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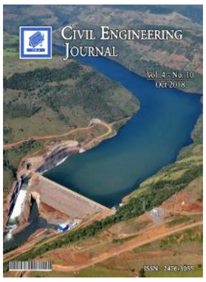

Application of Nor Sand Constitutive Model in a Highway Fill Embankment Slope Stability Failure Study

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Abstract

This paper presents a case study of a static load induced liquefaction in a simple roadway widening project constructed in north eastern part of Ohio in 2008. The widening required an embankment fill, which moved nearly 4 feet vertically and 1 foot laterally after two days of installation. The main objective of the work is to demonstrate how a simple Constitutive model, in this case Nor Sand model, can represent the static liquefaction in loose sand layers under specific conditions. A set of parameters is assumed based on the soil properties and an Excel Spreadsheet is used for simulations of triaxial compression of sand. It was considered that the situation which led to the failure, and the situation after the solution adopted. Moreover, slope stability analysis is provided for validation of the original results using a commercial software. It was found that the model can represent through stress strain curves and stress paths the behavior of the soil layer which led to the embankment fill movement. As the original work considered only slope stability analysis to explain this phenomenon, the present study shows a different approach for the case study, and this is the main contribution of this research.

Keywords: Static Liquefaction; Nor Sand; Slope Stability Analysis; Road Embankment Failure.

1. Introduction

In October 2008 a simple roadway widening constructed in northern Summit County, Ohio, had some issues during the construction. The situation is presented in [1]. It was a 25-foot wide and 15-foot high embankment. Right after the conclusion of fill placements activities, some shear cracking started in the embankment fill (Figure 1). After two days the placed fill had considerably moved (3.5 ft vertically and more than 1.0 ft horizontally) (Figure 2).

The two initial premises for the problem were vertical settlement and rotational slope failure. Four (4) soil borings were made utilizing both Standard Penetration Testing (SPT) and Cone Penetration Testing (CPT). After finishing the exploration program, an analysis was done to estimate the settlement, and it was concluded that this settlement could not be the cause of the failure. Therefore, a slope stability analysis was provided considering a typical section (Figure 3) and was able to simulate static liquefaction caused slope failure for this situation.

Two loading conditions were evaluated to simulate the static liquefaction in the sand layers: undrained condition (during failure) and drained condition (after installation of internal drainage). The undrained condition could be considered for this case because of the confining of loose sand layers between two clay layers (Figure 4). The factors of safety demonstrated that the undrained situation was not in a stable situation, and that a drained situation could change this. Therefore, the only required solution would be a way to change the condition of the loose sands from undrained to drained. The best solution presented in [1] was the installation of wick drains, therefore relieving the pore water pressure

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in the underlying sands. The mechanism that caused the rotational slope failure was the liquefaction of the sand layers beneath the road embankment, and the results corroborate this proposal.

Figure 1. Shear Cracking begins in embankment fill at the end of fill placement activities [1]

Figure 2. A 3.5-foot high scarp formed in the 15-foot fill over a 2-day period [1]

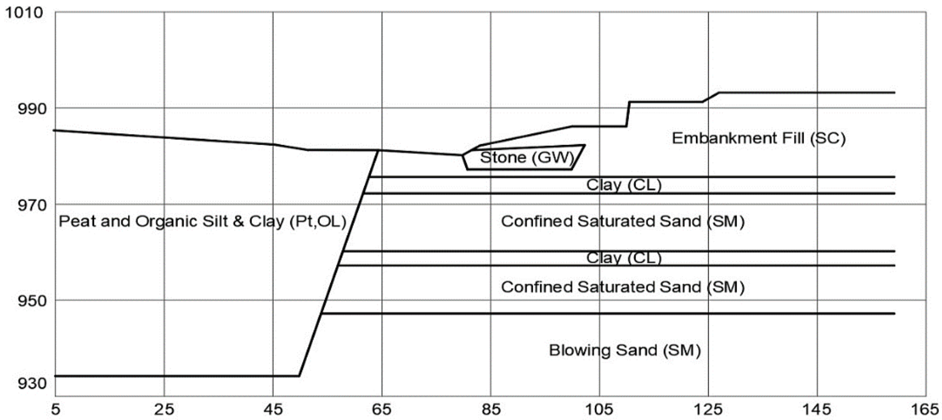

Figure 3. Typical section for Slope Stability Analysis (Adapted from [1])

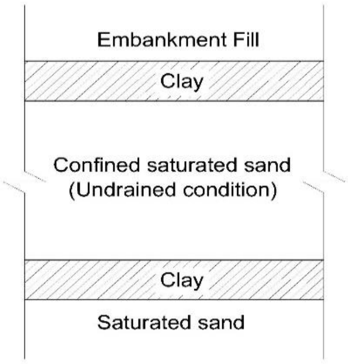

Figure 4. Layers beneath the road embankment

The study described above considered slope stability analysis to describe the failure due to static liquefaction. Another approach to this case study would be applying a constitutive model in the loose sand layers. Numerous constitutive models have been developed for modeling the stress/strain behavior of soils [2]. Some of them are also able to represent static liquefaction of loose sands.

It is important to understand the evolution of soil behavior models that has led to more realistic and complex models in the last years. The first model that included the void ratio as a variable to represent sand was developed in the Cambridge school. The term 'critical states' was included, leading to the framework of soil behavior called 'critical states soil mechanics' [3]. In this framework, the coupling of yield surface size to void ratio explains why and how soil behavior changes with density. Three models were developed: Cam clay [4], modified Cam clay [5] and Granta gravel [3]. However, these models were rarely used for sands. One of the reasons is that they cannot reproduce observed softening and dilatancy for sands that are on the dense side of critical, virtually all practical situations.

The limitations of early Cambridge models in the representation of sands and an increasing interest in topics such as sand liquefaction and flow slides led a great number of researches to develop advanced constitutive models for sands [6-11]. Each model has its particularities and experimental validation, but the idea always relies in mechanical theories and variables that affect the sand behavior. After reviewing some related works, it was concluded that the Nor Sand model [6] would be a simple and useful way to analyze the liquefaction of loose sand through a conventional triaxial data. Nor Sand constitutive model presents a theoretical framework capable of represent loose sands during liquefaction and is being used in this work.

The reported highway fill embankment study is a very important geotechnical engineering case study which has been utilized by article [12] in highlighting the importance of geotechnics encompassing not only rock and soil mechanics but also engineering geology. Li et al. (2018) [13] has been utilizing the case study in synthesizing what and how they have used the case study in enhancing under graduate senior design project engineering education. Li et al [12, 13] have been emphasizing the great importance of engineering geology in any civil engineering curriculum engineering education.

The aim of this technical paper is different from the objectives of [12, 13]. The purpose is to demonstrate how an adequate constitutive model can somehow reliably represent the soil behavior during a real engineering problem, in this case the liquefaction of the sand layers that led to a road embankment failure. It is very important to understand all the principles, main characteristics and limitations of the constitutive models before choosing the most adequate engineering properties and parameters for a specific case. Nor Sand model proposed by Jefferies is then applied to explain the liquefaction that occurred at the confined sand layers. The proposal of new models or theories is beyond the scope of this work. Also, a slope stability analysis is provided to validate the results presented in [1].

The paper is organized as follows. Section 1 presents basic concepts related to loose sands and static liquefaction phenomena, and also introduces one advanced constitutive model for sand: the Nor Sand model. Section 2 presents the methodology used both in Nor Sand and slope stability analysis. Section 3 shows the results of these simulations for the sand layers beneath the embankments. Section 4 presents some concluding remarks.

2. Literature Review

This section presents a brief review of some pertinent topics related to liquefaction of soils and the Nor Sand model. Nor Sand [18] was a first state parameter-based model and generalized critical state theory. Original Cam Clay model is a special case of Nor Sand model. This model has been effectively used in soil liquefaction research study [18].

2.1. Dilatancy

An important concept of soil mechanics that must be properly considered in a constitutive model for sands is the dilatancy of soils. Dilatancy is the tendency of soils to change volume while shearing. When soil is sheared, its volume may increase (dilate) or decrease (contract) depending on its dense density state and the magnitude of the effective stress applied on the soil [14]. Figure 5 [15] presents the dilatancy for two types of soil. Loose sands (Type I) compress until reach the critical void ratio. Dense sands (Type II) expand toward a critical void ratio after an initial compression at low shear strains.

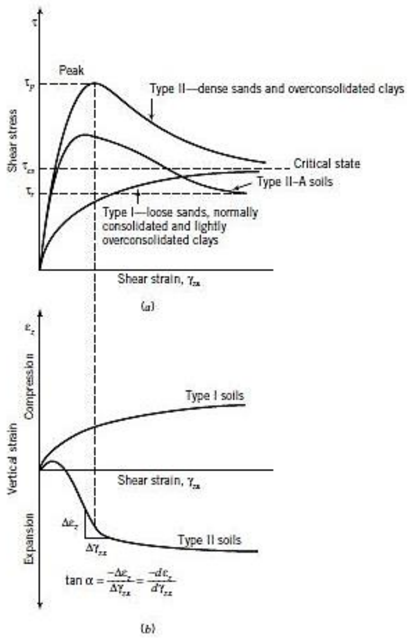

Figure 5. Response of soil to shearing [15]

2.1.1. Static Liquefaction

Liquefaction is simply defined as the act or process of transforming any substance into a liquid. Liquefaction of saturated sand is caused by a substantial reduction in its shear strength which, in turn, is caused by the development of high pore pressures induced by monotonically or cyclically applied strains [16]. In this paper, only static liquefaction by monotonic loading is presented, and it can also be named as flow liquefaction. In the past decades, the experimental and theoretical studies were more concentrated in cyclic liquefaction of soils. However, in recent years, more and more researchers also realized the severity of the failure caused by static liquefaction. One important aspect of liquefaction study is to predict the stress-strain relationship of granular materials that are susceptible to liquefaction [17].

Liquefaction occurs when the total stress remains constant and the pore pressure increases such that the normal effective stress becomes zero, or when the pore pressure remains constant and the total stress decreases such that the normal effective stress becomes zero [18]. In undrained condition increasing pore-water pressure bends an effective stress path towards failure condition (critical stress line) (Figure 6). The term critical state was firstly introduced by Casagrande [19] with reference to a critical void ratio.

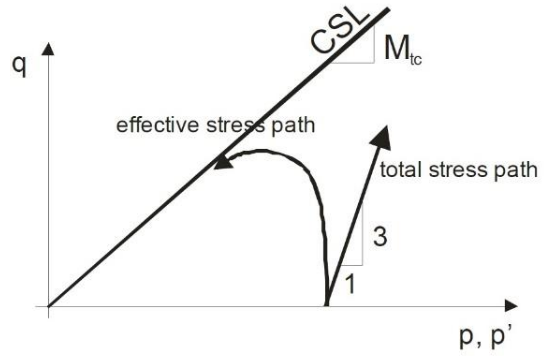

Figure 6. Stress path in conventional triaxial compression [20]

The type of sand plays an important rule in the behavior before, during and after liquefaction. Figure 7 presents the curves for three different sands under undrained shearing in triaxial apparatus usually after isotropic consolidation). Curve A represents dense sand, Curve B represents initially loose sand and Curve C represents loose sand in static liquefaction. What happens in liquefaction during monotonic loading is an increase in pore-water pressure and reduction in the effective mean stress. The stress path passes the peak deviatoric value (peak strength) and then goes down. At this point, the flow instability starts. Both deviatoric and effective mean stress eventually reach residual values during constantly increasing deformation.

Once the material undergoes the phase transformation line (PT line in Figure 7), liquefaction cannot occur (Curves A and B). After this phase transformation, the material must undergo hardening, so the deviatoric stress starts increasing again until reach the failure line.

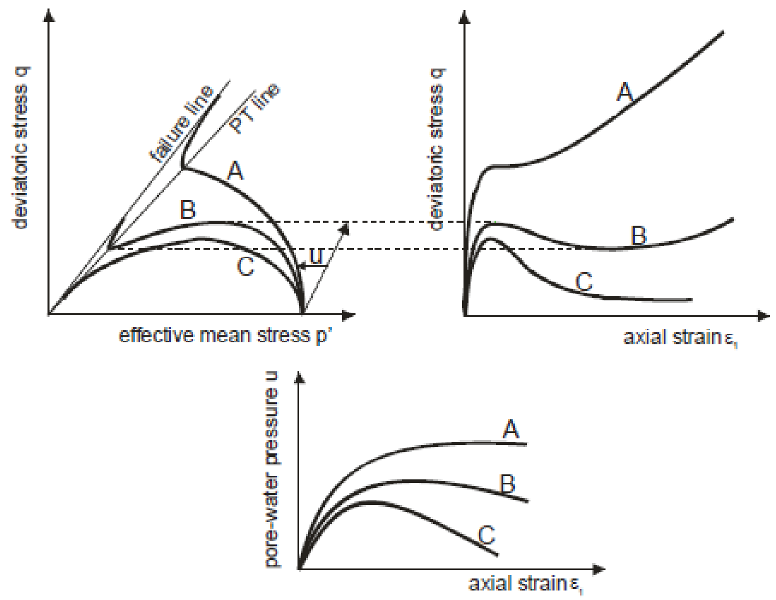

Figure 7. Curves for various types of sands [20]

Another property influencing static liquefaction is the confining pressure. Very loose compressible sands exhibit static liquefaction at low pressures rather than higher *pressures* dictated by normal soil behavior [21]. Figure 8 presents the response of five specimens isotropically consolidated to the same specific volume at different effective confining pressures. Samples C, D and E undergo flow liquefaction. The development of pore pressures under undrained conditions is directly related to the compressibility of the soil, and loose silty sands exhibit significant volumetric contraction at low pressures [21]. The onset of instability delimits the so-called instability line [11]. All the specimens reach the failure line, which is different than the instability line. Tests at high confining pressures indicate that this line intersects the stress origin [14].

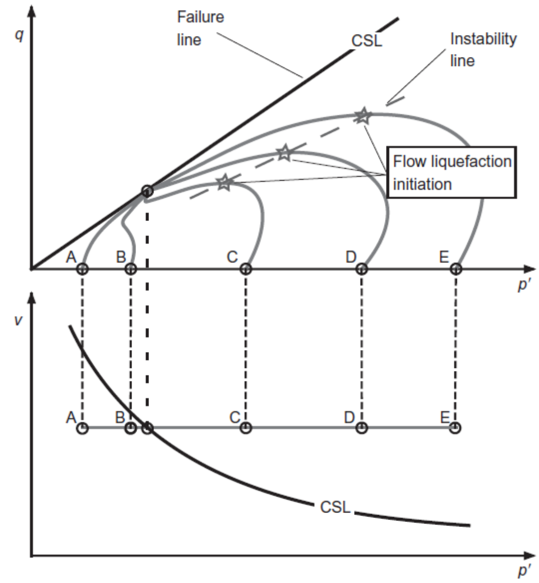

Figure 8. Response of five specimens (Instability and failure line) [11]

2.2. Nor Sand Constitutive Model

A full constitutive model should predict not just when liquefaction occurs but also the evolution of pore pressures and strains [18]. All realistic constitutive models for soils also require both elastic and hardening/softening plastic behavior in their framework. There are basically two main reasons for a model [18]:

Firstly, the literature abounds with liquefaction concepts developed based on incomplete information or poorly formed theory. Secondly, it is a fact of life that civil engineering is an activity with little opportunity for full scale testing.

The first theory offered that captured the dilatancy of soils was what became known as critical state soil mechanics, popularized by Schofield and Wroth [3] with the Cam Clay idealized theoretical model of soil.

Nevertheless, Cam clay and Granta Gravel are not able to reproduce dilation and yielding of real sands. According to Jefferies and Been [18], this inability arises from the assumption that a yield surface intersects the critical state line. Nor Sand model was one of the first constitutive models capable to overcome these limitations.

Nor Sand Constitutive mode was proposed by Jefferies M.G. in 1993 [6]. It is based on critical state theory also used in Cambridge models (Cam clay). Nor Sand used an associate flow rule. The two fundamental axioms of critical state theory considered are:

Axiom 1. A unique locus exists in q, p, e space such that soil can be deformed without limit at constant stress and constant void ratio; this locus is called the critical state locus (CSL).

Axiom 2. The CSL forms the ultimate condition of all distortional processes in soil, so that all monotonic distortional stress state paths tend to this locus.

The main difference between Nor Sand and Cam clay model is that Nor Sand assumes a yield surface and stress-dilatancy relationship known from Cam clay but disassociated hardening from CSL which gives the model capability to reproduce real sand behavior. This consists of using similar equation for the yield surface as for Cam clay but not using in it the mean stress at the critical state p'_c nor the critical stress ratio M. Instead, Nor Sand yield surface incorporates p'_i and M_i which correspond to the image state. Another difference is the consideration of infinity of normal compression loci (NCL) in $e - p'$ plane, not parallel to the critical state line neither are they straight [22].

The image state refers to the condition where dilatancy vanishes temporarily and soil behavior converts from contractive to dilative. This is the peak point of the logarithmic spiral known from the yield surface of Cam clay.

Nor sand introduces a new parameter: State parameter ψ (Figure 9). The state parameter is simply the void ratio difference between the current state of the soil and the critical state at the same mean stress. The critical state void ratio varies with mean effective stress and is usually referred to as the critical state locus (CSL). Dense soils have negative ψ and loose contractive soils have positive ψ. Soil constitutive behavior is related to ψ, and liquefaction behavior is no different from other aspects of stress-strain response.

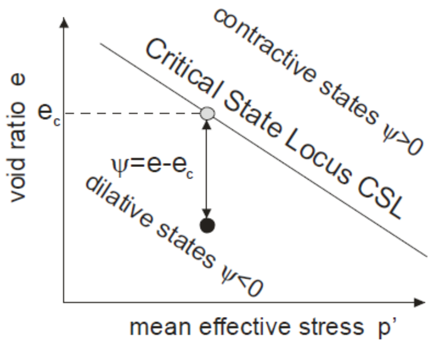

Figure 9. State parameter [20]

The basic equations of Nor Sand model are listed below and in [18] with some parameters explained in Table 1:

Stress notation: $\bar{\sigma}_m = p'$

Internal model parameters:

$$\Psi_i = \Psi - \lambda \ln(\bar{\sigma}_{m,i} \;/\; \bar{\sigma}_m) \quad \text{where} \quad \Psi = e - e_0 \tag{1}$$

$$\chi_i = \chi_{tc}/(1 - \chi_{tc}\lambda/\, M_{tc}) \tag{2}$$

$$M_i = M(1 - \chi_i N|\Psi_i| \;/\; M_{tc}) \tag{3}$$

Critical state:

$$e_c = \Gamma - \lambda \ln(\sigma_m \tag{4}$$

$$\eta_c = M = M_{tc} - \left({M_{tc}}^2 \;/\; (3 + M_{tc})\right) \cos\left(3\frac{\theta}{2} + \frac{\pi}{4}\right) \tag{5}$$

Yield surface and internal cap

$$\frac{\eta}{M_i} = 1 - \ln\left(\frac{\bar{\sigma}_m}{\bar{\sigma}_{m,i}}\right) \tag{6}$$

$$\left(\frac{\bar{\sigma}_m}{\bar{\sigma}_{m,i}}\right)_{max} = \exp(-\chi_{tc}\, \Psi_i \;/\; M_{tc}) \tag{7}$$

Hardening rule

$$\frac{\dot{\bar{\sigma}}_{m,i}}{\bar{\sigma}_{m,i}} = H \frac{M_i}{M_{tc}} \left(\frac{\bar{\sigma}_m}{\bar{\sigma}_{m,i}}\right)^2 \left[\exp\left(\frac{-\chi_i \Psi_i}{M_{tc}}\right) - \frac{\bar{\sigma}_{m,i}}{\bar{\sigma}_m}\right] \dot{\varepsilon}_q^p \tag{8}$$

Stress dilatancy

$$D^p = M_i - \eta \tag{9}$$

Elasticity

$$I_r = \frac{G}{\bar{\sigma}_m} \tag{10}$$

Conventional critical state models (e.g., Cam clay) derive the formulation of a yield surface from the assumption of normality (associated flow rule). Nor Sand controls dilatancy through the value of the mean stress at the image point pi. The evolution of yield surface is detailed below for a dense sand (Figure 10). The 3D plot is show in four steps for a better clarification. First, the yield surface starts in its initial configuration (a). Shear stress increases and the yield surface expands until stress ratio reaches the image state ($p'_i = p'$) at the configuration (b). During this stage volumetric strain decreases due to normality (void ratio decreases). Since the state parameter at the image point $\Psi_i < 0$ expansion of the yield surface continues (p'_i increases), but now accompanied by increasing volume. Expansion of the yield surface stops when the internal cap is reached (c). Dilatancy continues until the image state and critical state coincide. The yield surface stops changing its configuration at (d).

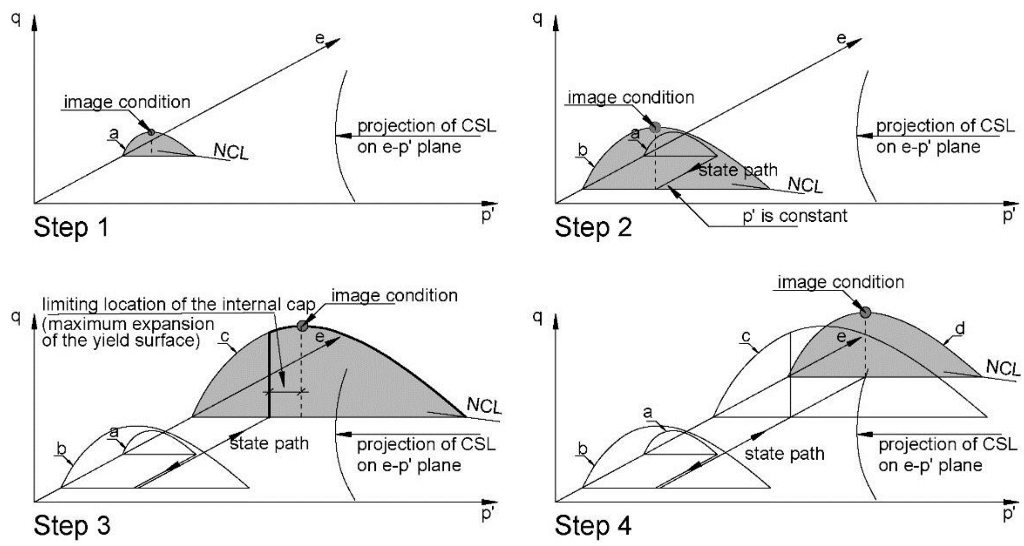

Figure 10. Evolution of yield surface (Adapted from [6])

There are some additional models originated from several rigid-plastic models proposed by Jefferies which presented even better improvements. The model presented in [23] included crucial features such as: a hyperelastic region, bulk and shear moduli dependent on the effective pressure, dependence of plastic flow on all three stress invariants, nonassociativity, and large deformations capabilities.

3. Methodology

The Nor Sand constitutive model is used in the confining loose sand layer to understand the road embankment failure. The parameters were chosen based in the SPT results and from typical values provided by Jefferies and Been [18] (Table 1). The contractive behavior of the sand is represented by the state parameter Ψ_i. Four simulations are provided, with Ψ_i being changed from 0.1 to 0.01 compared to the response for different initial values. Simulations of triaxial compression were performed with the assumption that the tests start from the initial effective mean stress p'_0= 100 kPa (pressure of water in a triaxial apparatus cell). This simulation is based on the work described by Sternik [20].

Table 1. Parameters Nor Sand Model

Parameter	Remark	Typical Range [6]	Value Considered
Γ	"Altitude" of CSL, defined at 1 kPa	0.9 - 1.4	1.2
λ	Slope of CSL, defined on natural logarithm	0.01 – 0.07	0.01
Mtc	Critical friction ratio, triaxial compression as the reference condition	1.2 – 1.5	1.07 (φ= 27°)
H	Plastic hardening modulus for loading, often f(ψ)	50 - 500	200
N	Volumetric coupling coefficient for inelastic stored energy	0.2 – 0.5	0.2
χ_{tc}	Relates maximum dilatancy to ψ. Triaxial compression as the reference condition	2.5 – 4.5	4.0
Ir	Dimensionless shear rigidity (G/p')	100 - 600	200
ν	Poisson's ratio	0.1 – 0.3	0.25

All the formulation of Nor Sand model is implemented in an Excel spreadsheet developed by Jefferies [18]. The software is for modelling a set of drained and undrained triaxial tests to determine, and validate, soil properties from conventional triaxial data. It is required as input data only the soil parameters presented in Table 1. The output generated are the plots of stress strain curves and stress paths. The change from undrained to drained is simply done by clicking in a button. Program coding is open-source VBA which contain all the formulations and equations presented in the literature review.

For the slope stability analysis, the software Slope/W 2007 is used for undrained and drained situations. The idea is to check the factor of safety before and after the solution of wick drains. This analysis was provided in the original work presented in [1], therefore it is being done again in current study for validation of previous findings and results. The values used were obtained from the SPT and CPT data and are presented in Table 2.

Bishop Method of slices is being considered for all the calculations. The methods of slices have become the most common method due to its ability to accommodate complex geometries and variable soil and water pressure conditions [24]. The typical section is being analyzed for both situations. It is being modeled in the simulation of eight different layers of soil. The liquefaction is simulated in the confined sand layers through the consideration of a pore pressure ratio of 1.0 due to the particular engineering geology of Kettle lake condition. When based on pore pressure-related criteria, soil liquefaction has often been defined as the state at which the excess pore water pressure ratio (r_u) equals 1.0 [25]. For the drained situation, it is being used the value of 0.5 for this ratio.

Table 2. Parameters considered in Slope Stability Analysis

Soil Description	Total unit Wt. (psf)	Saturated unit Wt. (psf)	Cohesion Intercept (psf)	Friction angle (deg)	Pore pressure Parameter (Undrained)	Pore pressure Parameter (Drained)
Peat/OL	65.0	67.0	0.0	5.0	0.50	1.00
Embankment fill	125.0	135.0	420.0	32.0	0.00	0.00
Stone	110.0	112.0	0.0	0.0	0.00	0.00
Clay	125.0	138.0	230.0	28.0	1.00	0.50
Saturated sand	85.0	90.0	0.0	27.0	1.00	0.50
Clay	125.0	138.0	230.0	28.0	1.00	0.50
Saturated sand	85.0	90.0	0.0	27.0	1.00	0.50
Blowing sand	85.0	90.0	0.0	27.0	1.00	0.50

4. Results and Discussion

In this section, the results of all simulations are presented. The first two simulations are undrained and drained triaxial response using Nor Sand model. The last two simulations are the slope stability analysis for undrained and drained condition.

4.1. Nor Sand – Undrained Triaxial Compression of Loose Sands

During the undrained triaxial compression, the four different loose sand soils presented unstable behavior. Two simulations have exhibited complete loss of strength (ψ_i = 0.1, 0.07) and two exhibited a residual strength (ψ_i = 0.04, 0.01) (Figure 11a). The state parameters $\psi_i = 0.1$ and $\psi_i = 0.07$ correspond to the value of initial void ratio e_0= 1.254 and 1.224. The state parameters $\psi_i = 0.04$ and $\psi_i = 0.01$ correspond to the value of initial void ratio e_0= 1.194 and 1.164. The results show that when the void ratio reduces, there is an increase in the peak deviator stress. The peak value is 52.8 kPa for $\psi_i = 0.01$ and reduces to 34.7 kPa for $\psi_i = 0.1$.

When the stress path passes through the instability line, there is a reduction in the deviator stress with an increase of the axial strain. The final value of deviator stress when it reaches the failure line in the situations with a residual strength are 2.6 kPa for $\psi_i = 0.04$ and 41.7 kPa for $\psi_i = 0.01$. This corresponds to a reduction compared to the peak values of 94.3% for $\psi_i = 0.04$ and 21.1% for $\psi_i = 0.01$. In the other two situations, the final axial strain after the total loss of strength is approximately 1.6% for $\psi_i = 0.1$ and 2.8% for $\psi_i = 0.07$.

All the simulations have exhibited full achieved liquefaction (Figure 11b). This could clearly be seen through the curves showed in the stress path. There is no phase transformation during the decrease of the deviator stress, therefore the material does not undergo hardening.

The variation of the state parameter between 0.1 and 0.01 represents a small change in the initial void ratio (from e_0=1.164 to e_0=1.254). The results showed in Figure 10a proved that a small perturbation in void ratio can cause large changes in postpeak behavior and undrained strength at large strains. These results present similar tendencies also showed in experimental and theoretical results in other researches [20, 26, 27]. The study presented in [26] considered another constitutive model (MIT-S1), and part of the results are highlighted below [26]. This study presented numerical simulation and experimental data from undrained shearing of Toyoura, and the stress strain curves and stress path contain a similar behavior of Nor Sand simulation (Figure 12a and b).

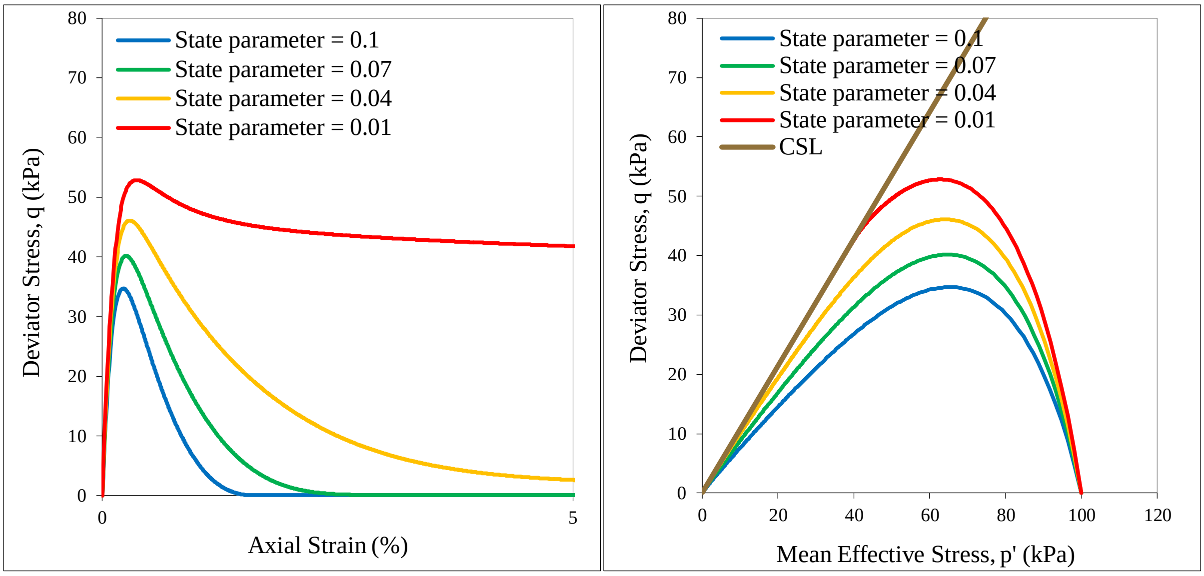

Figure 11. Results for loose sands in undrained condition (a) Stress strain curves. (b) Stress path

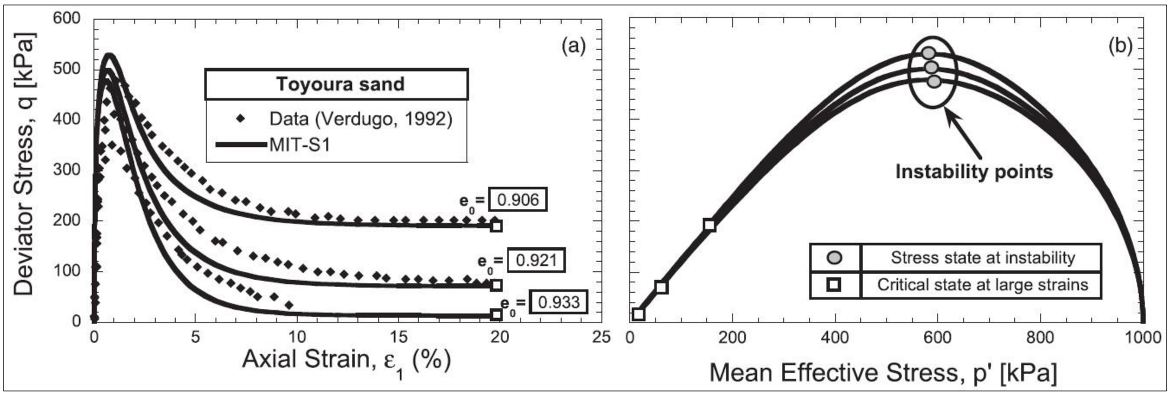

Figure 12. Results for Toyoura sand [21] (a) Stress strain curves. (b) Stress path

4.2. Nor Sand – Drained Triaxial Compression of Loose Sands

During the drained triaxial compression, the static liquefaction did not occur, as expected for this condition (Figure 13a). In the stress path (Figure 13b), it could be clearly seen that there is no increase in the pore pressure during the triaxial compression, as expected for a drained condition. Therefore, the mean effective stress increases approximately at the same rate for all the four loose sands until reaches the CSL.

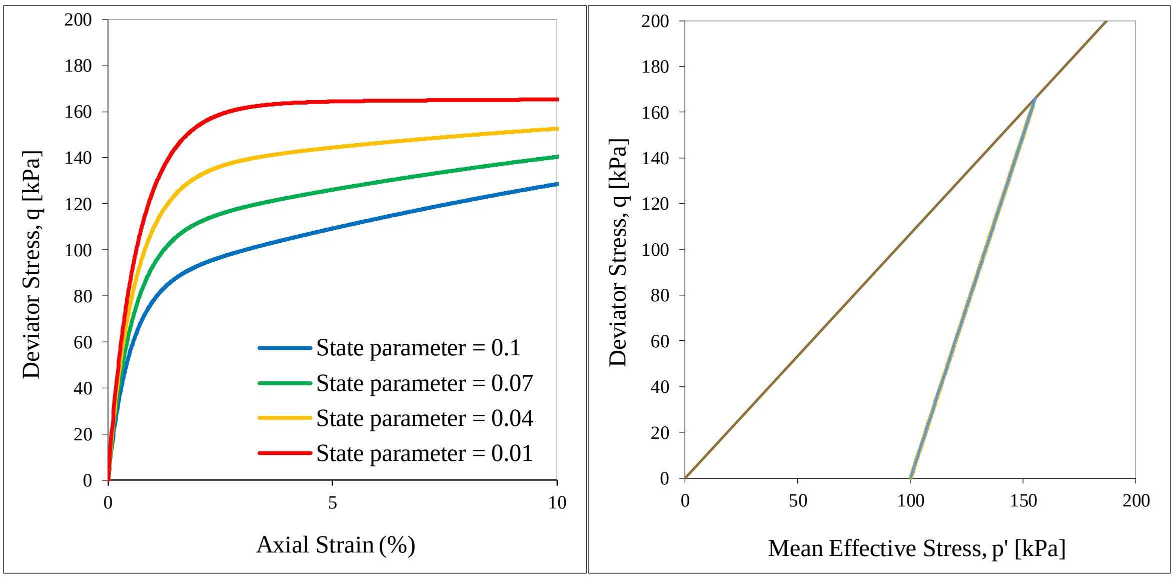

Figure 13. Results for loose sands in drained condition (a) Stress strain curves. (b) Stress path

4.3. Slope Stability Analysis – Undrained Condition

The results of slope stability analysis in the typical section during the failure are presented in Figure 14 and Table 3. The values obtained for the factor of safety are very close to those presented in [1]. This condition is unstable, as the minimum factor of safety of 0.835 shown in the figure. This minimum factor of safety obtained from the current study for undrained condition is approximately 5.7% larger than that reported in [1].

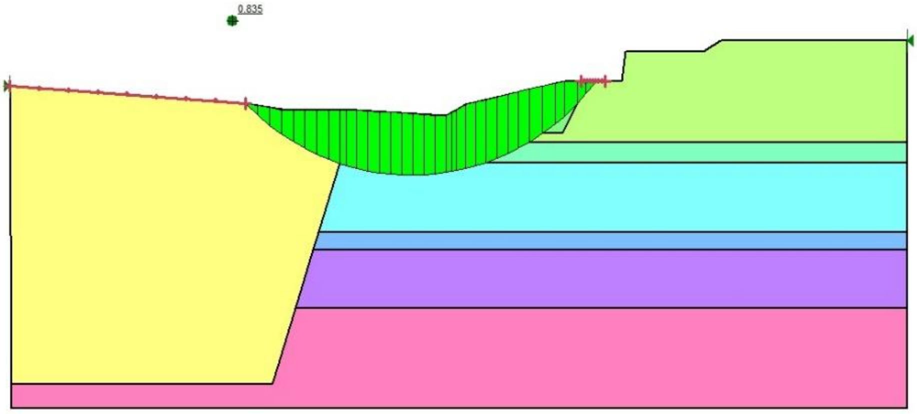

Figure 14. Slope stability analysis (During failure)

Table 3. Factor of safety - Undrained condition

FS obtained in current work	FS obtained at [1]	FS required
0.835	0.790	1.500

4.4. Slope Stability Analysis – Drained Condition

The results of slope stability analysis in the typical section after the solution with wick drains, and consequently in a drained condition are presented in Figure 15 and Table 4. The values obtained for the factor of safety are also very close to those presented in [1]. Now the factor of safety is greater than that required for a stable situation. Likewise, the minimum factor of safety obtained from the current study for drained condition is also larger than that reported in [1], approximately 2.8%. Therefore, we can validate the results presented in [1] for both situations.

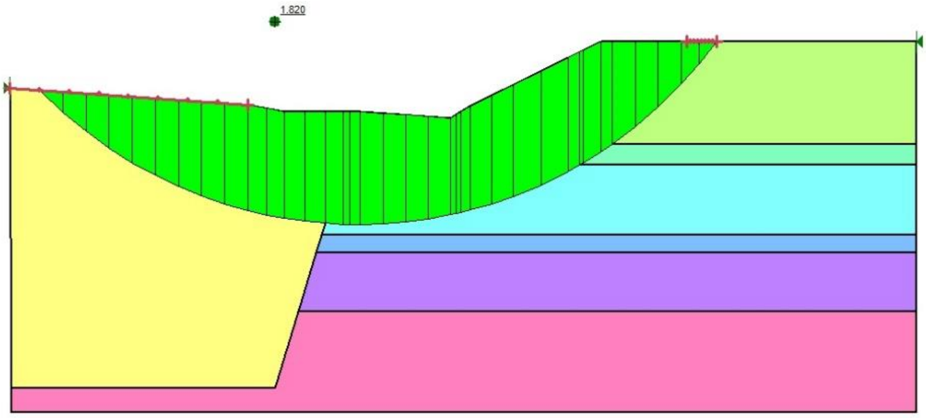

Figure 15. Slope stability analysis (After wick drains)

Table 4. Factor of safety - Drained condition

FS obtained this work	FS obtained from [1]	FS required
1.820	1.770	1.500

5. Concluding Remarks

This paper presented a static liquefaction study case of a highway fill embankment failure resulted from the liquefaction of foundation soils and how its failure could be analyzed using both a constitutive model (Nor Sand) and with a slope stability analysis. The soil conditions and rapid construction possibly led to the failure of the loose sand layer beneath the road embankment due to liquefaction partly caused by its unique engineering geology conditions [12, 13]. Unlike articles [12, 13], this technical paper however focuses more on the advanced Nor Sand constitutive model to the static liquefaction study. This paper also presented an overview of constitutive models for static liquefaction. A constitutive model able to represent static liquefaction must reproduce dilation and yielding of real sands. Nor Sand model has these features and provides a simple computable model that captures the salient aspects of liquefaction in all its forms. This critical state view is easy to understand which is characterized by a simple state parameter (ψ) with a few material properties (which can be determined on reconstituted samples) and lends itself to all soils. The results showed that minor changes in the initial void ratio can considerably affect the behavior after the peak strength in undrained conditions. The slope stability analysis could explain how a change in the condition of the loose sand layer from undrained to drained could improve the factor of safety due to the avoidance of the liquefaction to occur in this case.

6. Acknowledgements

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