Available online at www.CivileJournal.org

Civil Engineering Journal

Vol. 4, No. 10, October, 2018

Determination of Multilayer Soil Strength Parameters Using Genetic Algorithm

Seyyed Mohammad Hashemi [a], Iraj Rahmani [b*]

[a] *Master of Geotechnical Engineering, Faculty of Engineering, Islamic Azad University Central Tehran Branch, Tehran, Iran.*

[b] *Assistant Professor, Department of Geotechnical Engineering, Road, Housing and Urban Development Research Center (BHRC), Tehran, Iran.*

Received 15 June 2018; Accepted 13 October 2018

Abstract

This paper employs a back analysis method to determine soil strength parameters of the Mohr-Coulomb model from in situ geotechnical measurements. The lateral displacement of a soil nailed wall retaining an excavation in Tehran city used as a criterion for the back analysis. For this purpose, a genetic algorithm is applied as an optimization algorithm to minimize the error function, which can perform the back analysis process. When the accuracy of modeling is verified, the back analysis is performed automatically by creating a link between genetic algorithm in MATLAB and Abaqus software using Python programming language. This paper demonstrated that the genetic algorithm is a particularly suitable tool to determine 9 soil strength parameters simultaneously for 3 soil layers of the project site to decrease the difference of lateral displacement between the results of project monitoring and numerical analysis. The soil strength parameters have increased, with the most changes in Young's modulus of the first to third layers as the most effective parameter, 49.45%, 61.67% and 64.35% respectively. The results can be used in advanced engineering analyses and professional works.

Keywords: Excavation; Back Analysis; Parameters Determination; Mohr-Coulomb Model; Genetic Algorithm; Python Programming Language.

1. Introduction

Geotechnical in situ tests do not permit the identification of the soil parameters directly, which is considered as a limitation in engineering works. Given the recent advances, finite element method can be used to design geotechnical structures and developed as a numerical analysis technique since 1960. In geotechnical engineering, this method is often used to model and simulate problems for the prediction of geotechnical behaviour. But, this method is limited by the mechanical properties of the soil. So, it is possible to overcome the limitation by back analysis method [1, 2].

Optimization methods have been used in the back analysis method and divided into two general categories, such as optimization is based on the math basics and iterative methods (iterative algorithms). In this paper, the strength parameters of the Mohr-Coulomb model are determined by the back analysis based on genetic algorithm. A number of studies on back analysis using genetic algorithm of various problems have been already carried out, among which the following can be mentioned:

Gao et al (2016) used a back analysis procedure based on immunized genetic algorithm for parameter identification of elastic–plastic model for rock surrounding an underground excavation. The results showed that this model identification algorithm can significantly improve the computation efficiency and the computation effect [3].

* Corresponding author: i.rahmani@bhrc.ac.ir

http://dx.doi.org/10.28991/cej-03091167

Jin et al (2016), in order to obtain soil parameters, they employed the inverse analysis using an optimization method based on a genetic algorithm (GA). This study aims to develop a new efficient hybrid real coded genetic algorithm (RCGA) being applied to identify parameters of soils. They proposed new hybrid is evaluated by identifying soil parameters based on both laboratory tests and field tests, for different soil models. All results demonstrate that the proposed RCGA has a meritorious performance of inverse analysis in identifying soil parameters [4].

Levasseur et al (2013) used a back analysis method based on genetic algorithm optimization (iterative method) to identify the parameters of an earth and rock fill dam. In their study, a genetic algorithm was used as an optimization strategy to search for error minimization. They introduced this method as a time-consuming and efficient method for optimization [5].

Hui et al (2015) conducted a study to optimize the dynamic design of tunnels in shallow rock masses. They used Python programming language to set up a dynamic model of tunnel span that could be analyzed and implemented via Abaqus software. They also used MATLAB software to develop a genetic algorithm optimization. They imported the dynamic computational model into the genetic algorithm optimization program by MATLAB software. They introduced this method for stability and design improvement in tunnels [6].

Bartlewska-Urban and Strzelecki (2018) employed the inverse analysis using genetic algorithms (GA) to determine value of the coefficient of hydraulic conductivity. The commonly used method for the determination of coefficient of hydraulic conductivity based on Terzaghi consolidation leads to an underestimation of the value of coefficient of hydraulic conductivity. They presented an alternative methodology based on genetic algorithms for the determination of the basic parameters of Biot consolidation model. The results showed that genetic algorithms are a highly effective tool enabling automatic calibration of model based on simple rules [7].

Zhang et al (2014) presented the applications of the differential evolution (DE) algorithm in back analysis of soil parameters for deep excavation problems. They used Python programming language based on DE to develop and incorporate into the commercial finite element software ABAQUS, a synthetic case and a well-instrumented real case (Taipei National Enterprise Center) is used to demonstrate the capability of the proposed back-analysis procedure. For the synthetic excavation case, the back-analyzed parameters are basically identical to the input parameters that are used to generate synthetic response of wall deflection. For the TNEC case with a total of nine parameters to be back analyzed. Results show that multiple soil parameters are well identified by back analysis using a DE optimization algorithm for highly nonlinear problems [8].

Qinghui Jiang et al (2017) proposed a multi-objective inverse analysis method for slope excavation in orthogonal design which includes numerical simulation, back propagation neural network (BPNN) and elitist non-dominated sorting genetic algorithm (NSGA-II) were integrated. The multi-objective model is constructed by minimizing a set of multi-objective error functions between the time series of observations and corresponding calculated values. They used the obtain inversion parameters in a forward analysis to predict displacements. The results indicate that the proposed method can more precisely and reliably predict the slope deformation induced by excavation [9].

The estimated values of soil strength parameters are used as inputs in Abaqus software to simulate the problem directly. It is attempted to minimize the difference between the results of numerical calculations and measured values. Since the difference is reduced by changes in three parameters including the elastic modulus (E), cohesion (C) and soil friction angle (ϕ), it should be stated that there is no restriction on the geometry of problem, the number of parameters and the type of soil constitutive models in this optimization.

In fact, back analysis methods allow the comparison of the results calculated by computational methods and the measured values. Back analysis methods minimize the difference between the calculated and the measured values through the monitoring of projects, back analysis methods can be used as effective ways to determine the soil strength parameters [2]. Back analysis methods have been used practically to determine soil strength parameters in geotechnical problems since 1980 by Peck. Optimization methods have been used to accelerate the achievement of ultimate goal and increase the accuracy of predictions in the back analysis over time [4]. Back analysis can be simply explained but there are fundamental questions about the existence and uniqueness of the solutions [11]. As it is assumed that the solutions are not unique, how can achieve all solutions via minimum iterations? A variety of methods can be employed to optimize the back analysis used in geotechnical problems, such as genetic algorithm [1]. The genetic algorithm is properly known as an algorithm which can solve complex optimization problems [12]. This method is very robust and effective, but does not guarantee the exact identification of optimal solution. However, this method allows the optimal results to be concentrated near optimal solutions [9]. The genetic algorithm is based on iteration and search. So, this method can be used to perform the back analysis. In this research, the genetic algorithm is used as an effective method for the back analysis of a soil nailed wall by establishing a link between MATLAB and Abaqus software using Python programming language.

2. Objective of Back Analysis in Geotechnical Problems

Back analysis is mainly aimed to predict the events that are going to happen in next steps. For example, the events observed in step i must be able to predict the next events occurring in steps i+1 and i+2 [2].

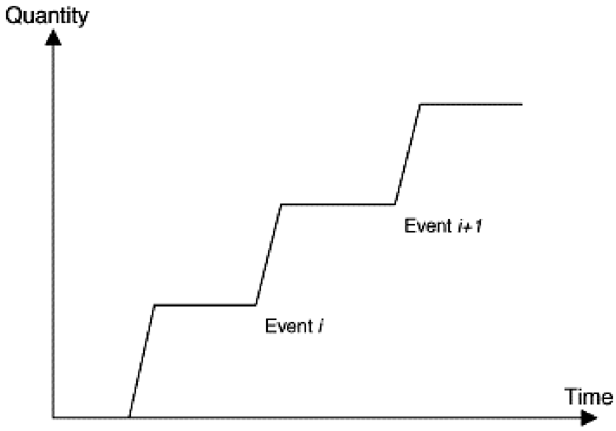


Figure 1. Behavior of measured quantity with respect to construction stages [2]

3. Genetic Algorithm (GA)

Genetic algorithm is an optimization method inspired by Darwin's theory of evolution. This method was originally introduced by John Holland and his students at University of Michigan in 1970. Later, the basic principles of genetic algorithm were developed by Goldberg and Renders. This method is one of the most well-known methods for minimization of objective (error) function in linear and nonlinear problems [14, 15]. This paper is conducted to optimize 3 parameters (E, Ø, C) for 3 layers of the soil (in total 9 parameters) simultaneously. However, researchers such as levasseur, malecot, boulon and flavigny have optimized three parameters of soil like shear modulus, dilation angle and soil friction angle step by step (non-simultaneously) using genetic algorithm [1].

3.1. Stages in the Genetic Algorithm Optimization Method

The main stages of genetic algorithm including:

1) Defining the research space: The research space defined by genetic algorithm, the problem is solved as an optimization problem in the N_p-dimension space. If each parameter is considered as P, restricted to authorized values of P between a minimum (P_{min}) and a maximum (P_{max}) [10]. In this study, the soil strength parameters such as Young's modulus (E), cohesion (C) and soil friction angle (ϕ) vary in ranges where the genetic algorithm approximates the values of each parameter for each soil layer according to the minimum error in terms of the error function and satisfying one of the convergence criteria. When the error function is less than the specified value and satisfies one of the convergence criteria, the genetic algorithm stops and introduces the optimal values. For example, E restricted to authorized values of E between E_{min} and E_{max}, C between C_{min} and C_{max} and ϕ between ϕ_{min} and ϕ_{max}.

2) Encoding individuals and populations: Each encoded parameter with a number of bytes (N_b) are forming a gene. The research space is meshed into $(2^{Nb})^{Np}$ elements and the choice of N_b is linked to the parameter values. Several genes form an individual (one point of the research space). A set of N_i individuals creates a population.

3) Generating an initial population: Initial population is chosen within the research space. The objective function of each individual of this population is evaluated by FEM calculation.

4) Selection, crossover and mutation: These mechanisms provide a set of solutions to be concentrated around the optimal solution in the research space.

Selection: Depending on the minimum value of objective function, only the best $N_i/3$ individuals is selected for the next population. The best individuals will be considered as parents in the next population.

Reproduction and crossover: Parents are randomly crossed over to generate new pairs of individuals. This process will continue until 2 $N_i/3$ new individuals (children), are generated.

Mutation and generation of a new population: The parents and children create a new population together with N_i children. To limit the convergence problems and to diversify the population, some children will randomly mutated and change. Then, the objective function is evaluated for each child of new population by FEM calculation [10, 14, 16, 17].

4. Error Function

In this study, the error (objective) function determines the difference between measured and calculated values.

$$\text{Error} = \frac{\sum_{i=1}^{N}[u_i - u_i^*]}{\sum_{i=1}^{N}[u_i^*]} \tag{1}$$

Where u_i is displacement calculated by finite element method in point i; u_i* is displacement measured by monitoring operations in point i; and N is the number of measuring points for a vertical section of the wall [21, 22].

5. Convergence Criteria

In this study, the principle of the convergence criterion for determining when the process of the back analysis should be terminated is based on one of the three components of the convergence criterion is reached at two successive iterations.

$$|f(x^{k+1}) - f(x^k)| \leq \varepsilon_a \tag{2}$$

$$\frac{|f(x^{k+1}) - f(x^k)|}{f(x^k)} \leq \varepsilon_b \tag{3}$$

$$\frac{|x^{k+1} - x^k|}{x^k} \leq \varepsilon_c \tag{4}$$

Where ε_a, ε_b, and ε_c are tolerances (equal 0.01).

The first convergence criterion Equation 2 is based on the absolute value of change of the objective (or error) function obtained between two successive iterations is less than the specified tolerance. The second convergence criterion Equation 3 is employed for the percentage of the change of the objective function. The third convergence criterion Equation 4 is used for the percentage of change of target parameters. When one of the convergence criteria is satisfied at two successive iterations, the optimization analysis is terminated [18].

6. Verification

In this paper, the modeling and analysis were done by Abaqus software in comparison with the numerical results of the studies by Babu and Singh for verification. They analyzed a soil nailed wall in five stages by the excavation of 2 m by Plaxis software [19]. The lateral displacement of the soil nailed wall is affected by excessive uplift and has unreasonable lateral displacement, this problem is due to the soil elasticity modulus in the reloading situation is equal to the loading condition (the Mohr-Coulomb model) in Abaqus, which may lead to overestimation of uplift in the excavation bottom. To overcome this problem, the research of Hajialilue Bonab and Razavi has been selected. According to their research, the geometry should slightly change in order to achieve a more reasonable lateral displacement of the soil nailed wall via modification of the height of excavation bottom to the model bottom between 5 and 7 m [20]. In this paper, after modification of the geometry, the results of modeling and analysis of the soil nailed wall by Abaqus indicate a good agreement with the results of the studies by Babu and Singh in Figure 2 (error rate less than 5%).

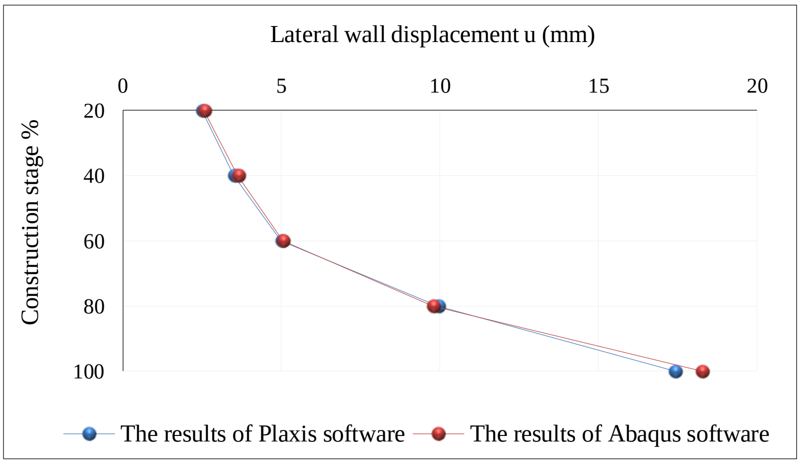


Figure 2. Comparison of lateral displacement

7. Introduction of Case Study

The project consists of three layers of homogeneous CL-type gravel-clay as well as SC- and GC-type clayey gravel and sand, classified by according to USCS, which is modeled by five parameters of the Mohr-Coulomb model. The groundwater level is not considered in the soil geometry. Three parameters of the soil such as friction angle (ϕ), elastic modulus (E) and cohesion (C) are selected for the back analysis and the other parameters that do not have much effect are ignored.

- Poisson's ratio $\nu = 0.3$
- Dilation angle $\psi = \phi - 30^o$

Jacky's formula is also used to calculate the coefficient of lateral earth pressure.

$$K_o = 1 - \sin(\emptyset) \tag{5}$$

In Table 1, a summary of geotechnical parameters used in the calculation of stability is presented.

Table 1. Summary of soil physical and mechanical parameters of the project site

Depth of soil layers (m)	Angle of friction (°)	Cohesion (KPa)	Elastic modulus (MPa)	Specific weight (KN/m^3)	Poisson's ratio
0-2 Backfill	30	15	60	16.2	0.3
2-8	30	30	60	15.6	0.3
8-15	30	30	60	19.6	0.3

7.1. Excavation Protection System and Loading

The west side of the project is selected for the back analysis. The west side was excavated in 7 lifts to a depth of 15 meter. To stabilize the wall, a nailing system is implemented by angle of 15 degrees and 2 m distances between boreholes. The load on the wall is considered in accordance with Table 2.

Table 2. The load assumed in analysis of west side

Wall	Location	Load (KN/m^2)
West	container	10

7.2. Results Based on Project Monitoring (Observation Measurements)

The monitoring operations are carried out to obtain and control the lateral displacement of the west side. In this paper, the value of displacement obtained by reflector No.23 (on the upper section of the wall) are used to perform the back analysis (according to Table 3).

Table 3. Displacement reported at point 23 on west side

Measurement point number	Lateral displacement (mm)
23	8

8. Numerical modeling via Python programming language by Abaqus software

According to the geometric properties presented in Table 4, the two-dimensional geometry of model is created. The excavation consists of 7 stages (lifts) to a depth of 15m and 30m in width. The lateral displacement of the west wall u_x is used as a criterion for determining the soil strength parameters. Moreover, Python programming language is used to link the genetic algorithm and Abaqus software, which causes the genetic algorithm to complete the cycle of the back analysis. This cycle creates an automated process which includes all modeling, determination of the soil strength parameters (Input Abaqus parameters), analysis of the soil nailed wall, outputs, calculation and achieve the least value of objective function, satisfy a convergence criterion and introduction of the best-fit parameters as optimal parameters without human interference. The process is shown in Figure 3.

Table 4. Geometric properties of the project

Nails	1	2	3	4	5	6	7
Nails length (m)	12	11.6	10	7	6	6	4
7 lifts (stages) of excavation	1	2	3	4	5	6	7
Depth of lifts (m)	2	3	2	2	2	2	2
Total depth of excavation (m)	15						
Thickness of shotcrete and Young's modulus	0.1 m, 21GPa						

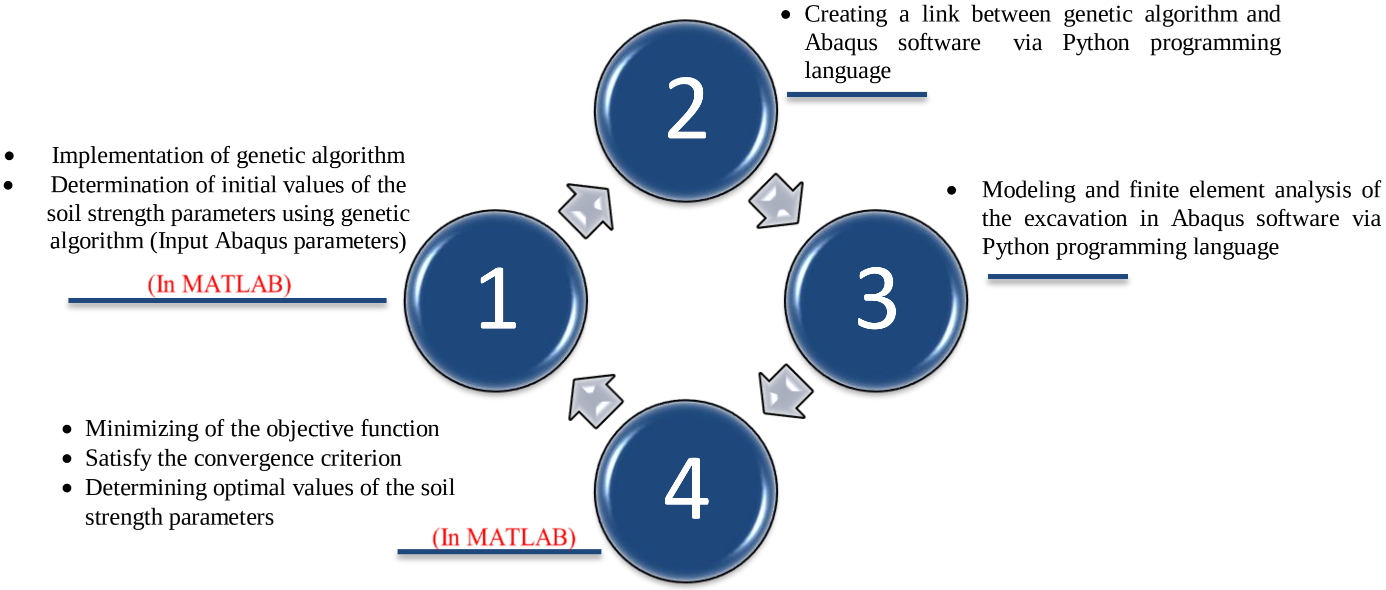


Figure 3. The back analysis process

9. Parameters of Genetic Algorithm

The population size (npop) and the maximum number of iterations (maxiter) are quantitative parameters, while rates of crossover and mutation are qualitative parameters. The increase and correct selection of the population size and the maximum number of iterations may improve the results; but exact rates of crossover and mutation cannot be determined to improve the results [22].

10. Investigation into Effective Parameters of Genetic Algorithm

In this paper, the role of effective parameters of genetic algorithm as a synthetic example is investigated by increasing the population size (npop), while the maximum number of iterations (maxiter) is constant. The assessments demonstrate that better results can be obtained by increasing the population size, according to Figure 4, which reduces the error rate.

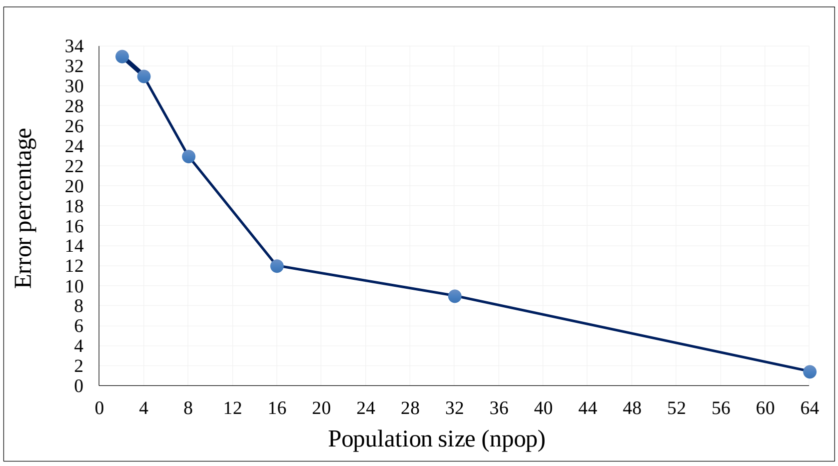


Figure 4. The process of error reduction versus population size

The number of objective (error) function evaluations increases as the population size (npop) or the maximum number of iterations (Maxiter) rises, while more finite element analyses are needed for further evaluation of the objective function in this study. Therefore, increasing the number of objective function evaluations is a time-consuming process.

11. Back Analysis Results

The results of project monitoring and numerical analysis show 73.7% difference in lateral displacement. The process of the back analysis in real problem should be implemented to reduce this difference (error rate), optimization of the soil strength parameters and satisfy one of the convergence criteria. In this paper, 9 soil strength parameters are determined simultaneously for 3 soil layers of the excavation by 254-times finite element analysis. The results of last 10 evaluations are presented in Table 5.

Table 5. Estimation process of soil strength parameters

No. of analysis	Error percentage F%	E1 (MPa)	E2 (MPa)	E3 (MPa)	C1 (KPa)	C2 (KPa)	C3 (KPa)	Ø$_1$ (deg)	Ø$_2$ (deg)	Ø$_3$ (deg)
245	3.14	89.1	98.4	95.6	18.2	38.9	39.7	30.2	31.9	34.8
246	1.48	89.5	98.2	98	18.8	34.7	40.4	30.3	31.5	35.1
247	1.90	89.5	97.8	97.3	18.8	33.9	40.7	30.1	30.2	35.2
248	2.85	89.7	97.6	96.6	18.1	36.7	38.6	29.9	29.6	35.2
249	3.40	90.6	97.4	95.6	17.9	34.4	38.6	30.7	29.9	35.1
250	2.36	89.5	95.4	97.6	18.4	36.3	39.8	29.9	29.7	35.5
251	3.46	88.6	95.4	96.6	18.4	35.5	38.4	29.9	29.5	35.5
252	1.88	98.6	97.4	98.8	18.6	36.6	40.3	30.5	31.2	34.2
253	1.68	90.7	98.5	98.8	18	36.2	39.9	28.4	30.3	34.1
254	1.61	89.7	97	98.6	18.8	32.8	39.9	30.5	31.4	35.2

The convergence criterion is satisfied using Equation 2. In Table 6, the overall process of error reduction in the back analysis is shown from the maximum error to the minimum error (11 out of 254 from 66% to 1.61%) in the back analysis as well as the soil strength parameters.

Table 6. The process of error reduction in the back analysis

Error percentage F%	E1 (MPa)	E2 (MPa)	E3 (MPa)	C1 (KPa)	C2 (KPa)	C3 (KPa)	Ø$_1$ (deg)	Ø$_2$ (deg)	Ø$_3$ (deg)
66	69.6	85.3	52.3	12.6	30.5	41.5	33.5	25.4	26.3
45	82.5	78.1	60.2	25	28.2	29.7	35.4	26.7	36.3
33	86	82.5	68.2	25	28.2	29.7	35.4	26.7	36.2
26.9	87.2	54.4	88.4	24.1	33	28.4	30.2	30.9	34.8
18.8	81.2	84.4	85.7	18.8	29.9	30.7	29.5	35.7	34.6
15	80.5	87.2	86.7	19.5	29.5	39	29.5	34.4	33.5
11.6	93.2	96.7	90	18.2	40	36.3	32.6	28.7	29.9
6.3	89.8	97.2	94.3	22.4	33.6	35.8	29.6	30.8	33.5
3.4	90.6	97.4	95.6	17.9	34.4	38.6	30.7	29.9	35.1
1.68	90.7	98.5	98.8	18	36.2	39.9	28.4	30.3	34.1
1.61	89.7	97	98.6	18.8	32.8	39.9	30.5	31.4	35.2

Figures 5 to 7 illustrate the process of error reduction in the back analysis using genetic algorithm versus the estimation of the soil strength parameters. The back analysis shows a good convergence between the results in accordance with the reduction of objective (error) function along the process.

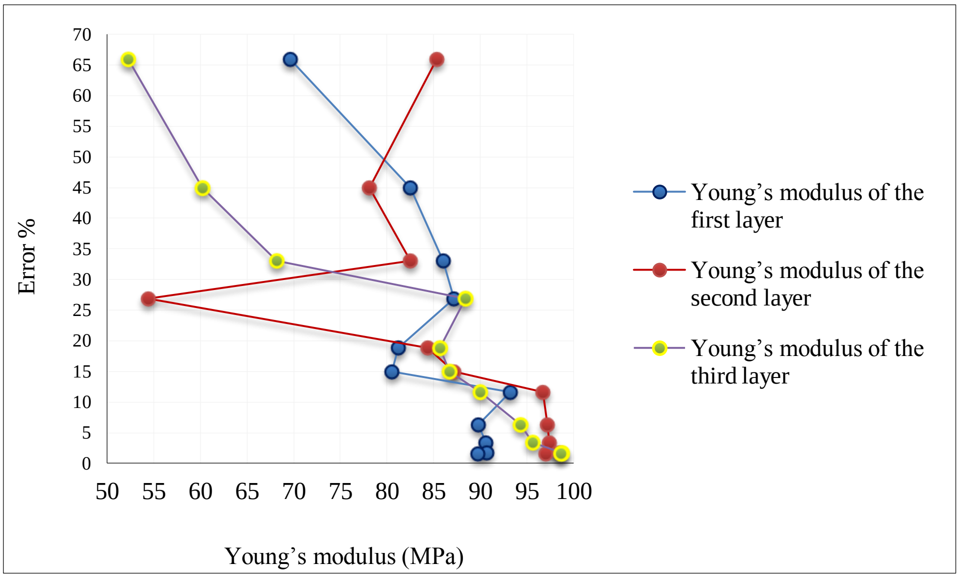


Figure 5. The estimation process of Young's modulus for 3 layers by genetic algorithm

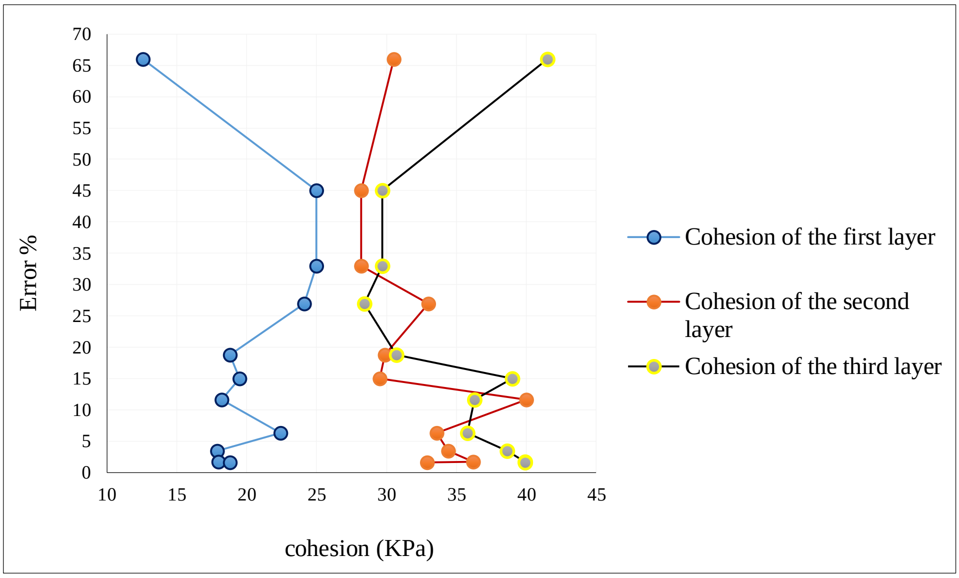


Figure 6. The estimation process of cohesion for 3 layers by genetic algorithm

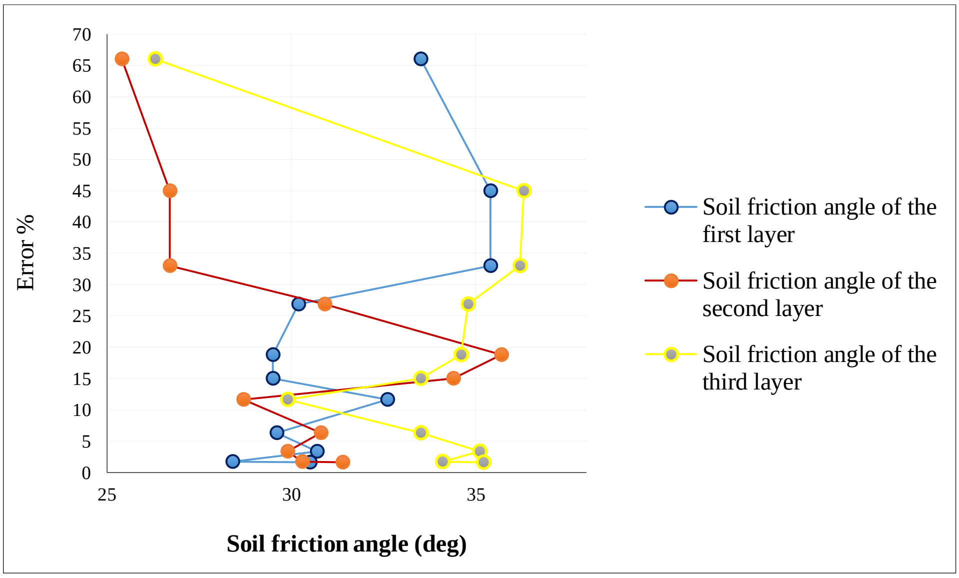


Figure 7. The estimation process of soil friction angle for 3 layers by genetic algorithm

The optimal parameters obtained so far are considered as equivalent soil parameters; because many factors such as previous stresses, depth and area of excavation, overheads, consolidation, drainage operations, etc. So, these parameters cannot be regarded as actual soil parameters. In Figures 8 to 10, the initial and optimized values of the soil strength parameters are compared. Table 7 also presents the increase of optimal values versus initial values in percentage.

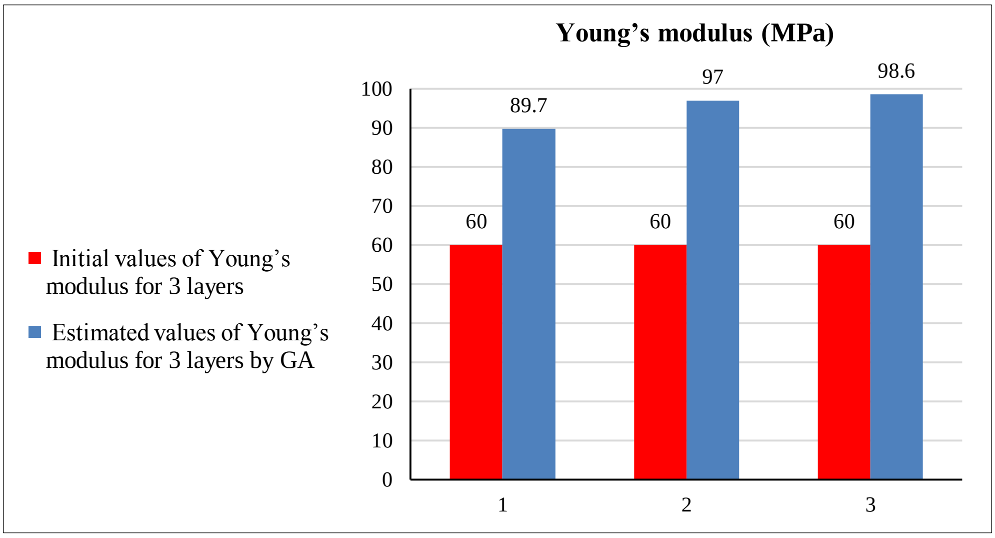


Figure 8. The increased optimal values versus initial results of Young's modulus

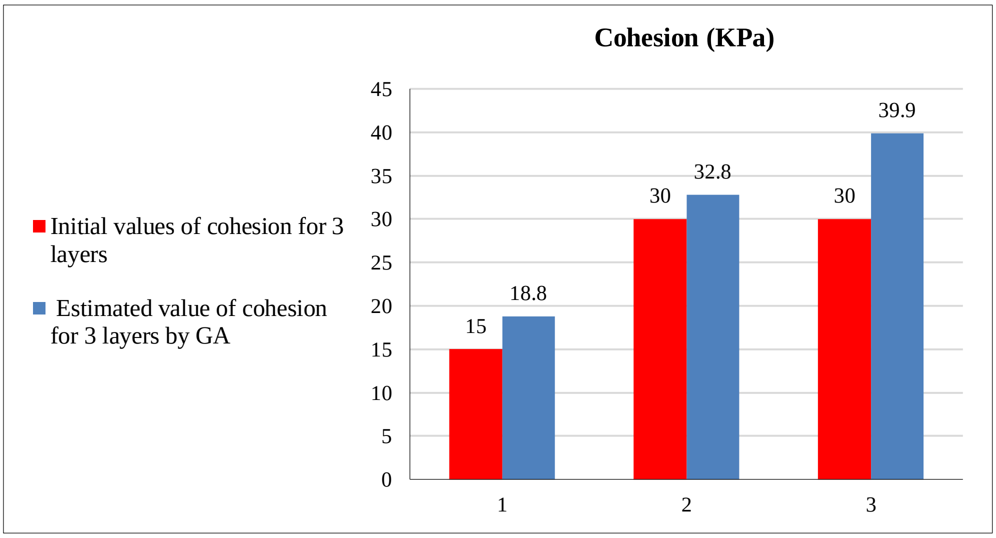


Figure 9. The increased optimal values versus initial results of cohesion

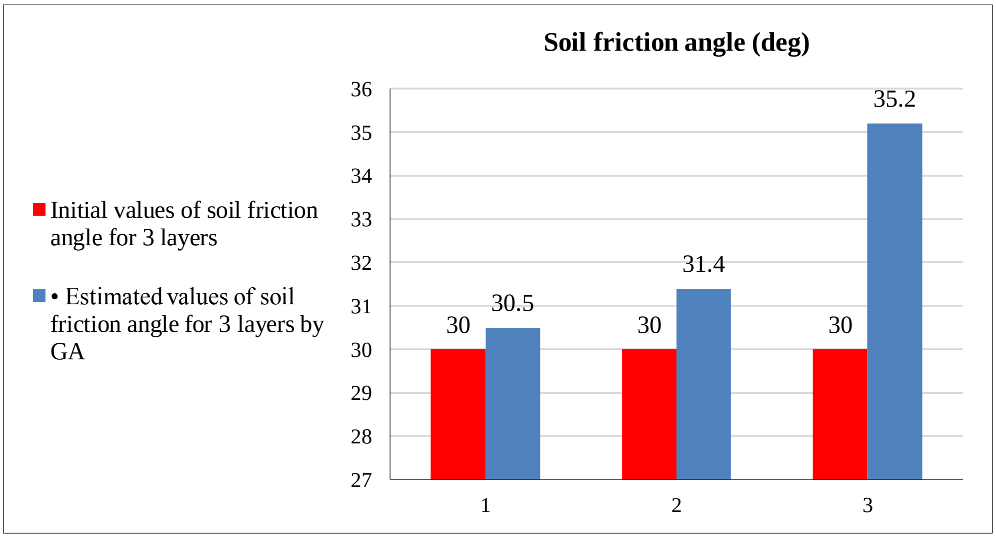


Figure 10. The increased optimal values versus initial results of soil friction angle

Table 7. The percentage increase of optimal values versus initial values

E1 (MPa)	E2 (MPa)	E3 (MPa)	C1 (KPa)	C2 (KPa)	C3 (KPa)	ϕ_1 (°)	ϕ_2 (°)	ϕ_3 (°)
49.45 %	61.67 %	64.35 %	25.6 %	9.47 %	33.1 %	1.67 %	4.67 %	17.23 %

The results show that the highest increase in optimal values is related to the elastic modulus. Underestimated values are applied in designs due to the limitations in field or laboratory tests. These limitations lead to inaccurate estimation of these parameters. These values may result in overestimated designs which are lead to non-economic projects.

Figure 11 illustrates the process of decreasing lateral displacement of the soil nailed wall in three positions by the genetic algorithm, in accordance with error reduction from 66% to 1.61% differences.

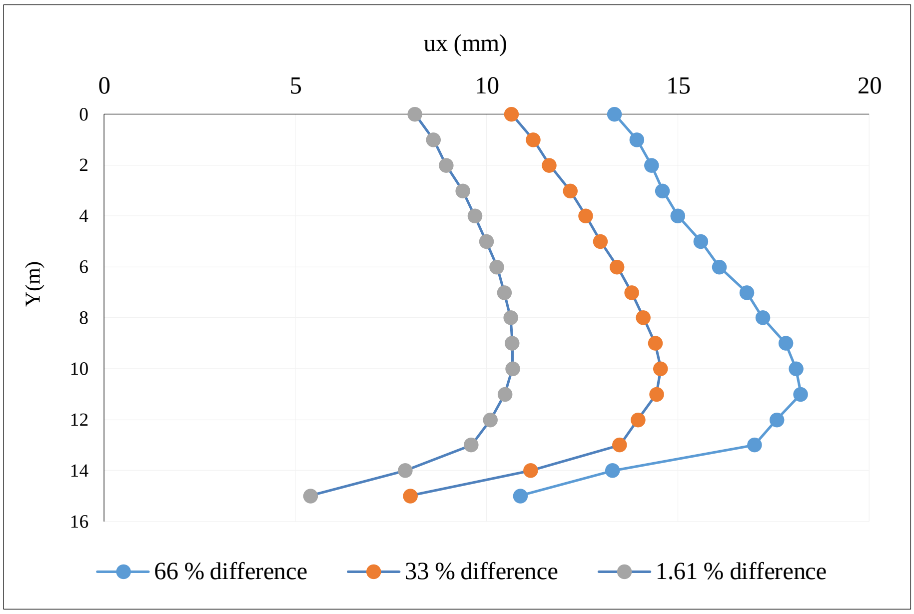


Figure 11. The estimation process of lateral displacement versus wall depth (Y) by GA

Figure 12 also shows the value of lateral displacement of the wall and the value estimated by the genetic algorithm via determining the soil strength parameters at the final iteration (1.61% error).

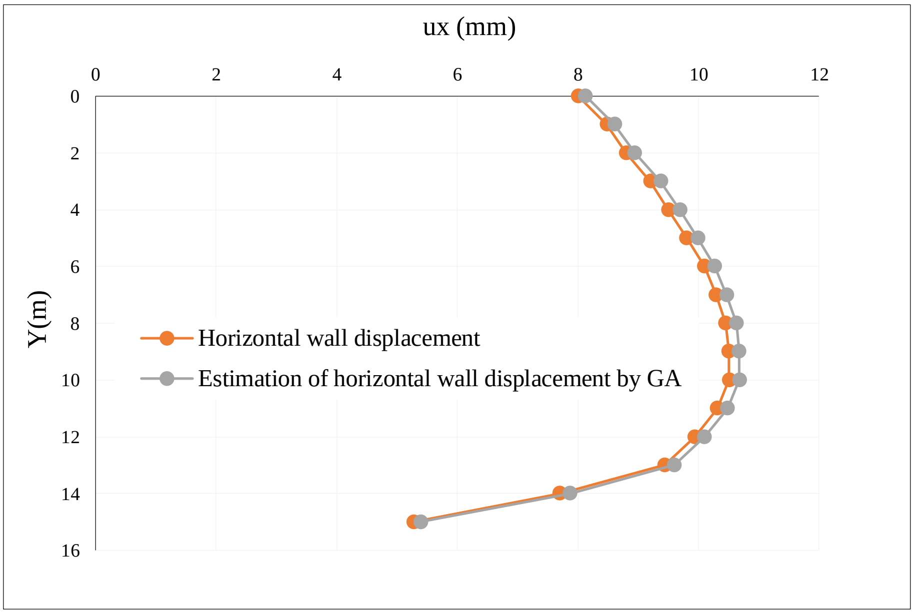
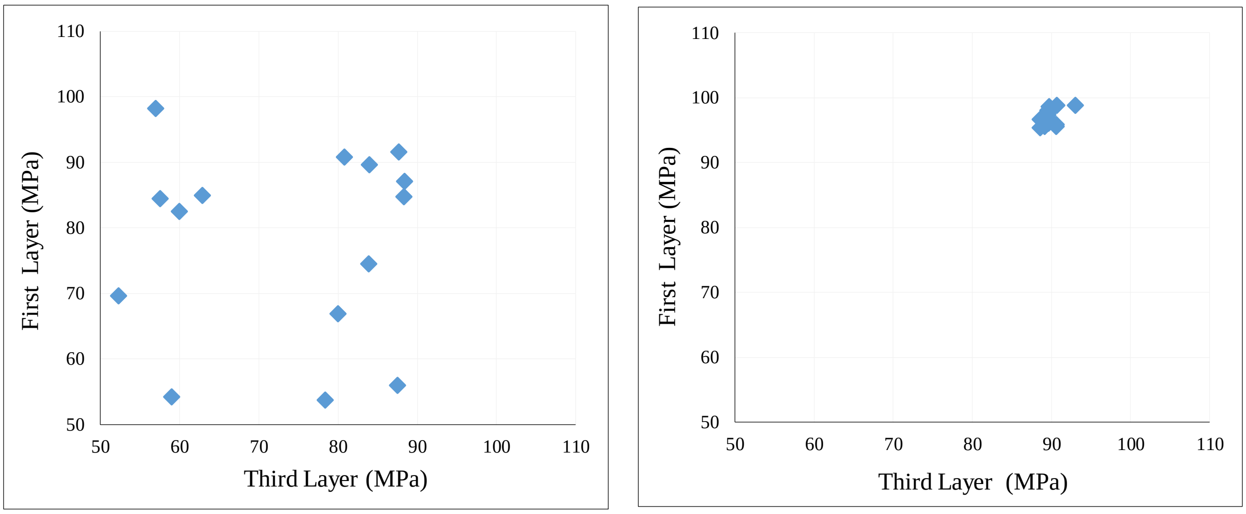


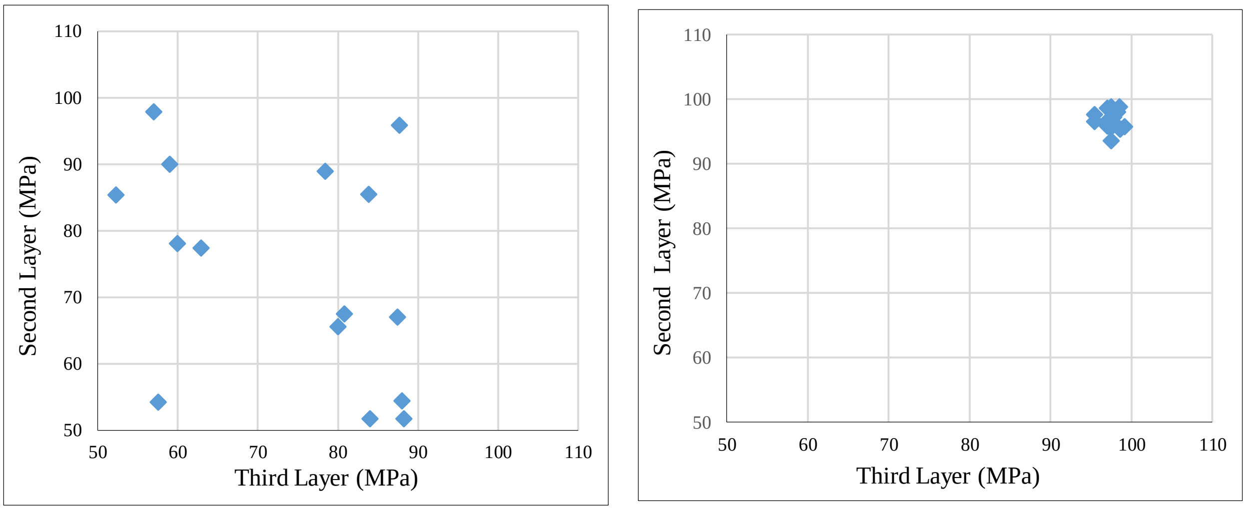
Figure 12. The comparison of lateral wall displacement versus wall depth (Y)

The estimation process of Young's modulus according to the initial population versus the final population of the genetic algorithm (npop=15) for 3 soil layers is compared in Figures 13 to 15.



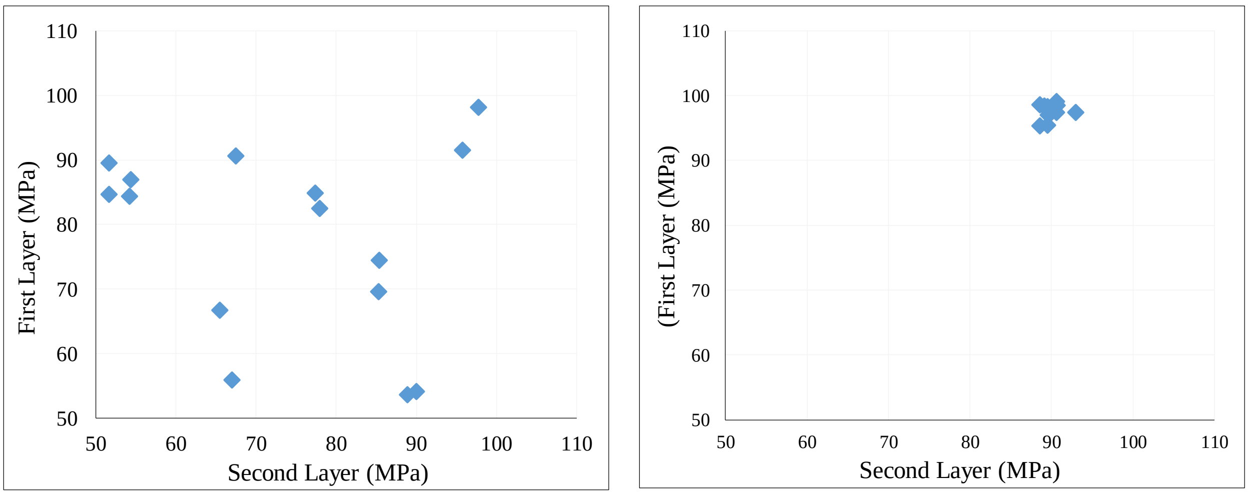
a) Initial population **b) Final population**

Figure 13. The estimation process of Young's modulus for the first layer versus the third layer in accordance with population



a) Initial population **b) Final population**

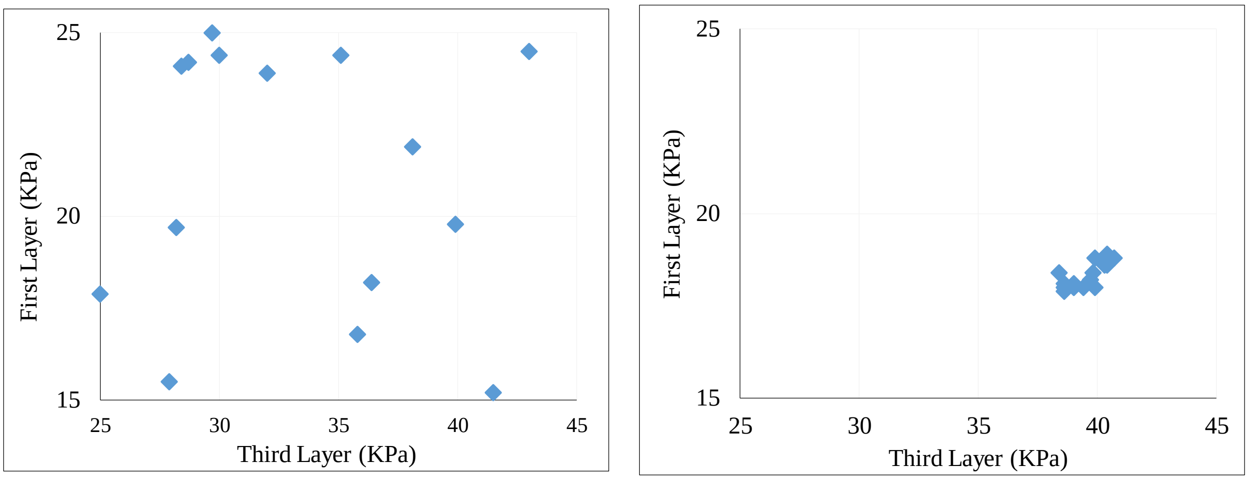
Figure 14. The estimation process of Young's modulus for the second layer versus the third layer in accordance with population



a) Initial population **b) Final population**

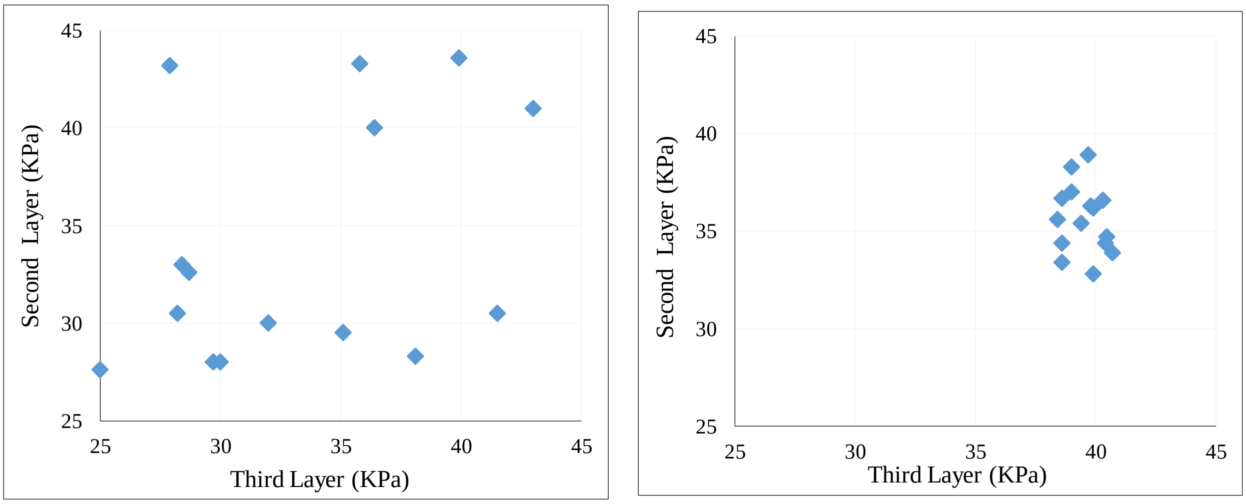
Figure 15. The estimation process of Young's modulus for the first layer versus the second layer in accordance with population

The initial population includes the first 15 results and the final population includes the last 15 results. As seen Figures 13 to 15, the initial population is very scattered in part a, but the population becomes concentrated around an optimum point in part b using the genetic algorithm. This trend can be observed for other parameters, i.e. cohesion and soil friction angle, in Figures 16 to 21.



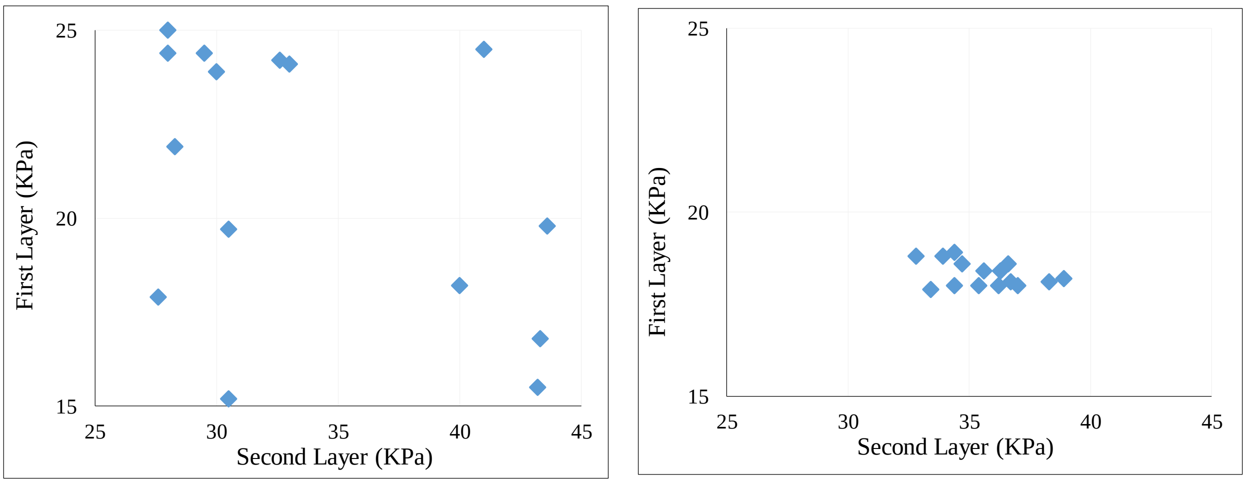
a) Initial population **b) Final population**

Figure 16. The estimation process of cohesion for the first layer versus the third layer in accordance with population



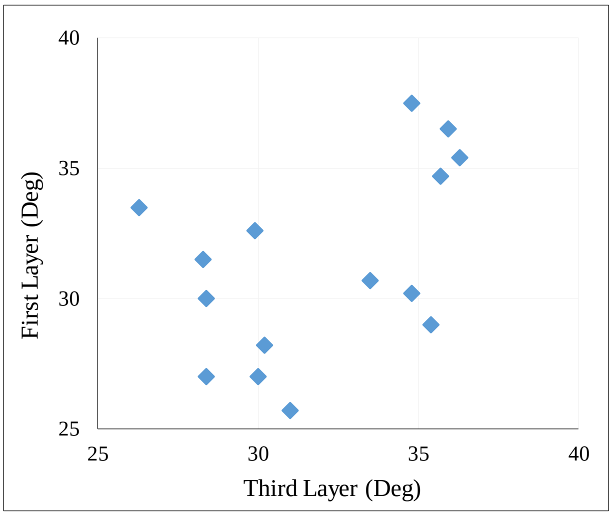
a) Initial population **b) Final population**

Figure 17. The estimation process of cohesion for the second layer versus the third layer in accordance with population

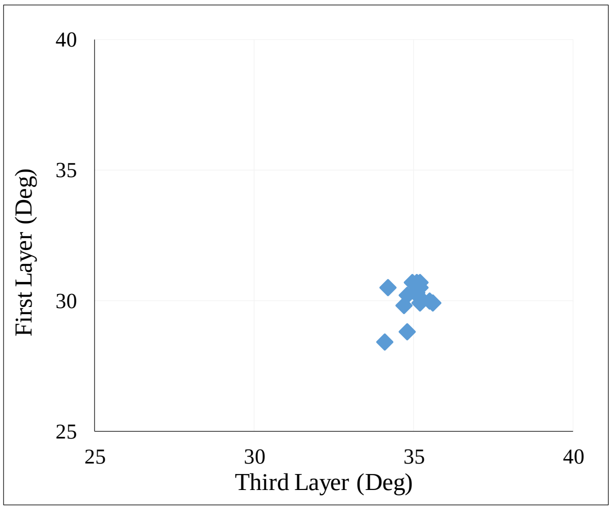


a) Initial population **b) Final population**

Figure 18. The estimation process of cohesion for the first layer versus the second layer in accordance with population

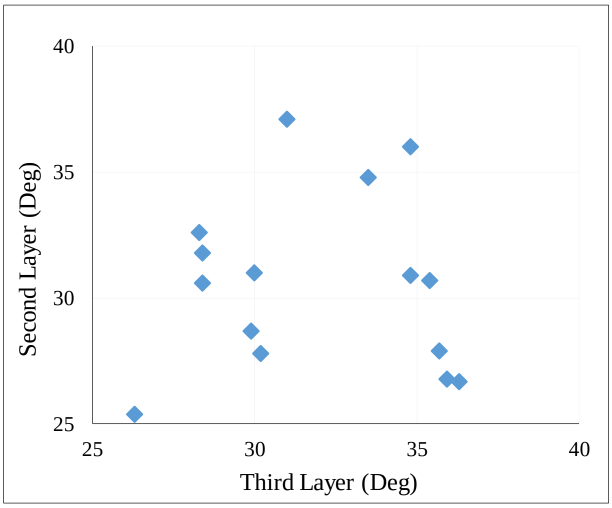


a) Initial population

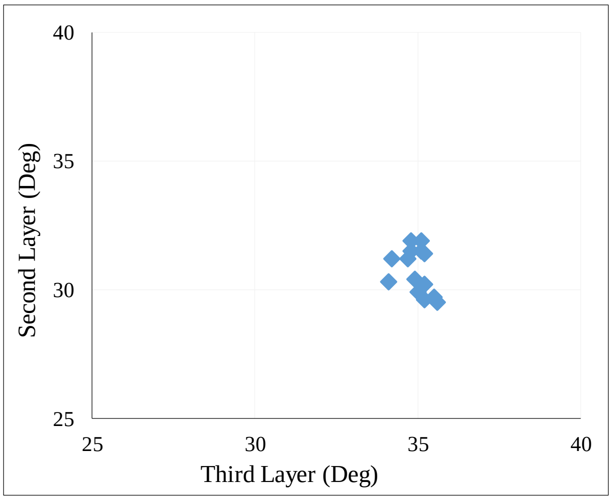


b) Final population

Figure 19. The estimation process of soil friction angle for the first layer versus the third layer in accordance with population

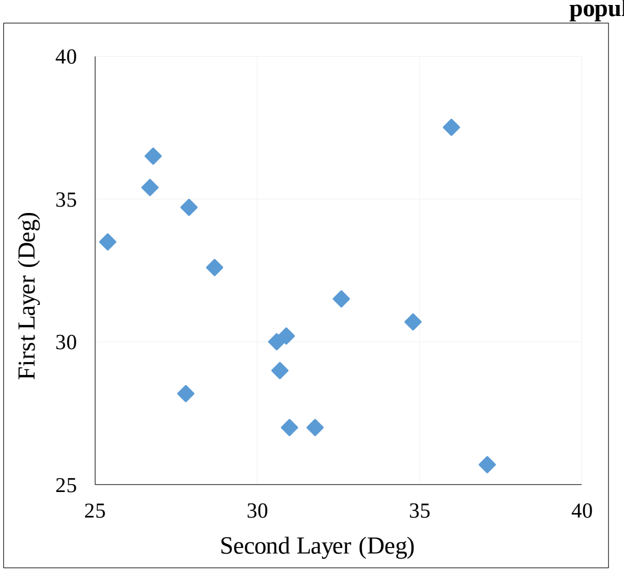


a) Initial population

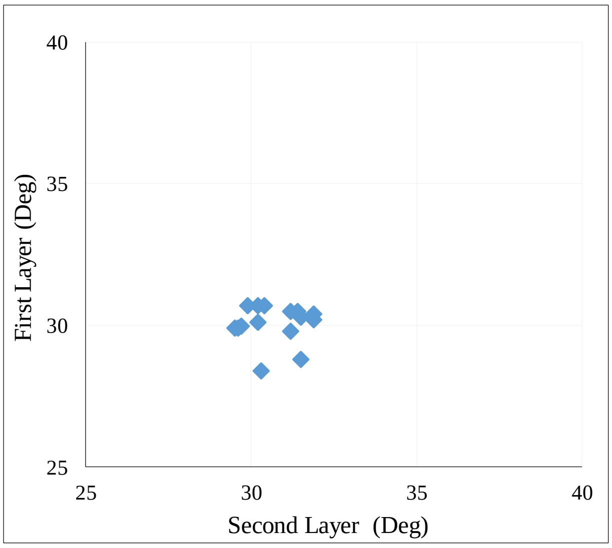


b) Final population

Figure 20. The estimation process of soil friction angle for the second layer versus the third layer in accordance with population



a) Initial population



b) Final population

Figure 21. The estimation process of soil friction angle for the first layer versus the second layer in accordance with population

12. Conclusion

In this paper, the back analysis using genetic algorithm can improve the values of unknown parameters introduced into the finite element code in order to achieve the least difference between the results of monitoring and numerical analysis. The parameters of Mohr-Coulomb model with the greatest effect on the lateral displacement of the soil nailed wall are selected, for which the back analysis process is performed and the elastic modulus is determined as the most influential parameter. The investigations show that the correct selection of the population size (npop) and the maximum number of iterations (maxiter) as effective parameters can shorten the process until the optimal results are obtained. All the information can be used in engineering fields for further analysis in the future. However, this method is time-consuming and involves a great deal of computations, which results in vast finite element calculations for determination of the objective (error) function. Given the idea of this paper, the back analysis using genetic algorithm can calculate 9 parameters simultaneously for 3 soil layers in the excavation problem and achieve the convergence criterion and the minimum value of error. This achievement indicates that the genetic algorithm is a highly suitable tool and can be effective for a wider range of geotechnical problems.

13. Acknowledgements

The authors would like to thank Dr. Mojtaba Salehi Dezfuli for his support and valuable comments in this research.

14. References

[1] Levasseur, S., Y. Malécot, M. Boulon, and E. Flavigny. "Soil Parameter Identification Using a Genetic Algorithm." International Journal for Numerical and Analytical Methods in Geomechanics 32, no. 2 (2008): 189–213. doi:10.1002/nag.614.

[2] Sakurai, Shunsuke, Shinichi Akutagawa, Kunifumi Takeuchi, Masato Shinji, and Norikazu Shimizu. "Back Analysis for Tunnel Engineering as a Modern Observational Method." Tunnelling and Underground Space Technology 18, no. 2–3 (April 2003): 185–196. doi:10.1016/s0886-7798(03)00026-9.

[3] Gao, Wei, Dongliang Chen, and Xu Wang. "Elastic–plastic Model Identification for Rock Surrounding an Underground Excavation Based on Immunized Genetic Algorithm." SpringerPlus 5, no. 1 (July 11, 2016). doi:10.1186/s40064-016-2726-z.

[4]Jin, Yin-Fu, Zhen-Yu Yin, Shui-Long Shen, and Dong-Mei Zhang. "A New Hybrid Real-Coded Genetic Algorithm and Its Application to Parameters Identification of Soils." Inverse Problems in Science and Engineering 25, no. 9 (November 24, 2016): 1343–1366. doi:10.1080/17415977.2016.1259315.

[5] Vahdati, Pooya, Séverine Levasseur, Hans Mattsson, and Sven Knutsson. "Inverse soil parameter identification of an earth and rockfill dam by genetic algorithm optimization." In ICOLD European Club Symposium (2013): 10/04/2013-12/04/2013.

[6] Hui, Wang, and Wang Qing-biao. "Dynamic Optimization Research on the Section Morphology of Large-Span Tunnels in Shallow Rock Mass." The Open Construction and Building Technology Journal 9, no. 1 (September 10, 2015): 210–213. doi:10.2174/1874836801509010210.

[7] Bartlewska-Urban, M., and T. Strzelecki. "Application of Genetic Algorithms for the Estimation of Hydraulic Conductivity." Studia Geotechnica et Mechanica 0, no. 0 (August 4, 2018). doi:10.2478/sgem-2018-0013.

[8]Zhao, B. D., L. L. Zhang, D. S. Jeng, J. H. Wang, and J. J. Chen. "Inverse Analysis of Deep Excavation Using Differential Evolution Algorithm." International Journal for Numerical and Analytical Methods in Geomechanics 39, no. 2 (May 9, 2014): 115–134. doi:10.1002/nag.2287.

[9]Sun, Yang, Qinghui Jiang, Tao Yin, and Chuangbing Zhou. "A Back-Analysis Method Using an Intelligent Multi-Objective Optimization for Predicting Slope Deformation Induced by Excavation." Engineering Geology 239 (May 2018): 214–228. doi:10.1016/j.enggeo.2018.03.019.

[10] Rechea, C., S. Levasseur, and R. Finno. "Inverse Analysis Techniques for Parameter Identification in Simulation of Excavation Support Systems." Computers and Geotechnics 35, no. 3 (May 2008): 331–345. doi:10.1016/j.compgeo.2007.08.008.

[11] Tikhonov A, V. Arsenine. Methodes de Resolution des Problemes mal Poses, Translation of Russian by V. Kotliar, Mir ed., Moscow, (1976).

[12] Haupt RL, Haupt SE. "Practical Genetic Algorithms". Wiley: New York, 1998.

[13] Gallagher, Kerry, and Malcolm Sambridge. "Genetic Algorithms: A Powerful Tool for Large-Scale Nonlinear Optimization Problems." Computers & Geosciences 20, no. 7–8 (August 1994): 1229–1236. doi:10.1016/0098-3004(94)90072-8.

[14] Holland JH. "Adaptation in Natural and Artificial Systems". University of Michigan Press: Ann Arbor, MI, (1975).

[15] Renders JM. Algorithmes Genetiques et Reseaux de Neurones. Hermes; Paris, (1995).

[16] Goldberg, DE, editor. Genetic algorithms in search, optimization and machine learning. Adisson-Wesley; (1989).

[17] Pal, Surajit, G. Wije Wathugala, and Sukhamay Kundu. "Calibration of a Constitutive Model Using Genetic Algorithms." Computers and Geotechnics 19, no. 4 (January 1996): 325–348. doi:10.1016/s0266-352x(96)00006-7.

[18] Tang, Yu-Geng, and Gordon Tung-Chin Kung. "Application of Nonlinear Optimization Technique to Back Analyses of Deep Excavation." Computers and Geotechnics 36, no. 1–2 (January 2009): 276–290. doi:10.1016/j.compgeo.2008.02.004.

[19] Singh, Vikas Pratap, and G. L. Sivakumar Babu. "2D Numerical Simulations of Soil Nail Walls." Geotechnical and Geological Engineering 28, no. 4 (December 25, 2009): 299–309. doi:10.1007/s10706-009-9292-x.

[20] Razavi, Seyyed Kazem, and Masoud Hajialilue Bonab. "Study of Soil Nailed Wall Under Service Loading Condition." Proceedings of the Institution of Civil Engineers - Geotechnical Engineering 170, no. 2 (April 2017): 161–174. doi:10.1680/jgeen.16.00006.

[21] Dehghan, A., F. Rezaee, and A. Ghanbari, "Back analysis of Karaj metro tunnel to determine geomechanical parameters of the encompassing soil." Geological Engineering Journal, No.2 (2009).

[22] Hashemi, S. M. "Back analysis of soil strength parameters by genetic algorithms", Master thesis (2017).