the consolidation process is known as Mandel-Crayer's effect in geotechnical literature. Later, Booker et al. [5] and Carter et al. [6] studied stability of numerical solutions of Biot equations in 1975 and numerical solutions of Biot equations in elastoplastic condition in 1977. Plotkoeit [7] made a comparison between the results of Abaqus and Rockflow finite element software for a foundation over a single layer of soil in 2005.

In 2005, Korsawe et al. [8] used the least-squared finite element method to analyze coupled flow and deformation of a porous medium. Ai and Wang (2008) [9] developed numerical analysis of consolidation behavior of soil mass in an axisymmetric sphere based on Biot's theory. Verruijt (2010) [2] performed a study of wave propagation in soil media by considering flow theory based on Biot consolidation equations. Furthermore, Osman (2010) [10] compared coupled and uncoupled analysis results in spherical conditions of a linear elastoplastic infinite medium under uniaxial loading. Sadeghi et al. [11] investigated the impact of train dynamic load on increasing of displacement and Palassi et al. [12] indicated that lateral confinement can effect on point load test.

More recently, Tall et al. (2015) [13] studied the numerical solution of saturated soil pore pressure and validated their approach by analytical solution. For this research, a saturated soil layer between two drainage layers subjected to a strip uniform load is considered. For validation of numerical analysis, the analytical solution of Terzaghi's one-dimensional consolidation is used. This research can be used to estimate pore water pressure in a compressible and saturated soil layer subjected to uniform load. Moreover, Fox et al. (2015) [14] demonstrated the numerical solution of two benchmark problems that can be used to check other numerical and analytical solutions. For this study, the effect of large strain, nonlinear constitutive relationship, soil self-weight, and variation of material properties during the consolidation are considered.

In previous researches both coupled and uncoupled consolidation approaches have been used to solve and define the consolidation problems. However, it is far less common to find a work that compares these two approaches. This comparison is very desirable because it can show the accuracy of the uncoupled equations in estimating consolidation for a soil mass.

The current work presents a finite element approach to solve the consolidation problems using couple and uncouple equations. In this approach, a saturated and elastic porous medium in plane-strain condition is considered. The results of these two approaches are compared. In this context, a computer program was developed using Matlab programming software, in which the soil mass was modeled and analyzed for its consolidation response as a two-dimensional poroelastic medium based on both coupled and uncoupled approaches. Conducting both coupled and uncoupled consolidation analyses through the developed program, we compared the obtained results to demonstrate the effectiveness and accuracy of the proposed approach.

2. Coupled Consolidation Equations

Biot presented coupled consolidation equations in 1941 [3]. These equations can be used to compute excess pore pressure and deformations in a porous medium throughout the time simultaneously. To develop these equations, flow through a two phase's saturated porous medium (solid and fluid) were considered. In this medium, unknown variables are fluid pressure (p) and displacements of the solid phase (u). To determine these variables, Biot considered several basic assumptions including:

(i) Soil is fully saturated, uniform, and homogeneous.

(ii) Seepage flow is governed by Darcy's

(iii) Poroelastic constitutive model and small strains assumptions are valid

(iv) Effective stress principles are valid. Biot then developed coupled consolidation equations as follows:

$$\alpha\frac{\partial\varepsilon}{\partial t}+s_p\frac{\partial p}{\partial t}=\nabla.(\frac{\mathrm{k}}{\gamma_w}\nabla p) \tag{1}$$

$$s_p=nc_f+(\alpha-n)c_s \tag{2}$$

$$\alpha=1-c_s/c_m \tag{3}$$

Equation 1, known as storage equation, is the most important equation of consolidation theory. In this equation, ε is volume strain, s_p is coefficient of the porous medium storage, n is porosity of medium, α is Biot's coefficient, k is permeability of the porous medium, γ_w is the water unit weight, c_s is compressibility of solid particles, c_f is compressibility of pore fluid, and c_m is compressibility of the porous medium. Since compressibility of solid particles (c_s) in comparison with compressibility of fluid (c_m) is very low, the amount of coefficient α is considered equal to 1, which is compatible with the practical value of this coefficient obtained through laboratory testing for soft clays [1]. Thus, as the value of compressibility of solid particles is very low for soft soils, the term of $(\alpha-n)c_s$ in Equation 2. is disregarded. Furthermore, as the model is water-saturated and compressibility of the fluid phase is the inverse of its modulus,$(c_f=1/\mathrm{K}_w)$, the bulk modulus of water K_w is used in the Equation 2. Thus, the value of s_p is obtained as follows:

$$s_p=nc_f=n/\mathrm{K}_w \tag{4}$$

By considering Equation 1, the consolidation theory may be interpreted as an approach in which the soil consolidation consists of both pore fluid and solid particles consolidation associated with the discharge of fluid from the soil mass. Since bulk modulus of water is significant, it is possible to ignore water compressibility in comparison with compressibility of the porous medium. Thus, in this study, the following equation is used in order to calculate the value of $\frac{n}{K_w}$ by default.

$$\frac{\mathrm{K}_w}{n}=\frac{3(\upsilon_u-\upsilon)}{(1-2\upsilon_u)(1+\upsilon)}\mathrm{K} \tag{5}$$

Where υ and K are respectively the Poisson's ratio and the bulk modulus of the medium, and υ_u is the undrained Poisson ratio of the soil mass, which is assumed 0.495 by default [2]. Because the constitutive equations of Biot's consolidation in bulk strain conditions are based on small strains theory, the effective stress principle and stress-strain relationships are considered respectively, as Equation 6 and 8:

$$\underline{\sigma}=\underline{\sigma'}+\alpha\underline{\mathrm{m}}\mathrm{p} \tag{6}$$

$$\underline{\sigma}=[\sigma_x\ \ \sigma_y\ \ \sigma_{xy}]^T,\underline{m}=[1\ 1\ 0]^T \tag{7}$$

$$\underline{\sigma'}=\underline{\underline{\mathrm{M}}}\ \underline{\varepsilon} \tag{8}$$

$$\underline{\varepsilon}=[\varepsilon_x\ \ \varepsilon_y\ \ \varepsilon_{xy}]^T \tag{9}$$

In these equations, $\underline{\sigma}$ represents total stress vector, $\underline{\sigma'}$ is effective stress vector, $\underline{m}$ is a vector with unit value for normal stresses and 0 for shear stresses. $\underline{\varepsilon}$ and $\underline{\underline{\mathrm{M}}}$ are strain vector, and material stiffness matrix, respectively.

Using the effective stress principle (Equation 6), stress-strain relationship (Equation 8), and storage equation (Equation 1) with each other, it is possible to calculate excess pore pressure and deformations of soil mass in coupled consolidation analysis simultaneously.

3. Uncoupled Consolidation Equations

Coupled consolidation equations are quite complex because they combine elasticity problems and fluid diffusion procedure in a porous medium. On the contrary, if the fluid diffusion is analyzed independently (thus providing the values of excess pore pressure), the porous medium deformations may be calculated separately by performing stress-strain analysis. Such uncoupled consolidation equations may be developed only in special conditions, which will be discussed in the sections 3.1 and 3.2.

3.1. Uncoupled Two-Dimensional Consolidation Equations

Generally, the coupled equations can be simplified if the first term in the storage equation (Equation 1) is expressed as:

$$\frac{\partial \varepsilon}{\partial t} = C^{2D}\frac{\partial p}{\partial t} \tag{10}$$

To determine the constant coefficient C^{2D} in homogeneous materials, volume strain ε should be considered as a function of effective stress(σ'). Considering the total stress remains constant over time, as a primary assumption, the coefficient C^{2D} may be derived as:

$$C^{2D} = \alpha C_m^{2D} \tag{11}$$

Substituting Equation 9 and 10 in Equation 1. the differential equation to calculate excess pore pressure independent of deformations is derived as follows:

$$\frac{\partial p}{\partial t} = \mathrm{c}_v^{2D}\nabla^2 p \tag{12}$$

$$\mathrm{c}_v^{2D} = \frac{k}{(\alpha^2 C_m + s_p)\gamma_w} \tag{13}$$

As already mentioned, the essential assumption to develop this equation is to consider the total stress constant during time. In most cases, consolidation occurs under constant loading conditions. Thus, there is a certain stress redistribution in which changes in the total stress may be considered as negligible. Moreover, in the plane-strain condition, the compressibility of the porous media C_m^{2D} is as follows:

$$C_m^{2D} = \frac{(1-2\upsilon)(1+\upsilon)}{2E} \tag{14}$$

In such cases, the excess pore pressure may be determined independently using the Equation 12. Deformations may then be calculated based on the predetermined excess pore pressure through stress-strain analysis using Equation 6 and 8.

3.2. Uncoupled One-Dimensional Consolidation Equation

One-dimensional consolidation equations in uncoupled form were first developed by Terzaghi in 1925 [1]. He presented these equations by considering a $2h$ thick soil layer of and constant distributed load q over the soil surface throughout the time. In this model, both upper and lower boundaries of the soil layer are assumed drained boundary conditions, thus the pore pressure (p) remains zero in these boundaries during the full consolidation process. Terzaghi presented his theory in one-dimensional form of Equation 12. as:

$$\frac{\partial p}{\partial t} = \mathrm{c}_v^{1D}\nabla^2 p \tag{15}$$

Considering the initial and boundary conditions, the exact solution of Equation 15. for excess pore pressure at any time step may be derived as:

$$\mathrm{p} = \sum_{n=0}^{\infty}\left(\frac{1}{\mathrm{h}}\int_0^{2\mathrm{h}} p_i \sin\frac{n\pi z}{2h}\,\mathrm{dz}\right)\sin\frac{n\pi z}{2h}\,\mathrm{e}^{\frac{-\mathrm{n}^2\pi^2 T_v}{4}} \tag{16}$$

In which, h is the longest drainage path, $2h$ is the thickness of the medium, and T_v is the time factor that can be calculated as follows:

$$\mathrm{T_v} = \frac{\mathrm{c_v^{1D}t}}{\mathrm{h^2}} \tag{17}$$

$$\mathrm{c}_v^{1D} = \frac{k}{C_m^{1D}\gamma_w} \tag{18}$$

$$C_m^{1D} = \frac{(1-2\upsilon)(1+\upsilon)}{E(1-\upsilon)} \tag{19}$$

Thus, to carry out consolidation analysis, the excess pore pressure should be firstly determined using Equation 16. and the resultant deformation may then be determined as follows:

$$\varepsilon_{zz} = -C_m^{1D}(\sigma_{zz} - \alpha p) \tag{20}$$

In this equation, the value of α is 1 and the value of stress σ_{zz} is equal to the applied stress or initial excess pore pressure. To carry out the one-dimensional consolidation analysis, several other parameters including elastic modulus, Poisson ratio, permeability coefficient, and dimension of model should also be known.

4. Finite Element Discretization

Since consolidation equations are involved with special complexity, analytical solution of these equations is only possible for simple cases while the application of numerical method seems to be inevitable for complex problems. In these cases, the numerical method of finite element is frequently used to discretize and solve the consolidation equations. The following equations have been used to discretize consolidation equations in the finite element framework [15].

$$\underline{u} = \underline{\underline{N}}\ \underline{u_n}\,,\quad p = \underline{\underline{N}}\ \underline{p_n}\,,\quad \underline{\varepsilon} = \underline{\underline{B}}\ \underline{u_n} \tag{21}$$

4.1. Finite Element Discretization of Coupled Consolidation Equations

Combining finite element equations with equilibrium equations, the following equations have been derived.

$$\underline{\underline{\mathrm{K}}}\frac{d\underline{\mathrm{u_n}}}{\mathrm{dt}} + \underline{\underline{\mathrm{L}}}\frac{d\underline{\mathrm{p_n}}}{dt} = \frac{d\underline{\mathrm{f_n}}}{dt} \tag{22}$$

Where, $\underline{\underline{\mathrm{K}}}$ is stiffness matrix, $\underline{\underline{\mathrm{L}}}$ is coupling matrix, and $\mathrm{d}\underline{\mathrm{f_n}}$ is load increment vector. These parameters are determined as follows:

$$\underline{\underline{\mathrm{L}}} = \int \underline{\underline{B}}^{\mathrm{T}}\ \underline{\mathrm{m}}\ \underline{N}\ \mathrm{dv} \tag{23}$$

$$\mathrm{d}\underline{\mathrm{f_n}} = \int \underline{\underline{N}}^{\mathrm{T}}\ \mathrm{d}\underline{\mathrm{f}}\ \mathrm{dv} + \int \underline{\underline{N}}^{\mathrm{T}}\ \mathrm{d}\underline{\mathrm{t}}\ \mathrm{ds} \tag{24}$$

Where $\mathrm{d}\underline{\mathrm{f}}$ represents the body-force vector and $\mathrm{d}\underline{\mathrm{t}}$ is surface traction vector. In addition, Equation 1 to 3. has been used to model seepage in the soil mass. Assuming soil permeability to be constant throughout the model, the storage equation has been derived using Galerkin finite element method, as follows:

$$-\underline{\underline{\mathrm{H}}}\,\underline{p_n} + \underline{\underline{\mathrm{L}}}^T\frac{d\underline{u_n}}{dt} - \underline{\underline{\mathrm{S}}}\,\frac{d\underline{p_n}}{dt} = 0 \tag{25}$$

$$\underline{\underline{\mathrm{H}}} = \int (\underline{\underline{\nabla N}})^T\ \underline{\underline{R}}\ (\underline{\underline{\nabla N}})/\gamma_w\ dv \tag{26}$$

$$\underline{\underline{\mathrm{S}}} = \int \frac{n}{K_w}\underline{N}^T\ \underline{N}\ dv \tag{27}$$

$$\underline{\underline{R}} = \begin{bmatrix} \mathrm{k}_x & 0 \\ 0 & \mathrm{k}_y \end{bmatrix} \tag{28}$$

Where $\underline{\underline{R}}$ is permeability matrix, k_x is permeability in x direction, and k_y is permeability in y direction. Combining Equation 22 and 25, the following matrix is derived:

$$\begin{bmatrix} \underline{\underline{\mathrm{K}}} & \underline{\underline{\mathrm{L}}} \\ \underline{\underline{\mathrm{L}}}^T & -\underline{\underline{\mathrm{S}}} \end{bmatrix} \begin{bmatrix} \frac{d\underline{u_n}}{dt} \\ \frac{d\underline{p_n}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \underline{\underline{\mathrm{H}}} \end{bmatrix} \begin{bmatrix} \underline{u_n} \\ \underline{p_n} \end{bmatrix} + \begin{bmatrix} \frac{d\underline{f_n}}{dt} \\ 0 \end{bmatrix} \tag{29}$$

Applying step by step time integration to Equation 29. the general matrix equation is derived as:

$$\begin{bmatrix} \underline{\underline{\mathrm{K}}} & \underline{\underline{\mathrm{L}}} \\ \underline{\underline{\mathrm{L}}}^T & -\underline{\underline{\mathrm{S}}}^* \end{bmatrix} \begin{bmatrix} \Delta\underline{u_n} \\ \Delta\underline{p_n} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & dt\,\underline{\underline{\mathrm{H}}} \end{bmatrix} \begin{bmatrix} \underline{u_{n0}} \\ \underline{p_{n0}} \end{bmatrix} + \begin{bmatrix} \Delta\underline{f_n} \\ 0 \end{bmatrix} \tag{30}$$

$$\underline{\underline{\mathrm{S}}}^* = dt\,\underline{\underline{\mathrm{H}}} + \underline{\underline{\mathrm{S}}} \tag{31}$$

As seen, the general matrix 30 should be solved to determine deformations and excess pore pressure. The time step in the above equations is considered equal to *dt,* while the essential problem returns to the primary time step. The primary time step is the first time step once loading has been applied: it starts from zero and its duration might be different from other time steps during the consolidation. To solve this problem and defining its effect on results, the load has been applied to the soil mass, as shown in Figure 1.

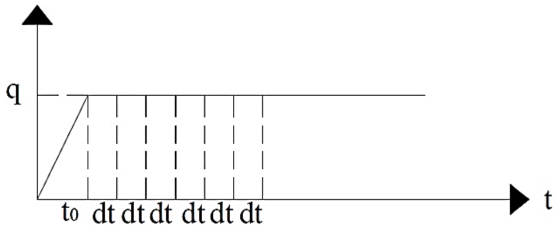

Figure 1. Loading applying through the time

Although the applied stress is assumed uniform and stable, for numerical analysis convenience, it is considered to increase from 0 to q within a very small duration of t_0. Then, the time steps *dt* remain constant throughout the consolidation time. The t_0 duration was considered equal to 0.001seconds in the developed program.

4.2. Finite Element Discretization of Uncoupled Consolidation Equations

As already mentioned, Equation 12. has been used for uncoupled consolidation analysis of the porous medium. To provide uncoupled consolidation equations in the finite element environment, the method of weighted residual has been employed as:

$$\underline{\underline{K}}\,\underline{p_n} + \underline{\underline{C^{2D}}}\,\frac{d\underline{p_n}}{dt} = 0 \tag{32}$$

$$\underline{\underline{K}} = \int \underline{\underline{B}}^T\,\underline{\underline{c_v^{2D}}}\,\underline{\underline{B}}\,dv \tag{33}$$

$$\underline{\underline{c_v^{2D}}} = \begin{bmatrix} c_{vx} & 0 \\ 0 & c_{vy} \end{bmatrix} \tag{34}$$

$$\underline{\underline{C^{2D}}} = \int \underline{N}^T\,\underline{N}\,dv \tag{35}$$

Taking step by step time integration of Equation 32. results in the following equation:

$$\underline{\underline{K}}\,dt\;\underline{p_n} + \underline{\underline{C^{2D}}}\,\Delta\underline{p_n} = 0 \tag{36}$$

It should also be noted that the parameters c_{vx} and c_{vy} are the consolidation coefficients of the porous medium, which are determined as:

$$c_{vx} = \frac{k_x}{(\alpha^2 C_m^{2D} + s_p)\gamma_w} \tag{37}$$

$$c_{vy} = \frac{k_y}{(\alpha^2 C_m^{2D} + s_p)\gamma_w} \tag{38}$$

Thus, excess pore pressure within the model throughout the time may be determined through Equation 36. To determine deformations, the excess pore pressure values should first be obtained, similar to Terzaghi's theory, and then regarding effective stress principle, the model deformations can be determined through stress-strain analysis.

To develop finite element equations for stress-strain analysis in an elastic condition, it is required to consider equilibrium equations. Moreover, the excess pore pressure is assumed to act as a potential body force. Considering effective stress principle (6) and substituting it in the equilibrium equation, the required approach for stress-strain analysis has been derived as follows:

$$\underline{F} = \underline{\underline{K}}\,\underline{u_n} \tag{39}$$

$$\underline{\underline{K}} = \int \underline{\underline{B}}^T\,\underline{\underline{M}}\,\underline{\underline{B}}\,dv \tag{40}$$

$$\underline{F} = \int \underline{\underline{N}}^T\,\underline{f}\,dv + \int \underline{\underline{N}}^T\,\underline{\underline{N}}\,\underline{t}\,ds - \int \underline{\underline{B}}^T\,\underline{m}\,p\,dv \tag{41}$$

In these equations, $\underline{f}$ is body force vector and $\underline{t}$ is surface traction vector. Thus, to perform uncoupled consolidation analysis, the excess pore pressure should be determined using Equation 36. Once known the excess pore pressure, the finite element stress-strain analysis based on Equation 41. is then used to determine the soil mass deformations.

The essential problem to calculate excess pore pressure through Equation 36. is that this pressure at the consolidation onset should be known. In this work, Henkel's equation (Equation 42) has been used to solve this problem [16]. To use this equation, the values of normal stresses are obtained using stress-strain analysis.

$$\Delta p = B\,\frac{\Delta\sigma_1+\Delta\sigma_2+\Delta\sigma_3}{3} + a\sqrt{(\Delta\sigma_1-\Delta\sigma_2)^2+(\Delta\sigma_2-\Delta\sigma_3)^2+(\Delta\sigma_3-\Delta\sigma_1)^2} \quad (42)$$

$$a = \frac{1}{\sqrt{2}}\left(A-\frac{1}{3}\right)$$

In this equation, σ is normal stress, and A and B are Skempton's excess pore pressure parameters. Skempton parameters are presented with the following equations.

$$\mathrm{B} = \frac{\Delta p}{\Delta\sigma} = \frac{1}{1+\frac{\mathrm{n}}{\mathrm{K_W}}(\mathrm{K})} \quad (43)$$

$$\mathrm{A} = \frac{\Delta p}{\Delta\sigma} = \frac{1}{\frac{3\mathrm{n}}{\mathrm{K_W}}(\mathrm{K})+3} \quad (44)$$

Thus, the excess pore pressure at $t = 0$ can be determined through Equation 42 to 44. that are employed to evaluate excess pore pressure in the model at any time duration using seepage analysis.

5. Model Geometry and Material Properties

A soil mass under a strip uniform loading is considered (Figure 2) as a typical problem. The 2 m width strip loading is applied on midsection of the medium surface.

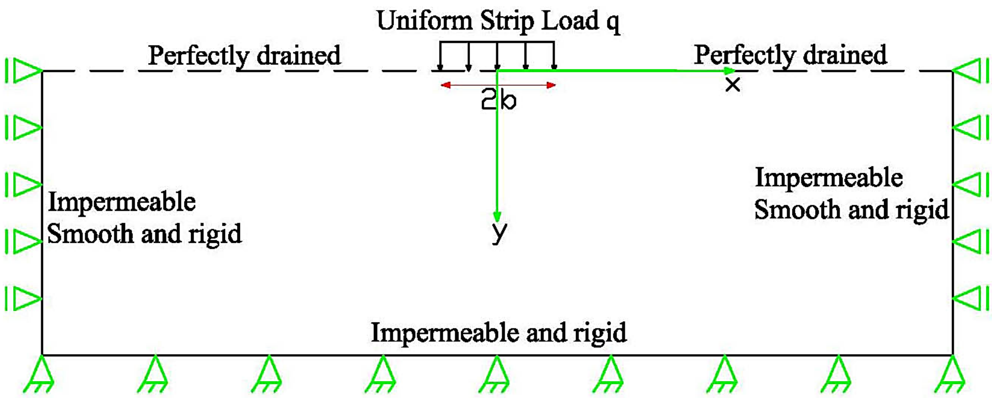

Figure 2. Geometry and boundary conditions of model

In this problem, p is excess pore pressure, u_x is deformation of model in the x direction, u_y is deformation of model in the y direction, H is model height, and q is the intensity of uniform applied loading. Furthermore, regular triangular elements are obtained for this model based on division of each length unit to 4 sections. Each triangular element is featured by 3 nodes. The model geometry and material properties are presented in Table 1 as adopted from references [7] and [8].

Table 1. Material properties of model

Parameters	Symbol	Value	Unit
Young's modulus	E	30	KPa
Poisson's ratio	υ	0.2	-
Permeability	K	1e-3	m/s
Load	q	1	KPa
Height	H	5	m
Width	W	16	m
Load width	2b	2	m

In the first step, the validity of the computations through the developed computer code has been verified based on the results of close-form-solution of Terzaghi's theory. Although the material properties and geometry dimensions are identical to what is presented in Table1, in the validation stage the applied load is assumed to be uniform and spread all over the model surface. Furthermore, the boundary and initial conditions are also considered as to match and satisfy Terzaghi's consolidation conditions. The two-dimensional consolidation computer program results for both coupled and uncoupled approaches are then compared with those of the close-form-solution of Terzaghi's consolidation.

6. Numerical Analysis Results

This section describes the results of the numerical analysis of the proposed model. In this case, at first the verification of prepared Matlab code for performing consolidation analysis using Terzaghi's analytical solution is developed. Then, the excess pore pressure and deformation of soil mass due to strip loading over the soil mass is extended. Finally, the results of two approaches are compared to each other.

6.1. Verification

To validate the developed computer program, the results of two-dimensional coupled and uncoupled consolidation analyses have been compared with those of Terzaghi's equations. The analyses were made for the selected model with geometry and material properties, as described in the previous section. It should be noted that, although the considered model is a two-dimensional medium, for validation purpose it was assumed to undergo a widespread surface uniform load to satisfy Terzaghi's one-dimensional consolidation conditions.

Based on the analyses results, the distribution of excess pore pressure across the model height within 100 seconds duration has been presented in Figure 3. Moreover, surface settlement variations with time have been shown in Figure 4.

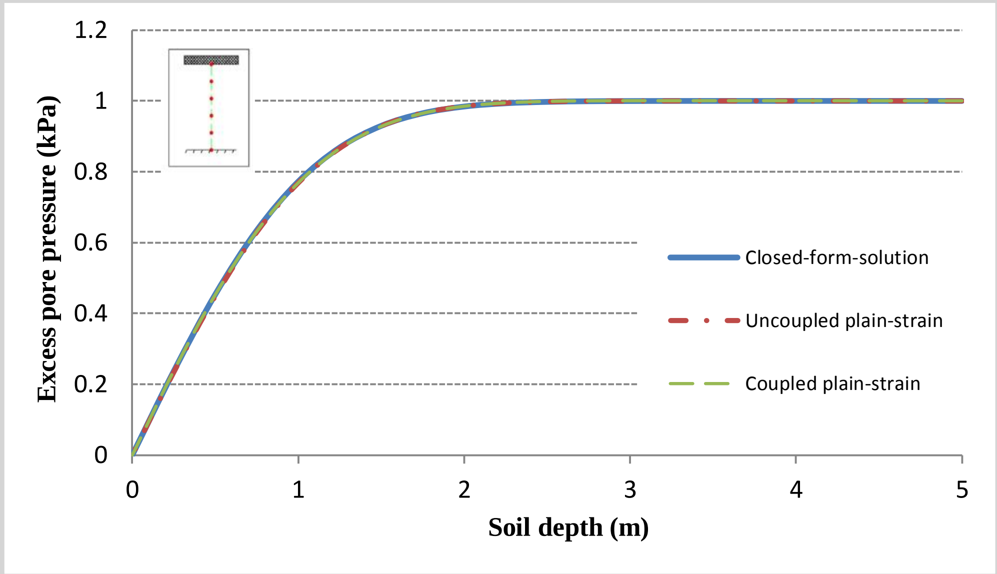

Figure 3. Comparison of excess pore pressure of coupled and uncoupled consolidation analysis with Terzaghi's consolidation analysis at t = 100 seconds

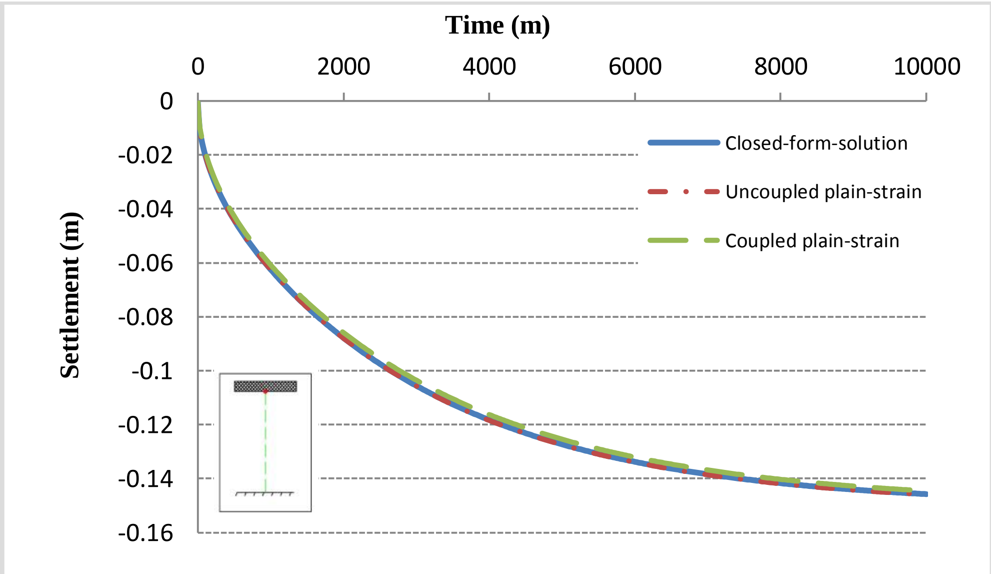

Figure 4. Comparison of surface settlement of coupled and uncoupled consolidation analysis with Terzaghi's consolidation analysis

As indicated in Figures 3 and 4, the results of both coupled and uncoupled consolidation have a perfect compatibility with Terzaghi's results. This can be considered as an acceptable evidence of the validity of the developed computer program.

6.2. Comparison of Coupled and Uncoupled Consolidation Results

For the assumed plane-strain model, as presented in Figure 2. both excess pore pressures and settlements obtained from coupled and uncoupled consolidation analyses are compared and discussed in this section. The excess pore pressure distributions across the footing centreline section, as well as another section with 2 meters distant from the centreline within 1000 seconds duration have been presented respectively in Figures 5 and 6. Also, the analyses results of excess pore pressure across the same sections for 5000 seconds duration have been shown in Figures 7 and 8.

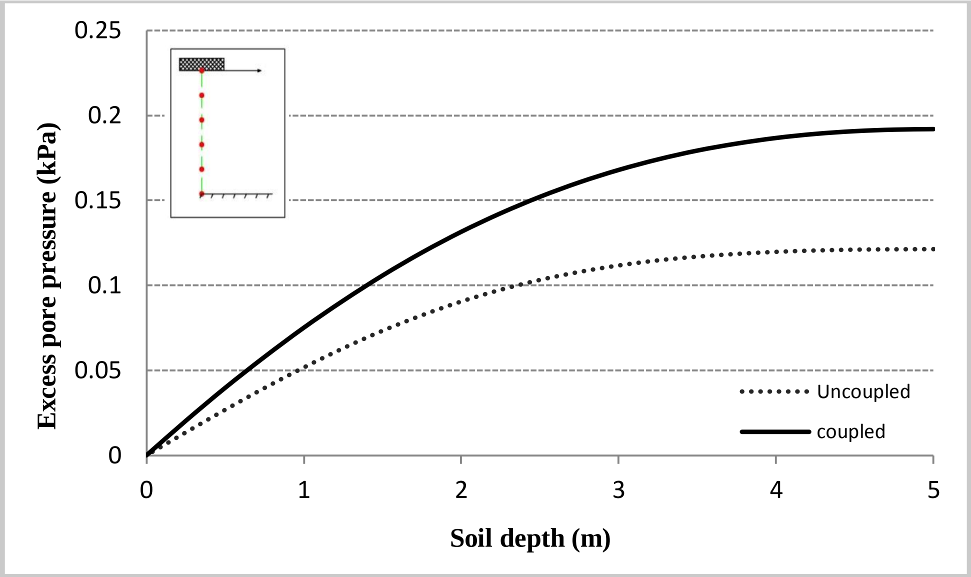

Figure 5. Comparison of excess pore pressure of coupled and uncoupled consolidation analysis along the center line of foundation at t = 1000 seconds

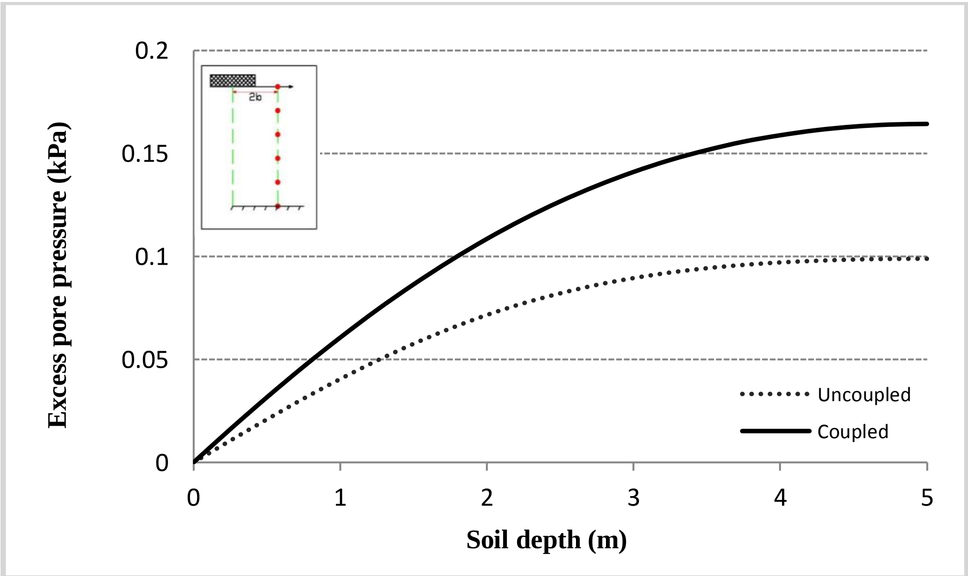

Figure 6. Comparison of excess pore pressure of coupled and uncoupled consolidation analysis along the distance of 2 meters from the center line of foundation at t = 1000 seconds

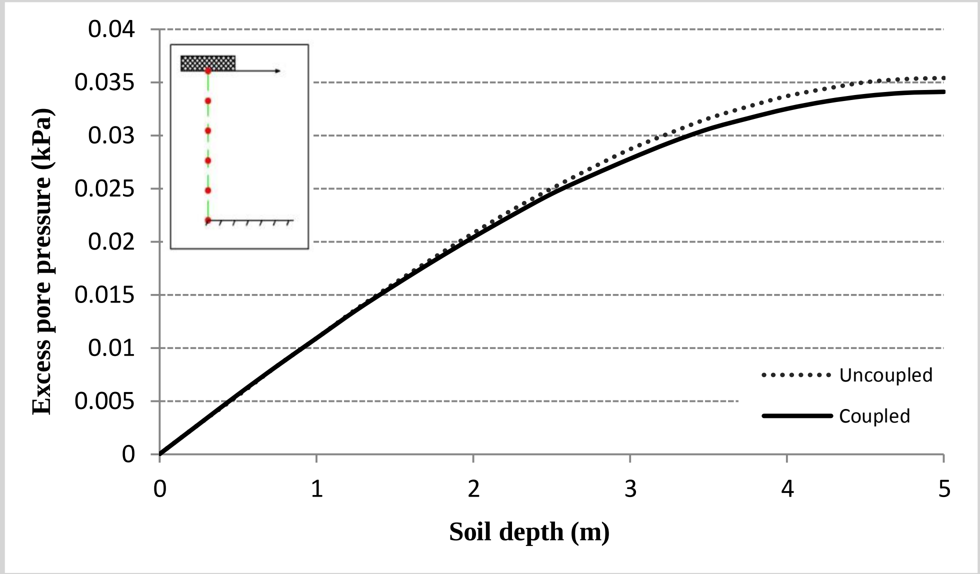

Figure 7. Comparison of excess pore pressure of coupled and uncoupled consolidation analysis along the center line of foundation at t = 5000 seconds

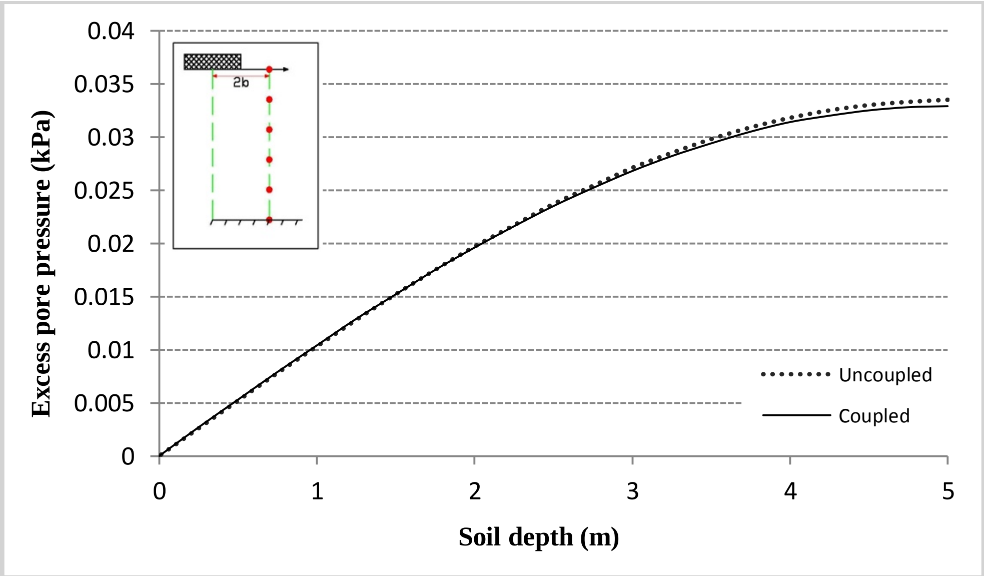

Figure 8. Comparison of excess pore pressure of coupled and uncoupled consolidation analysis along the distance of 2 meters from the center line of foundation at t = 5000 seconds

As observed in Figures 5 and 6, there is a considerable difference between excess pore pressure from coupled and uncoupled analyses in primary times of consolidation. In fact, coupled consolidation analysis has led to excess pore pressure higher than that of uncoupled analysis. This difference may be explained by the fact that the initial increase of excess pore pressure in the coupled analysis, before dissipation onset, may be regarded as the coupling effects of consolidation equations. In more detail, when excess pore pressure starts to distribute and create deformations in boundaries, it makes an immediate effect on other parts of the soil mass as the temporary increase of excess pore pressure. Nevertheless, in longer time, the excess pore pressures from both analyses converge to each other (Figures 7 and 8). Thus, it may be inferred that resulting excess pore pressure from both coupled and uncoupled consolidation analyses reach close compatibility with time elapsing. In other words, the initial additional excess pore pressure from coupled consolidation analysis dissipates within a relatively long time, so that the ultimate differences between coupled and uncoupled analysis would be insignificant. As seen in Figure 9, at primary short times of consolidation, the excess pore pressure in the middle of model from coupled consolidation analysis can be higher than the applied surface pressure q: such temporary increase, that tends to vanish with time elapsing is known as Mandel-Crayer's effect [1]. Of course, such effect is not observed by the uncoupled consolidation results.

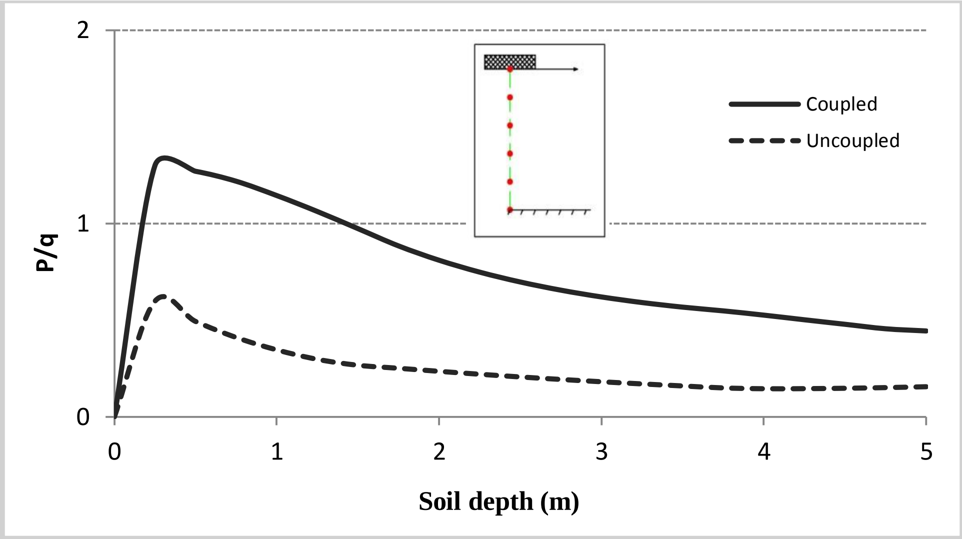

Figure 9. Comparison of coupled and uncoupled primary consolidation analysis

As another consolidation analyses finding, settlement values at various locations of the selected model have been plotted versus time in Figures 10 and 11. Figure 10. shows final consolidation surface settlement at the centreline of the footing while Figure 11. presents the surface settlement at 2 m offset of the footing centreline.

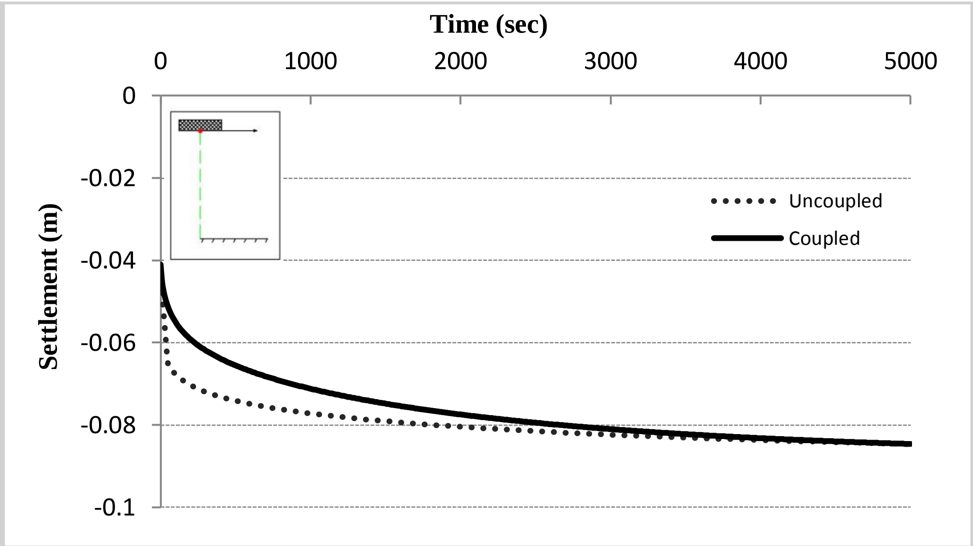

Figure 10. Comparison of consolidation settlement in surface of foundation centre

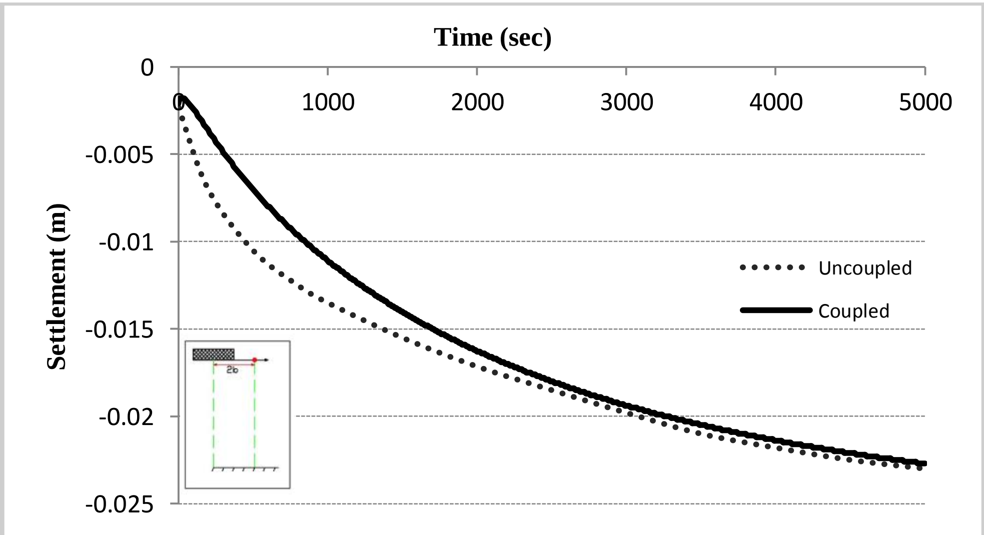

Figure 11. Comparison of consolidation settlement in surface and 2 meter distance of foundation centre

As can be seen in Figures 10 and 11, the initial settlement of uncoupled analysis agrees well with the coupled analysis. This occurs because in this study the elastic behaviour of soil is considered. On the contrary, during the first stage of consolidation the settlement calculated by the uncoupled analysis is higher; of course, such result may be attributed to the initial additional excess pore pressure resulting from coupled analysis. Despite the considerable difference in primarily times of settlement values, the final settlements are nearly the same from both the coupled and uncoupled approaches.

Furthermore, the distribution of settlement in the whole model has been presented in Figure 12. As expected, the maximum settlement is seen under the centre of footing, while minimal settlement values occur at far distance around the footing sides. To evaluate the effect of soil Poisson's ratio on the consolidation features, the analyses results for two different Poisson's ratios have been presented and compared in Figures 13 and 14.

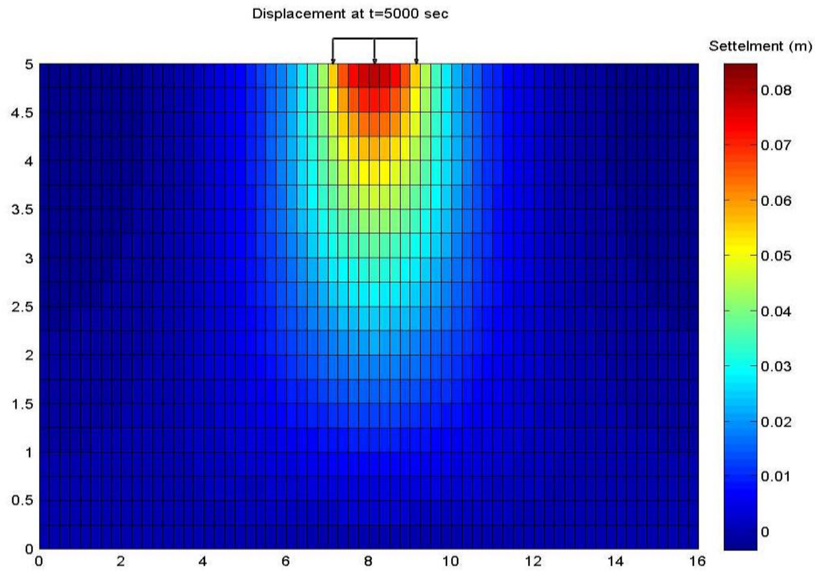

Figure 12. Distribution of settlement for the foundation at t = 5000s

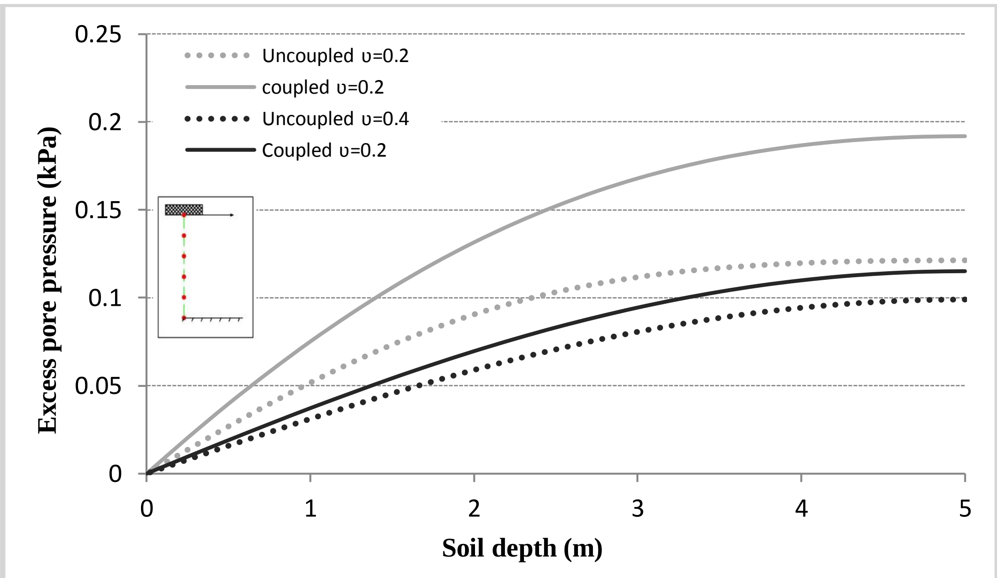

Figure 13. Excess pore pressure comparison of two different Poisson ratio's along the centre line of the foundation

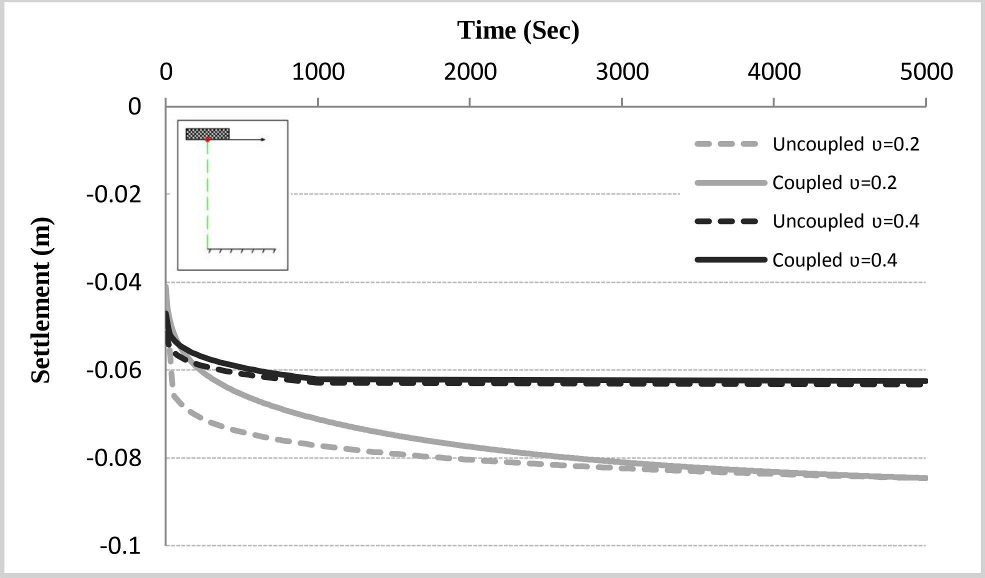

Figure 14. Settlement comparison of two different Poisson ratio's at surface of the foundation

As seen in Figure 14. an increase in the medium Poisson's ratio leads to a decrease in the produced excess pore pressure, which also dissipates in shorter time. Furthermore, Figure 15. shows that when Poisson's ratio increases, settlements decrease in both coupled and uncoupled analysis approaches. Nevertheless, with using higher Poisson's ratios, the settlements from the coupled and uncoupled consolidation analyses converge much faster.

7. Conclusion

The coupled and uncoupled consolidation equations have been solved and the comparison between their results has been performed in the current paper. In this context, the equations of both approaches have been solved using the finite element method. A consolidation computer program has been developed for plane-strain conditions in Matlab. Making several consolidation analyses through the developed computer program with both coupled and uncoupled approaches, the excess pore pressure and settlements have been compared. Altogether, the following conclusions may be outlined based on the obtained analyses results:

- Verification of developed computer program with exact solution of Terzaghi's theory shows that the developed computer program can appropriately simulate both coupled and uncoupled consolidation problems in poroelastic conditions.
- Excess pore pressures obtained from coupled and uncoupled consolidations, when the consolidation starts, are different. However, they show similar results in steady state situations.
- Although final settlements of both coupled and uncoupled consolidation analyses are approximately identical, there is a considerable difference between primarily settlements obtained from these approaches.
- Because it shows more deformation and settlement values, uncoupled consolidation analysis is more conservative that coupled analysis for design purposes. Moreover, the uncoupled consolidation analysis cannot show the Mandel-Crayer's effect in soil mass. Thus, coupled analysis is more practical for evaluating soil consolidation.
- Increasing Poisson's ratio leads to a decrease in the produced excess pore pressure and settlements. Further at higher Poisson's ratio, settlements from coupled and uncoupled consolidation analyses converge faster.

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