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Synthesis of Guaranteed Stability Regions of a Nonstationary Nonlinear System with a Fuzzy Controller

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Abstract

The paper proposes a method for constructing guaranteed regions of stability of nonstationary nonlinear systems on the plane of parameters of a fuzzy PID controller. It is shown that this method allows to determine the full stability areas, which are significantly larger than the areas determined by classical methods (frequency circle criterion, quadratic Lyapunov functions). This improvement is achieved by using the algorithm for constructing spline Lyapunov functions. This type of Lyapunov functions is based on the necessary and sufficient conditions of stability, while the classical methods are only sufficient conditions of stability. In this regard, on the basis of the proposed method, it is possible to calculate the maximum sizes of the sectors in which the nonlinear characteristics in the channels of the fuzzy PID controller should be located. Examples of the synthesis of fuzzy P, PI, PID controllers for a nonstationary control object of the third order are given. Numerical experiments show that the expansion of the boundaries of nonlinear characteristics allows to improve the accuracy in the steady state, and also to almost double the speed of the automatic control system with a nonstationary object. The advantages over linear controllers are demonstrated. The proposed method guarantees the stability inside the calculated stability regions for any character of the change in the nonstationary parameter, as well as for any character of the change in the nonlinear characteristics in the corresponding sectors.

Keywords: Nonstationary Nonlinear System; Stability Regions; Lyapunov Functions; Circle Criteria; Spine Functions; PID Controllers; Fuzzy Logic; Adaptive Systems.

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1. Introduction

In recent decades, there has been a rapid increase in interest in the study of control systems for nonstationary objects. This is due to the fact that with the help of nonstationary models it is possible to describe rather complex technological objects in industry [1]. To achieve the required values of accuracy and speed of such a systems, artificial intelligence technologies are used in practice [2]. However, the analysis of the stability of intelligent control systems of nonstationary objects is a difficult task. On the other hand, the use of active controllers represents a perspective way of isolating the structure from earthquake-induced vibrations [3]. Since there are many uncertain parameters in the buildings and system coefficients are varying in the time, methods of absolute stability theory are proposed for control system synthesis [4].

As part of solving the problem of the stability of systems with nonstationary nonlinear elements, algorithms for constructing special Lyapunov functions were developed in [5-7]. These algorithms are based on the necessary and sufficient conditions for absolute stability [8-10], which makes it possible to identify guaranteed (full) regions of stability in the system parameter space. This has a practical importance in the design of fuzzy control systems in which fuzzy controllers perform nonlinear transformations that ensure the improvement of the quality characteristics of automatic control system (ACS) [11]. In this connection, the problem arises of determining the permissible sector inside of which

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a nonlinear characteristic of a fuzzy controller should be located.

This paper studies the problem of synthesis of nonlinear characteristics in the channels of a fuzzy PID controller with a nonstationary control object. The study proposes a new method to construct stability regions in the system parameter space. The main difference with a classical methods is to use spline Lyapunov functions instead of frequency criterias and quadratic Lyapunov functions. This advantage allows significant expand of stability regions and thus, improve quality of the control as well as stability reserve.

2. Research Methodology

The purpose of this section is to present a method which is able to model a system with uncertain time-varying parameters and technique for sythesiz fuzzy PID controllers. The analysis of bibliography (see references in [1-5, 8]) shows that ACS with nonstationary control objects can be described by differential equations of the form:

$$\frac{dx}{dt} = Ax + \sum_{j=1}^{m} b^j \varphi_j(\sigma_j, t),$$
$$\sigma_j = (c^j, x) = \sum_{i=1}^{d} c_i^j x_i, \quad \varphi_j(0,t) \equiv 0, \tag{1}$$

Where $x = (x_1, x_2, \ldots, x_d)^T$ – is a d-dimensional column vector of state variables, A is constant $(d \times d)$ matrix, b^j and c^j $(j = 1,\ldots,m)$ are constant d-dimensional vector- columns, m – is the number of nonstationary nonlinear elements, $(\cdot,\cdot)$ is the scalar product of vectors. It is assumed that the nonlinear nonstationary elements $\varphi_j(\sigma_j, t)$ inherent in fuzzy controllers satisfy the sector constraints.

$$\delta_j^1 \le \frac{\varphi_j(\sigma_j, t)}{\sigma_j} \le \delta_j^2$$

$$(-\infty < \delta_j^1 \le \delta_j^2 < \infty, \ \ j = 1,\ldots,m)$$

For all σ_j and t. A particular case of nonlinear nonstationary elements $\varphi(\sigma, t)$ are linear nonstationary elements $u(t)\sigma$, for which the variable coefficient satisfies the constraints $\delta^1 \le u(t) \le \delta^2$. A system with nonstationary linear and nonlinear elements and nonlinear characteristics in the controller channels can be described using Equations 1.

Algorithms based on the construction of Lyapunov functions and ensuring the construction of guaranteed stability regions were developed in [5-7]. The algorithm for constructing spline Lyapunov functions [7] is the most accurate and fast. In accordance with this algorithm, 2^m matrices A_k are introduced for the analysis of system (1), then the level set of the homogeneous smooth Lyapunov function is constructed. Moreover, as shown in [8-10], if the constructed Lyapunov function has a everywhere negative derivative for all 2^m linear systems $dx/dt = A_k x$, then the original system (1) will be asymptotically stable. To construct the matrices A_k, it is necessary to replace each nonlinear nonstationary element $\varphi_j(\sigma_j, t)$ in system (1) by δ_j^1 or δ_j^2. In consequence, the number of matrices A_k is equal to the number of all possible combinations of δ_j^1 and δ_j^2. Below are the main stages of the methodology for constructing stability areas:

Step 1. The equations of the system are made up in the form (1);

Step 2. The number m of elements $\varphi_j(\sigma_j, t)$ (including both nonlinear nonstationary and linear nonstationary elements) and the boundaries of the sectors of change δ_j^1, δ_j^2 are determined.

Step 3. A rectangular grid of nodes is defined on the plane of system parameters. Coordinate axes of the plane can serve the usual coefficients of the system and the boundaries of the sectors δ_j^1, δ_j^2;

Step 4. Matrices A_k are compiled for each node of the grid. To compute A_k we need to replace $\varphi_j(\sigma_j, t)$ in (1) with δ_j^1 or δ_j^2. The number of matrices A_k is equal to the number of all possible combinations δ_j^1, δ_j^2, namely, 2^m;

Step 5. Matrices A_k are used in the algorithms for constructing Lyapunov functions from [7]. As the result, a conclusion about the stability or instability of a system with given parameters is drawn.

Step 6. After calculating all the nodes of the parameter plane, the obtained discrete stability regions are approximated by smooth functions.

Consider as an example a nonstationary control system, the structure of which is shown in Figure 1, where $T_1 = 0.5$, $T_2 = 0.1$.

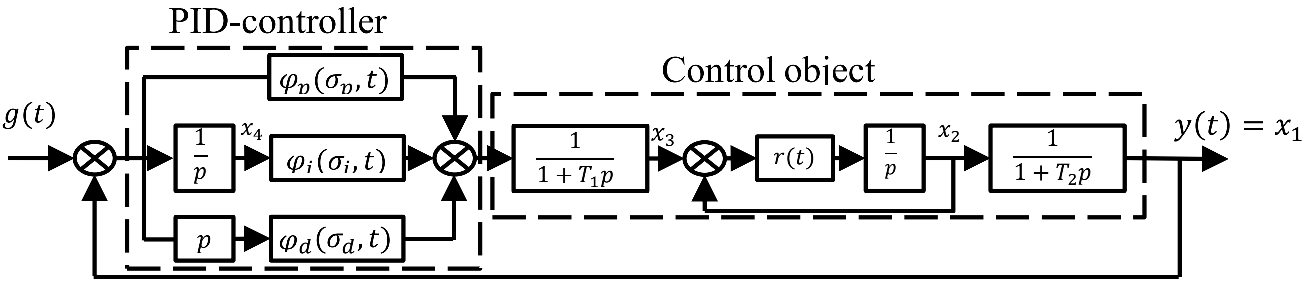


Figure 1. Structure of an ACS with a nonstationary control object and a fuzzy PID controller

As can be seen from the structural scheme, a third-order static control object contains a nonstationary element $r(t)$, which varies according to an unknown law within certain limits $\delta_r^1 \le r(t) \le \delta_r^2$, where $\delta_r^1 = 1$, $\delta_r^2 = 3$. Let us write the equations of system dynamics, shown in Figure 1, in the form (1):

$$\frac{dx}{dt} = Ax + b_p\varphi_p(\sigma_p, t) + b_i\varphi_i(\sigma_i, t) + b_d\varphi_d(\sigma_d, t) + b_r\varphi_r(\sigma_r, t),$$

$$A = \begin{pmatrix} -\frac{1}{T_2} & \frac{1}{T_2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_1} & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, b_p = b_i = b_d = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_1} \\ 0 \end{pmatrix}, b_r = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix},$$

$$c_p = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, c_i = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}, c_d = \begin{pmatrix} \frac{1}{T_2} \\ -\frac{1}{T_2} \\ 0 \\ 0 \end{pmatrix}, c_r = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \tag{2}$$

Where $\varphi_r(\sigma_r, t) = r(t)\sigma_r$, $\sigma_p = (c_p, x)$, $\sigma_i = (c_i, x)$, $\sigma_d = (c_d, x)$, $\sigma_r = (c_r, x)$.

For the ACS described by Equations 2, in the next three sections we study the guaranteed stability regions on planes whose coordinate axes are the sector boundaries of nonlinear characteristics in the channels of the fuzzy PID controller.

3. Synthesis of Stability Regions of ACS with a Fuzzy P Controller

Using the method described above, we calculate the stability regions on the plane of the parameters of the fuzzy P controller, in which the zero equilibrium position of system (2) is asymptotically stable. To do this we need to transform the Equation 2 as follows:

- Remove terms containing $\varphi_i(\sigma_i, t)$, $\varphi_d(\sigma_d, t)$;
- Delete the last row and last column from matrix A;
- Remove the last component from the vectors c_p and c_r.

Since the total number of nonstationary elements is two (one nonlinear element $\varphi_p(\sigma_p, t)$ and one linear $r(t)\sigma_r$), then the total number of matrices A_k is $2^2 = 4$. We define the boundaries of the nonlinear characteristic of the fuzzy P controller as $\delta_p^1 = k_p$, $\delta_p^2 = k_p + \Delta k_p$, then at $k_p \ge 0$ and $\Delta k_p \ge 0$ it follows that $\delta_p^2 \ge \delta_p^1$. To build stability regions, we will change the k_p value from 0.1 to 10, and Δk_p from 0.0 to 10 with a step of 0.1, taking into account the fact that for these k_p values the required accuracy is ensured in the steady state. The calculation results are illustrated in Figure 2.

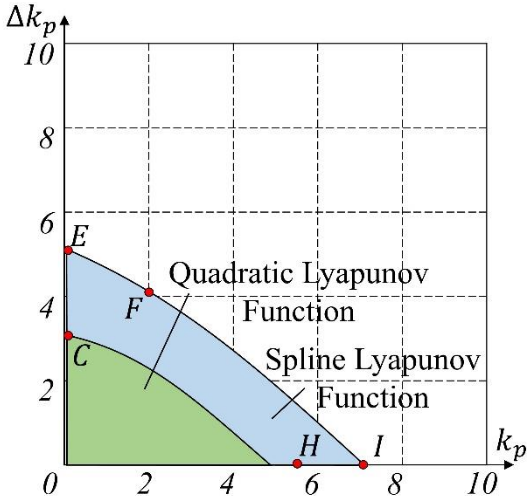
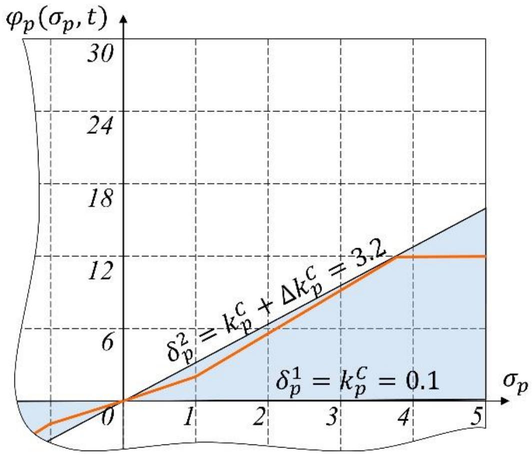


Figure 2. The sectors boundaries of the nonlinear characteristics of the fuzzy P controller, providing guaranteed stability of nonstationary system (2)

Figure 2 shows a comparison of the regions calculated on the basis of the algorithm for constructing spline Lyapunov functions and on the basis of quadratic Lyapunov functions. The choice of quadratic Lyapunov functions as an alternative method of analysis is due to the fact that frequency criteria in the presence of more than one nonstationary element ($m \geq 2$) lose their simplicity and graphic visibility [12] and require, generally speaking, the use of numerical methods for calculating parameters [13]. Moreover, for $m \geq 2$, the frequency criteria go over into sufficient conditions for the existence of a quadratic Lyapunov function [14, 15]. The last one means that the system may have a quadratic Lyapunov function, but the frequency criteria will not be realized. That's why the presence of more than one nonstationary element in the system, leads to usage of numerical methods of convex optimization for constructing quadratic Lyapunov functions [16, 17].

Analysis of Figure 2 shows that method proposed in this paper gives bigger regions of stability than method based on quadratic Lyapunov functions. This means that sector boundaries of the nonlinear characteristic sector of a fuzzy P controller can be significantly expanded. Indeed, with the same lower sector boundaries $\delta_p^1 = k_p^E = k_p^C = 0.1$, the upper limit calculated on the basis of Lyapunov's spline functions, $\delta_p^2 = k_p^E + \Delta k_p^E \approx 5.3$ (point E in Figure 2) is 1.6 times larger than the upper limit calculated on the basis of quadratic Lyapunov functions, $\delta_p^2 = k_p^C + \Delta k_p^C \approx 3.2$ (point C in Figure 2). Note that the upper limit of the sector can be further raised by raising the lower limit. For point F, the lower limit is $\delta_p^1 = k_p^F = 2.0$, and the upper one is $\delta_p^2 = k_p^F + \Delta k_p^F \approx 6.2$, which is 15% more than for point E. In this regard, the guaranteed stability region of the ACS is increased and the possibility to enhance the accuracy by rising k_p is created.

The nature of nonlinear transformations implemented by a fuzzy controller is studied in detail in [18-20], where it is shown that the generalized form of such transformations corresponds to Figure 3. The specific parameters of such a nonlinear relation are determined by the requirements for stability, accuracy and quality indicators. Below, from the standpoint of such transformations, the problem of the synthesis of fuzzy P, PI, and PID controllers for linear nonstationary objects is considered.



a)

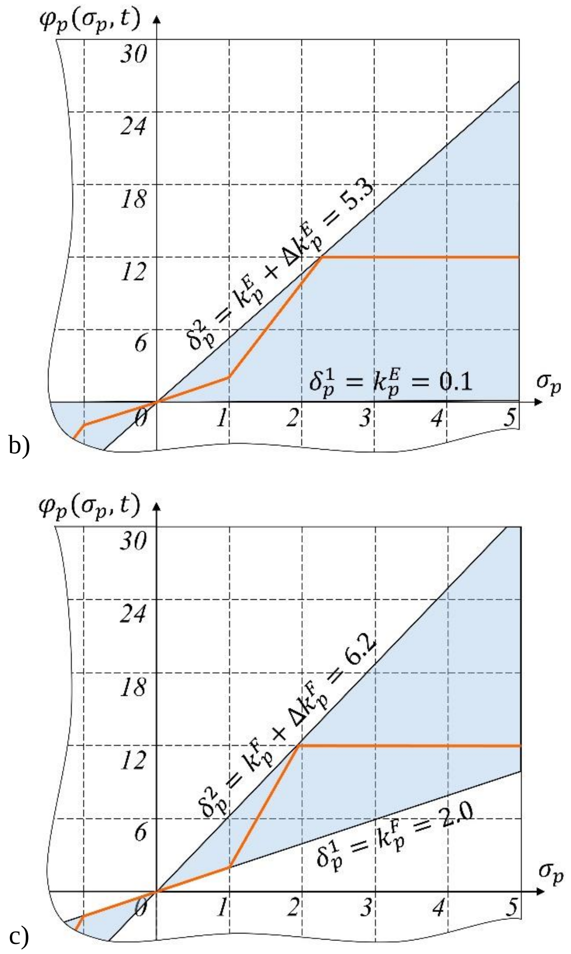


Figure 3. Nonlinear characteristics of the P channel inside the sectors, calculated on the basis of quadratic and spline Lyapunov functions

The use of Lyapunov spline functions allows to increase the size of the sector, inside which the nonlinear characteristic is located, and, accordingly, the inclination of the central part of the nonlinearity (with the same level of the horizontal part). Indeed, in Figure 3.a nonlinear characteristic is built inside the sector, calculated on the basis of quadratic Lyapunov functions (point *C* in Figure 2), and in Figure 3.b – inside the sector, calculated on the basis of spline functions (point *E* in Figure 2). The greater inclination of the central section has a positive effect on the accuracy of the ACS with a fuzzy P controller in the input range $g(t) > 5$, which is clearly seen in Figure 4.

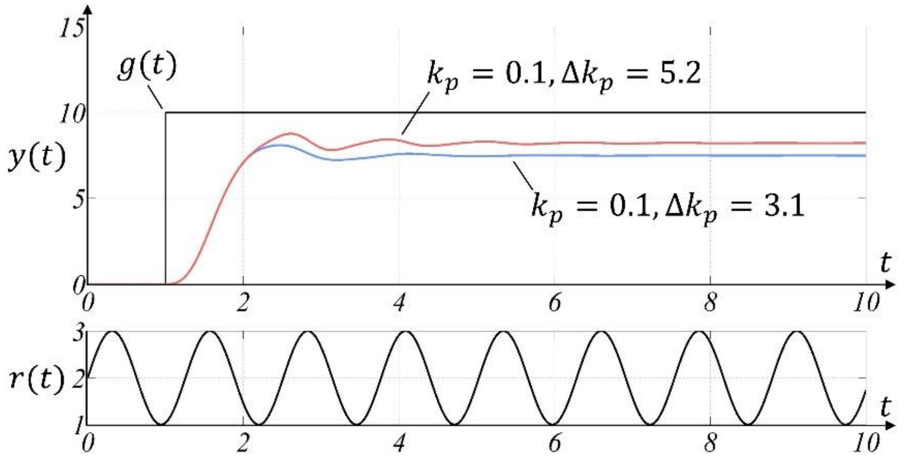


Figure 4. Transients in automatic control systems with different nonlinear characteristics in the P channel of a fuzzy controller

A further increase in the inclination of the central section is possible by raising the lower boundary of the nonlinearity sector. In Figure 3.c, the lower boundary of the sector coincides with the first part of the nonlinearity (point *F* in Figure 2). A study of the benefits of a fuzzy P controller with such a nonlinear characteristic over conventional linear P controller is of great interest. Comparison of transient processes in nonstationary ACS with linear and fuzzy P controllers is illustrated in Figure 5, where the gain of the P controller is equal to $k_p = 5.5$ (point *H* in Figure 2).

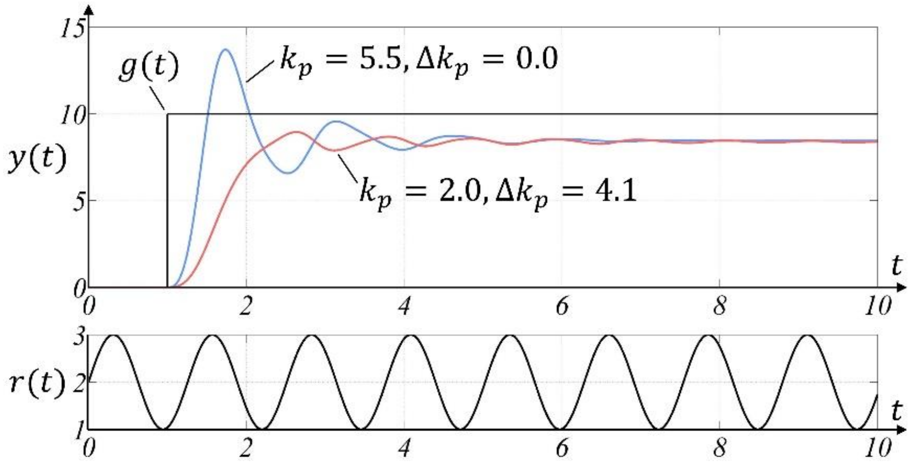


Figure 5. Comparison of transient processes in ACS with linear and fuzzy P controllers

Analysis of transient curves shows that ACS with fuzzy and linear P controller have the same error in steady state. However, an ACS with a fuzzy controller almost completely suppresses the influence of a nonstationary parameter and provides significantly better indicators of the quality of the transition process (no overshoot). It should be noted that the gain of the linear P-controller can be increased to $k_p = 7.1$ (point *I* in Figure 2), but together with an increase in the accuracy of such an ACS, the value of the overshoot will increase even more. In this connection, the usage of fuzzy P controllers instead of linear allows to improve the quality of control of nonstationary objects. At the same time, if the presence of an error in the steady state is unacceptable, then it is necessary to consider other modifications of the fuzzy controller. The next section focuses on the synthesis of a fuzzy PI controller based on guaranteed areas of stability

4. Synthesis of Stability Regions of ACS with a Fuzzy PI Controller

In accordance with the proposed method, we calculate the zero equilibrium point stability regions on the parameters plane of the fuzzy PI controller. In this case, it is necessary to remove the term containing $\varphi_d(\sigma_d, t)$ from Equations 2. Since the total number of nonstationary elements is three (two nonlinear elements $\varphi_p(\sigma_p, t)$ and $\varphi_i(\sigma_i, t)$, as well as one linear $r(t)\sigma_r$), the total number of matrices A_k is $2^3 = 8$. By analogy with the previous section, the boundaries of the nonlinear characteristic of the P channel are defined as $\delta_p^1 = k_p$, $\delta_p^2 = k_p + \Delta k_p$, and for the I channel as – $\delta_i^1 = k_i$, $\delta_i^2 = k_i + \Delta k_i$. To build stability regions, we will change the Δk_p value from 0.0 to 10, and Δk_i from 0.0 to 5 with a step of 0.1. The complete stability regions constructed on the basis of the Lyapunov spline functions for $k_p = k_i = 0.1$ and for $k_p = 2.0$, $k_i = 0.5$ are presented in Figure 6.

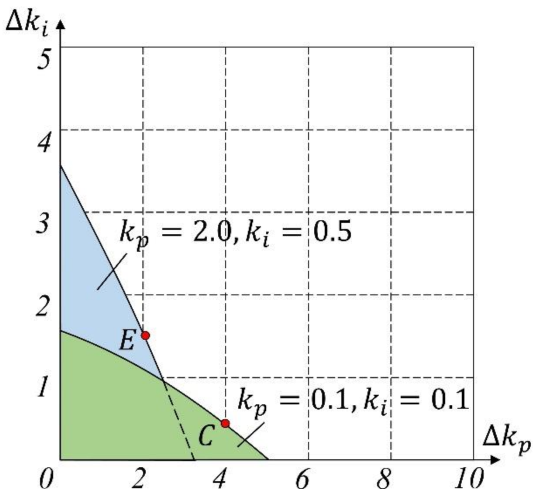


Figure 6. Sectors boundaries of the nonlinear characteristics of the fuzzy PI controller, providing guaranteed stability of nonstationary system (2)

Let us estimate the impact of the areas obtained on the quality characteristics of the ACS. So, for point *C* in Figure 6, the value of the lower sector boundaries is $\delta_p^1 = \delta_i^1 = 0.1$, while the value of the upper boundaries is $\delta_p^2 = 4.1$, $\delta_i^2 = 0.6$. Note that by changing k_p, k_i, we can increase the upper limit of the sector by almost 4 times. Indeed, for point *E* in Figure 6 $\delta_i^2 = k_i^E + \Delta k_i^E = 2.0$. It can also be noted that the upper boundary of the nonlinear characteristic sector in the P channel for points *C* and *E* (Figure 6) coincide $\delta_p^2 = k_p^C + \Delta k_p = k_p^E + \Delta k_p^E \approx 4.1$. That's why, we use the characteristic shown in Figure 3.a as a nonlinearity of the P channel of fuzzy controller. As the nonlinearity of the I channel, we use the characteristic from [18]. The general form of nonlinearities for points *C* and *E* (Figure 6) is illustrated in Figure 7.a and 7.b respectively.

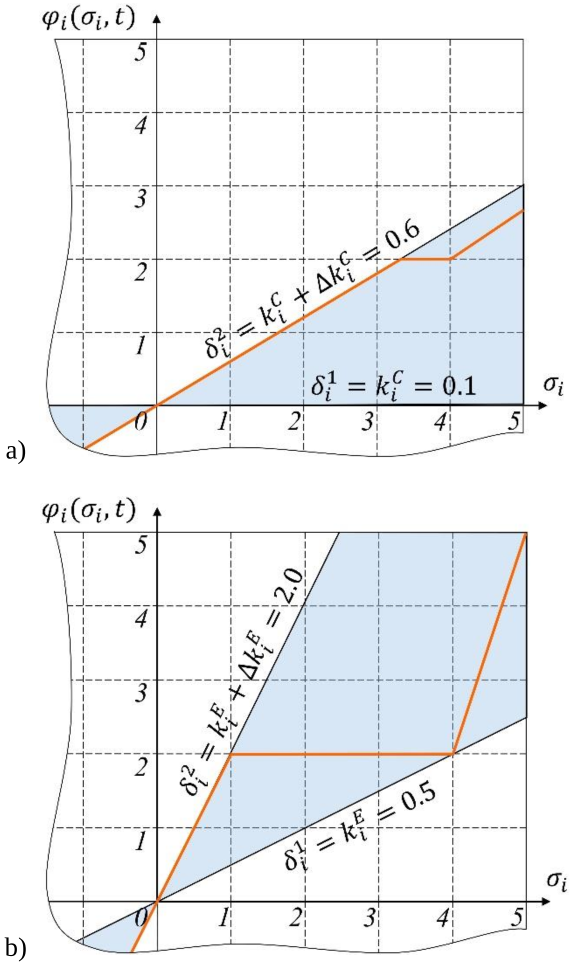


Figure 7. Nonlinear characteristics of the I channel inside the sectors, calculated on the basis of spline Lyapunov functions

From Figure 7, it is easy to see that raising the upper boundary of the nonlinearity sector allowed to increase the inclination of the first and last characteristic sections, leaving the horizontal segment at the same level. At the same time, the speed of the system also changes, as can be seen from the analysis of transients in Figure 8. If for the characteristic shown in Figure 7.a, the time of entry into the 5% zone of the established value is 9 s, then for the characteristic in Figure 7.b this time is reduced to 4 s.

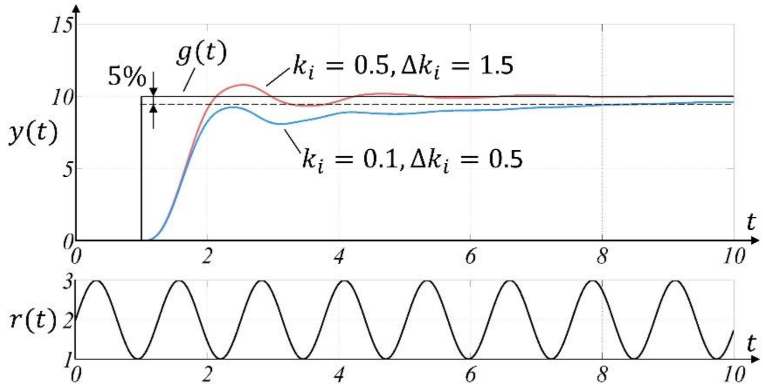


Figure 8. Comparison of transients of nonstationary ACS with different nonlinear characteristics in the I channel of a fuzzy PI controller

In this regard, the connection of the I channel allowed to get rid of the steady-state error, however, an overshoot occurred. In order to eliminate this effect, consider the possibility of synthesizing a fuzzy PID controller.

5. Synthesis of Stability Regions of ACS with a Fuzzy PID Controller

By the method described above, we calculate the stability regions on the plane of parameters of the fuzzy PID controller. In this case, the total number of nonstationary elements is four: three nonlinear elements $\varphi_p(\sigma_p, t)$, $\varphi_i(\sigma_i, t)$, $\varphi_d(\sigma_d, t)$ and one linear $r(t)\sigma_r$. Therefore, the number of matrices A_k is $2^4 = 16$. We define the boundaries of the sector of the nonlinear characteristic of the D channel in the form $\delta_d^1 = k_d$, $\delta_d^2 = k_d + \Delta k_d$. We calculate the full stability regions on the plane Δk_p, Δk_i, while setting $k_p = 2.0$, $k_i = 0.5$, $k_d = 0.3$. The results are presented in Figure 9.

Comparing the areas in Figure 6 and 9, it can be noted that the introduction of a linear characteristic into the D channel (the region with $\Delta k_d = 0.0$, i.e. when the upper and lower boundaries of the D channel sector coincide) makes it possible to almost double the region of the guaranteed stability of a nonstationary system. At the same time, raising the upper boundary of the D channel sector sharply narrows the regions of stability. Already at $\Delta k_d = 0.2$, the region in Figure 9 is smaller than the corresponding area in Figure 6.

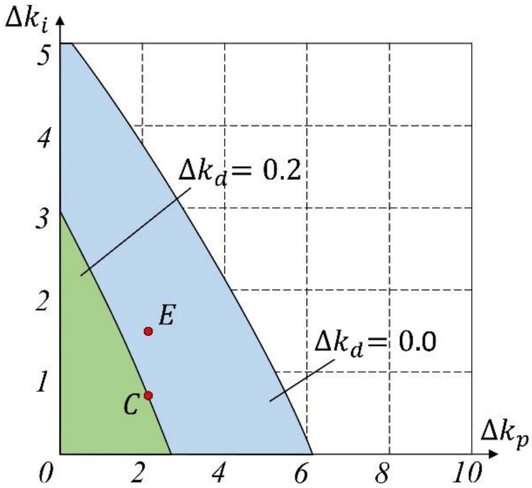


Figure 9. Sectors boundaries of the nonlinear characteristics of the fuzzy PI controller, providing guaranteed stability of nonstationary system (2)

However, for the given nonlinear characteristics in the previous sections (the characteristic in Figure 3.a for the P channel and the characteristic in Figure 7.b for the I channel), the introduction of a linear D channel with the coefficient $k_d = 0.3$ (point *E* in Figure 9), does not allow to get rid of overshoots. This circumstance is seen from the comparison of the corresponding curves of the transition process in Figure 9 and 10. A further increase of the coefficient k_d and the upper boundaries of the P and I channel will only increase the overshoot and oscillation.

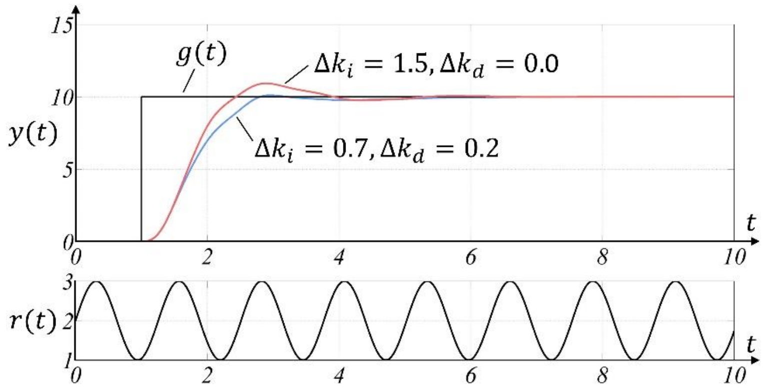


Figure 10. Comparison of transients of nonstationary ACS with linear and nonlinear characteristics in the D channel of a fuzzy PID controller

For point *C* in Figure 9, it is possible to synthesize nonlinearity in the limits $\delta_d^1 = k_d^C = 0.3$, $\delta_d^2 = k_d^C + \Delta k_d^C = 0.5$. As a nonlinear characteristic, it is proposed to use the characteristic from [18]. The specific form of nonlinearity for this case is presented in Figure 11. Note that to use a fuzzy D channel, it is necessary to reduce the upper limit of the I channel. Indeed, for point *C*, the value of the upper boundary is $\delta_i^2 = k_i^C + \Delta k_i^C = 0.7$. The transition process for this system is illustrated in Figure 10. Analysis of this transient process shows that the use of a fuzzy PID controller with nonlinear characteristics in all channels allows to achieve high quality and performance indicators for ACS with a nonstationary control object.

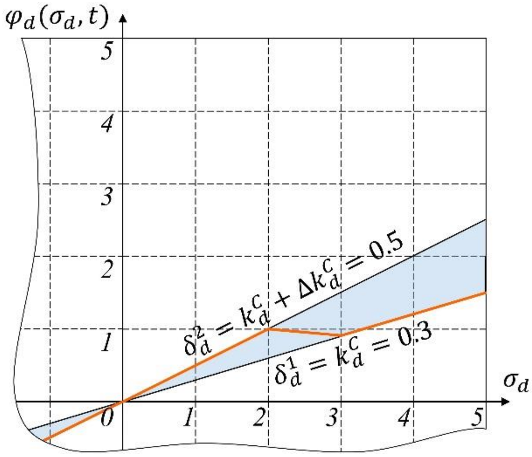


Figure 11. Nonlinear characteristics of the D channel inside the sectors, calculated on the basis of spline Lyapunov functions

In this paper, it was assumed that the nonstationary parameter $r(t)$ varies sinusoidally. It is worth noting that the method of calculating the stability regions proposed in this work guarantees stability for any character of a change in a nonstationary parameter within given limits [8-10]. Moreover, the nonlinear characteristics of fuzzy controllers may change arbitrarily within the calculated sectors over time. The last circumstance makes it possible to use the proposed methodology in the design of adaptive systems built on the basis of fuzzy logic, in which the parameters of a fuzzy controller are either changed by commands from a tactical level of control, or configured in the process of self-learning [21-23].

6. Conclusion

The developed method of constructing the stability regions of nonstationary ACS allows to determine the guaranteed sectors of nonlinear characteristics of a fuzzy PID controller. In contrast to the well-known frequency methods and quadratic Lyapunov functions, this advantage over traditional sufficient conditions is achieved by using spline Lyapunov functions, which determine the necessary and sufficient stability conditions. It is shown that the fuzzy PID controller has a number of indisputable advantages over a linear PID controller in the tasks of controlling nonstationary objects (in particular, it can improve the accuracy and quality in the steady state). The proposed method can be used to analyze systems with an arbitrary nature of a change in a nonstationary parameter in a given range. A promising objective of the study is the use of calculated regions of stability for the synthesis of intelligent ACS with self-learning algorithms.

7. Funding

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8. Conflicts of Interest

The authors declare no conflict of interest.

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