



## Fuzzy AHP Method for Selection of a Suitable Seismic Retrofitting Alternative in Low-Rise Buildings

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Received 11 January 2018; Accepted 17 May 2018

### Abstract

Decision making for selecting an appropriate alternative among nominated alternatives is still a problem among retrofit designers. It is clear that selected alternative should comply the current codes in terms of structural criteria, but the other criteria may not be considered. The main goal of this study is to introduce a suitable method for making a decision in order to find the best alternative considering the effective criteria in retrofitting of low-rise buildings. Analytic Hierarchy Process (AHP), as a technique of Multi-Criteria Decision Making (MCDM), is compatible to solve the problem. Effective criteria have been categorized to structural, operational, economic and functional criteria and sixteen sub-criteria considered as a pattern that satisfies the entire involved group including structural and architectural engineers, contractor, client, and authorities in retrofitting of low-rise buildings. Since most of the involved criteria such as aesthetic, durability, and compatibility have fuzzy nature and cannot be compared numerically, fuzzy AHP can be a compatible method for comparison different retrofitting alternatives among both fuzzy and non-fuzzy criteria. A matrix of pair-wise comparison (MPC) is used for determining the weight of criteria and also for scoring the alternatives respect to each criterion. A Fuzzy Importance scale with Triangular Fuzzy Numbers (TFN) is applied for comparing the criteria. The method is examined by a case study and the results show the used method can help designers for selecting the appropriate alternative.

*Keywords:* Multi-Criteria Decision Making; Seismic Retrofitting; Fuzzy AHP.

### 1. Introduction

According to codes such as FEMA 356 [1], ASCE /SEI 41-17 [2], NZSEE [3], BS EN 1998-3 [4] and IRI 360 [5], all the old buildings which do not meet the criteria of the codes, should be evaluated with regard to their resistances against earthquake. They probably need to be retrofitted due to some deficiencies related to their gravitational and lateral resistances, material and construction weaknesses. Although some alternatives have been proposed in codes and researches to retrofit vulnerable buildings, decision making for selecting an appropriate alternative is still an unsolved problem among retrofit designers and a few patterns are available to come up with this problem.

Many problems are involved in selecting the best seismic retrofitting alternative among lots of available alternative in low-rise buildings. Finding an appropriate framework among different engineering algorithms is the first problem of this research. This framework should satisfy all of the involved groups including structural engineers, architects, contractors, owners, and authorities. On the other hand, the framework should be applicable so that companies and retrofit designers can easily use it. After analyzing an exciting building most of the weakness in bearing against earthquake load should be cleared. Then, based on the output of analysis, retrofit designers consider some (at least three) alternatives for retrofitting of the building without any designing. It is clear that for each of the alternative they should

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 <http://dx.doi.org/10.28991/cej-0309157>

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consider a brief plan and specification of required equipment and materials. Finally, it should be decided which one is the most suitable for retrofitting of the building and then the selected alternative is designed, plotted, and estimated. It is clear that requirements of current codes should be compiled by selected seismic retrofitting alternative. Does the selected alternative satisfy the other economic, operational, and architectural criteria? Therefore, having an appropriate method considering all of the effective criteria can help designers to select the best alternative for seismic retrofitting of large numbers of low-rise buildings in earthquake-prone areas such as Iran.

A few available algorithms are based on analysis and design of all of the screened alternatives. Researchers such as Bostenaru Dan (2004) and Giovinazzi and Pampanin (2007) proposed methods which were based on analysis and design all of the screened alternatives, and the best alternative was selected through a comparison method with respect to some criteria [6,7]. These approaches are time-consuming and increase the costs of retrofitting design; however, they are beneficial for high-rise and important buildings. Besides, Moghadam and Azmoodeh (2011) proposed a binary approach procedure to optimize the limited seismic retrofitting alternatives for specific vulnerable buildings [8].

In spite of the fact that AHP is a convenient method in dealing with both quantitative and qualitative criteria of MCDM, fuzziness and vagueness in many decision-making problems may contribute to the imprecise judgments of decision makers in conventional AHP methods. Most of the criteria in seismic retrofitting such as availability, vulnerability during the performance, and downtime have fuzzy nature and other criteria such as drift, irregularity, cost of operation and maintenance can be evaluated by fuzzy algorithm without any designing and estimating of the alternatives. The first step in fuzzy AHP involves decomposing the problem into a hierarchical structure comprising of goal, criteria, sub-criteria, and alternatives to construct the model. Then, the elements are compared pairwise with respect to the importance to the goal, importance to the criterion, and importance to the sub-criterion [9-10]. In this study, the effective criteria are categorized with a pattern that consists of four main criteria; structural, operational, economic and functional (architectural) criteria, and for each of main criteria, related sub-criteria are extracted from codes and articles. A matrix of pair-wise comparisons (MPC) is used for determining the weight of criteria and also for scoring the alternatives respect to each criterion. A Fuzzy Importance scale with Triangular Fuzzy Numbers (TFN) is applied for comparing the criteria. Instead of eigenvalue method, the Normalization of the Geometric Mean (NGM) method is applied for computing weights from the fuzzy pairwise comparison matrices. Then Center of Area (COA) as a defuzzified method is employed to determine the Best Non-fuzzy Performance (BNP) value of weights.

## 2. Literature Review

Multi-criteria decision analysis (MCDA) methods can be applied to those problems where a decision maker ought to choose or rank a limited number of alternatives that are measured by several relevant criteria. There are four basic elements common to all MCDA problems include a finite set of alternatives, trade-offs among criteria, incommensurable units and decision matrix [11]. They introduced 17 major method classes and later updated these classes by adding three new methods. Some of these classes were considered not practical for using to real problems. Some methods can apply just for screening the alternatives or choosing the best alternative, while the other methods are used for multi-purposes. Methods such as additive weighting, TOPSIS, NCIC, and AHP are of this multi-purpose. Analytical hierarchy process (AHP) was developed primarily by Saaty [12]. It has been widely applied and has been discussed extensively in the peer-reviewed literature. He introduced a large number of subjects such as economic, social and also engineering fields which AHP can help decision makers to solve the problems. AHP has been applied in a wide variety of decision making including resource allocation, forecasting, total quality management, business process re-engineering, quality function deployment, and the balanced scorecard [13].

AHP is a MCDA method which falls within the broader class of methods known as additive weighting methods. A comprehensive survey of MCDA methods and applications have proved that additive weighting methods are probably the best known and most widely used to deal with MCDA problems by researchers [14-16]. AHP extends the basic additive weighting method by applying the principal eigenvector method for converting the paired comparison data into criteria weights. AHP specifies the use of one particular conversion method, involving the use of some basic concepts of linear algebra called eigenvalues and eigenvectors [12]. In Figure 1, a three-level hierarchy presentation is shown that at the top or the first level is the objective of the decision problem. The second level is the set of criteria or several sub-criteria levels to be considered in achieving the objective and the third level or the lowest level is the set of alternatives [13].

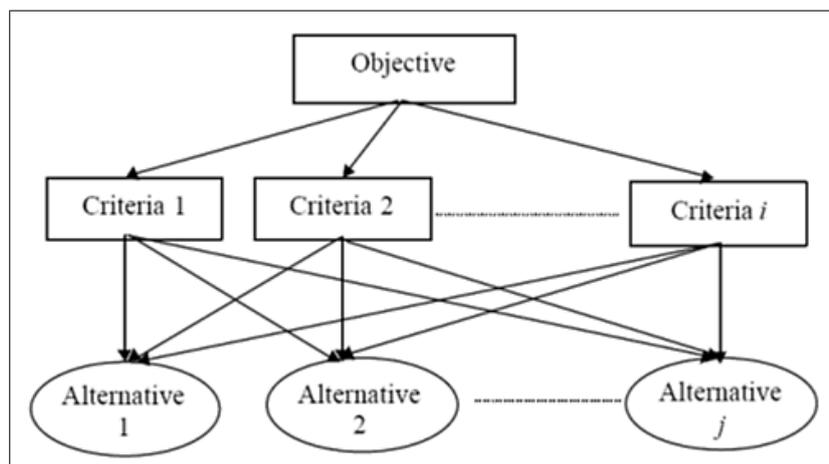


Figure 1. Hierarchical Pattern of a Decision

Many researchers who have reviewed and studied the fuzzy AHP which is the extension of Saaty's theory, have presented that fuzzy AHP shows relatively more sufficient description of these kind of decision making processes versus the traditional AHP methods. Zhu et al. [17] made a review and discussion on the analysis methods and applications of fuzzy AHP. Yu [18] employed the property of goal programming to solve group decision making fuzzy AHP problem. Kulak and Kahraman [19] used fuzzy AHP for multi-criteria selection among transportation companies. Mustaf B. Ayhan [20] applied the triangular fuzzy numbers (TFN) as a suitable method for selecting the best suppliers by considering five criteria and three alternatives and finally ranked the alternatives; however, the consistency ratio (CR) of judgment was not mentioned. Sunita.B et al. [21] applied fuzzy AHP for selection of most suitable construction method of green buildings. Two precast and cost-in situ were compared by considering economic, environmental, and social criteria as main criteria and 7, 5, 5 sub related criteria respectively. Saaty's method was used for weighting the criteria and four fuzzy scales considered for fuzzy relation between two alternatives and criteria.

In order to find the effective criteria in seismic retrofitting a few references are available. FEMA547 [1] consider five basic issues concern to building owners or users. This code considers irregularity (both in terms of stiffness and over strength distributions), modification of the strength, stiffness, and ductility as technical criteria such as NZSEE [3]. Some other factors affecting the choice of various retrofitting techniques include available workmanship, structural compatibility with the existing structural system, irregularity of stiffness, strength and ductility, the technology available, and sufficient capacity of foundation system [22]. Therefore many criteria are involved in making decision for retrofitting of buildings so classification of these criteria as main and sub-criteria are essential to weight and rank them. Bostenaru Dan [6] proposed a multi-criteria model for retrofitting existing building. His hierarchy framework consists of four levels that includes main problem (retrofit), actors (engineer, architect, investor, and user), main-criteria, and sub-criteria. His method is needed to allocate lots of time and budget for designing, drawing, and estimating all the considered alternatives, especially, in some cases using a nonlinear (static or dynamic) approach is indispensable. Giovinazzi and Pampanin [7] just considered eleven criteria under three major categories including a sustainable retrofit intervention, an effective emerging management, and a resilient post-earthquake reconstruction. This method is also needed designing and estimating all of the considered alternatives. Pashaei et al (2016) proposed an AHP model including four main criteria and some qualitative and quantitative related sub-criteria based on the Additive Weighting Method (AWD) by using a verbal rating scale for quantitative criteria, however for some structural sub-criteria analysis of alternatives should be calculated [23].

### 3. Research Methodology

#### 3.1. Fuzzy AHP

The first step in fuzzy AHP as shown in Figure 1 involves decomposing the problem into a hierarchical structure comprising of goal, criteria, sub-criteria, and alternatives to construct the model that proposed in Figure 3 in this research. Then, the elements are compared pair-wise with respect to the importance of the goal, importance to the criterion, and importance to the sub-criteria. A Fuzzy Logic System (FLS) consists of four main parts that including Fuzzification, Rules, Inference, and Defuzzification. A triangular fuzzy number (TFN) is the special class of fuzzy number (Fuzzification) whose membership is defined by three real numbers, expressed as  $(l, m, u)$ . Figure 2 displays the structure of a Triangular Fuzzy Number (TFN) according to Zhu et al [17].

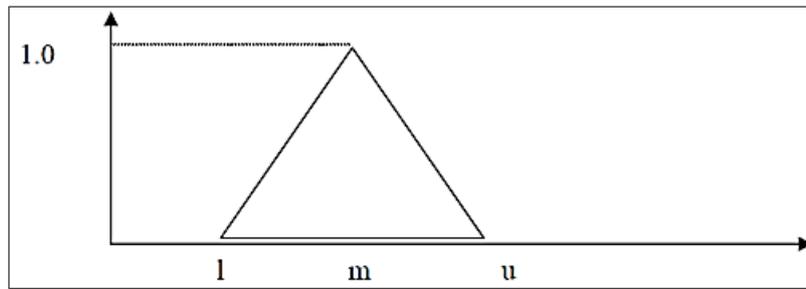


Figure 2. Triangular membership function

In Fuzzy-AHP, pairwise comparisons can be applied by using Triangular Fuzzy Numbers (TFN) corresponding to each linguistic variable by using a unique table such as Table 1 that arranged by Lamata [24]. This table is comparable with a crisp value that two lower limit and upper limit values are joined to the values.

Table 1. Fuzzy Importance scale with triangular fuzzy numbers

Saaty scale	Definition	TFS	Reciprocal TFS
1	Equally important	(1,1,1)	(1,1,1)
3	Weakly important	(2,3,4)	(1/4,1/3,1/2)
5	Fairly important	(4,5,6)	(1/6,1/5,1/4)
7	Strongly important	(6,7,8)	(1/8,1/7,1/8)
9	Absolutely important	(9,9,9)	(1/9,1/9,1/9)
2		(1,2,3)	(1/3,1/2,1)
4	The intermittent values between two adjacent scales	(3,4,5)	(1/5,1/4,1/3)
6		(5,6,7)	(1/7,1/6,1/5)
8		(7,8,9)	(1/9,1/8,1/7)

To reflect pessimistic, most likely and optimistic decision-making environment, triangular fuzzy numbers with minimum (or lower limit) value, most plausible (or most promising) value and maximum (or upper limit) value are considered and the fuzzy comparison matrix is defined as Equation 1.

$$\tilde{A} = \begin{pmatrix} 1 & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \dots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & 1 & \dots \\ \dots & \dots & \dots & 1 \end{pmatrix} \tag{1}$$

Where  $\tilde{a}_{ij} = (\tilde{\alpha}L_{ij}, \tilde{\alpha}M_{ij}, \tilde{\alpha}U_{ij})$  is the relative importance of each criteria in pair wise comparison and  $\tilde{\alpha}L_{ij}, \tilde{\alpha}M_{ij}, \tilde{\alpha}U_{ij}$  are the minimum value, most plausible value and maximum value of the triangular fuzzy number. The eigenvalue method, the geometric mean method, the linear programming method, and the lambda-max method are common methods which have been proposed to derive the weights using the AHP. Among these four methods, only the eigenvalue method is applied to handle the crisp numbers and the other methods are employed to deal with the fuzzy AHP numbers. The Normalization of the Geometric Mean (NGM) method that was proposed by Buckley et al. [25] is an easy method to derive weights from the fuzzy pair wise comparison matrices which is given by Equation 2:

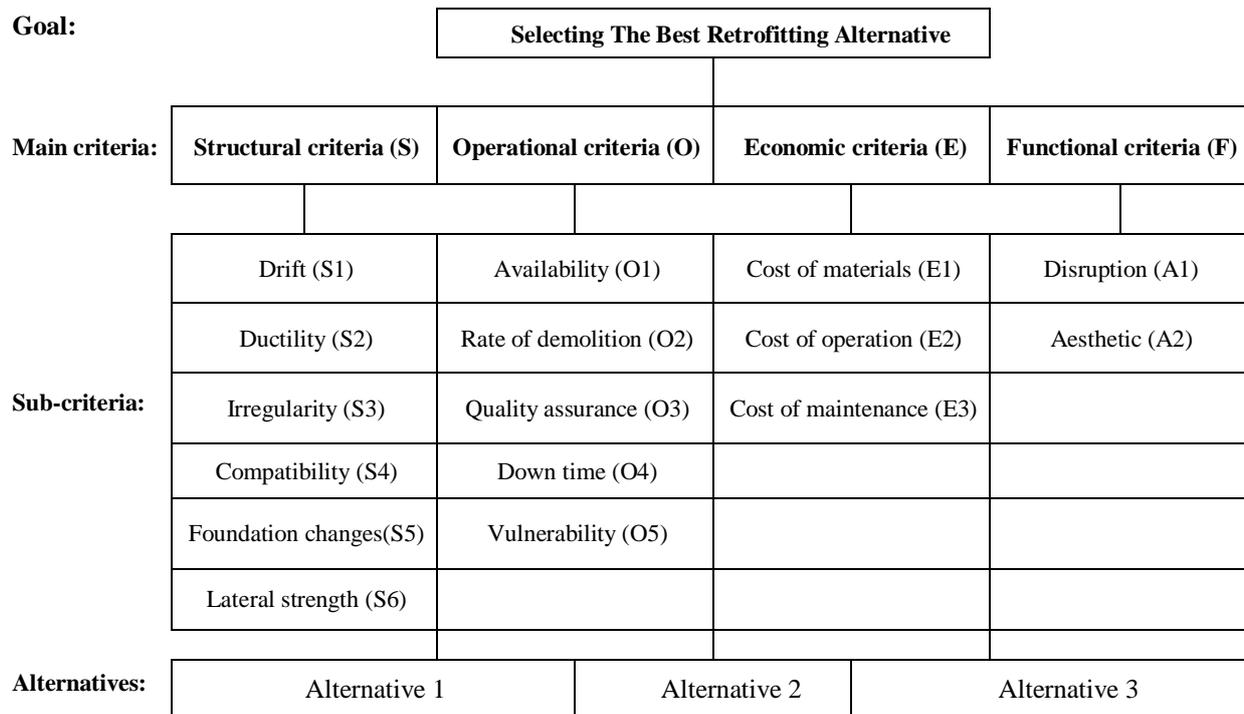
$$w_i = a_i / \sum_{i=1}^n a_i \quad \text{Where } a_i = (\prod_{j=1}^n a_{ij})^{1/n} \tag{2}$$

In the above equations,  $a_i$  is geometric mean of criterion and  $a_{ij}$  is the comparison value of criterion " i " to criterion " j ". The  $w_i$  is the ith criterion's weight, where  $w_i > 0$  and  $\sum_{i=1}^n w_i = 1$ .

Since  $w_i$  is a fuzzy number, a defuzzified method should be used to determine the Best Non-fuzzy Performance (BNP) value of weights. Saletic et al. [26] analyzed basic defuzzification techniques such as Center of Area (COA), Center of Gravity (COG), Fuzzy Mean (FM), and Quality Method (QM). The COA method is widely used to determine the Best Non-fuzzy Performance (BNP) value of weights and scores.

### 3.2. Finding and Categorizing the Effective Criteria

In this study, based on Figure1, goal, main criteria, sub criteria and alternatives stand as a hierarchal plan. Main criteria consist of structural, operational, economic, and functional criteria, and for each of main criteria, some appropriate sub-criteria are considered based on the AHP model (Figure 3).



**Figure 4. AHP model in this study**

Bansal et al. [21] mentioned that decision makers can simultaneously compare up to seven criteria, therefore the number of sub-criteria are limited less than seven criteria. For each of main and sub criteria an abbreviation is also considered for future calculations.

### 3.2.1. Structural Criteria

Structural criteria play an important role in decision-making and should satisfy the current codes. In order to compare the alternatives in terms of structural criteria, six indispensable criteria have been considered based on the current codes and a few researches related to this issue.

- Drift: Based on all of the codes for design of new buildings and retrofitting of buildings drift is an important role in stability and lateral loads capacity, as this parameter has been limited depending on the type of structures.

- Ductility and dissipation energy: This is one of the important structural criteria that dissipates and reduces the lateral earthquake loads on buildings. This criterion has been mentioned by NZSEE [3], BS EN 1998-3[4] and some researchers such as Bostenaru Dan M.D. [6] and Jong-Wha Bia [22] as the effective criterion related to the seismic behavior of structures. Innovation systems such as active, semi-active, and intelligent damper or base isolation dissipate earthquake energy, which was considered as “reduced demand” by M.D. Bostenaru Dan [6].

- Irregularity: According to FEMA 356 [1] Torsional Stiffness Irregularity and Vertical Stiffness Irregularity play an important role in the behavior of retrofitting alternatives and shall be evaluated for each story and direction of a building. Decreasing irregularity with considering new elements (such as kinds of walls and bracing) in the right location can decrease the earthquake loads on buildings.

- Compatibility: Compatibility of the new system with the existing structure is another criterion which is mentioned by FEMA 547 [1] and Jong-Wha Bia [22]. For example, adding reinforced concrete frames or shear walls to an existing reinforced concrete structure are more compatible than adding steel bracings.

- Foundation changes: Enlargement the size of foundation and adding new reinforcements are indispensable factors that caused by some alternatives such as shear walls and bracings considered by FEMA 547[1].

- Lateral Strength: It has been focused by FEMA 547[1] and also by NZSEE [3], Bostenaru Dan M.D [6] and Jong-Wha Bia [22] as the main factors to control drift and configuration problems by adding new lateral force-resistance. It is obvious that creating a shear wall in a bay cause more lateral strength of building than creating CBF or EBF.

### 3.2.2. Operational Criteria

Owners or clients, contractors and sponsors are the individuals who are involved in these criteria. Operational criteria are considered based on current codes and researchers. The most important sub-criteria for comparing the alternatives consist of:

- Availability: Availability of Material, equipment and skilled workers is an important criterion that should be considered for each nominating alternatives.
- Rate of Demolition: Some methods need to change and demolish some members such as beams, columns, and walls that were considered by Bostenaru Dan [6].
- Quality Assurance (Q.A): Rate of quality assurance is an essential criterion for all the selective alternatives that should be evaluated which is considered by FEMA547 [1].
- Down time: This sub-criterion will be more important for buildings such as hospitals, schools, and emergency buildings possibility of incremental retrofitting which is also considered by Giovinazzi and Pampanin [7] can be effective on down time.
- Vulnerability during Work: For some types of the alternatives, buildings need to be temporarily supported. For example, replacement of shear wall instead of masonry walls which is considered by FEMA547 [1].

### 3.2.3. Economic Criteria

Economic criteria are also focused by a few researchers such as Bostenaru Dan [6].

- Cost of materials and equipment: it is consist of providing materials and procurement of equipment.
- Cost of Operation: Cost of operation consists of the cost of demolition and retrofitting elements performance. Although operation's cost of alternatives can be exactly determined when design and drawing are completed, it can be approximately estimated and compared by fuzzy value among the nominated alternatives without completing the process of design and drawing.
- Cost of Maintenance: Some innovation systems need to be maintained and monitored by electronic equipment and skilled personals, and the other need to be inspected in a specific period. It can be exemplified by intelligent, smart damper and base isolation.

### 3.2.4. Functional Criteria

The most important Functional and architectural criteria are including:

- Disruption: The significance of conflicts with mechanical, electrical, or plumbing distribution systems or equipment should be considered during development of retrofitting schemes". Giovinazzi and Pampanin [7] also assessed this criterion as "disruption of use".
- Aesthetic aspects: Aesthetic sensitivity just in heritage and historic buildings has been mentioned by NZSEE [3] and FEMA547 [1]. Some retrofitting systems such as adding shear walls can influence on facade, and the other systems such as enlargement of columns and beams or adding bracing systems in the interior and exterior of buildings can be frequently evaluated as a negative aspect.

### 3.3. Weighting the Main and Sub-Criteria

Matrix of pair-wise comparisons (MPC) is used for determining the criteria and also for scoring the alternatives. A Fuzzy Importance scale with Triangular Fuzzy Numbers (TFN) is applied for comparing the criteria and alternatives. The Normalization of the Geometric Mean (NGM) method are applied for computing weights from the fuzzy pair wise comparison matrices. Then Center of Area (COA) as a defuzzified method is employed to determine the Best Non-fuzzy Performance (BNP) value of weights.

Pair-wise comparisons are generally applied to determine the weights of criteria; however, often decision makers find it difficult to determine uprightly the weights of criteria. Therefore, the problem is inverted to making a series of pair-wise comparisons as summarized in a Matrix of Pair-wise Comparisons (MPC). A judgment is required for each pair of criteria to figure out how much a criterion has priority over one another criterion. However, the diagonal of the matrix are members of units and the elements below the diagonal are equal to the reciprocals of the corresponding elements above it. Therefore, the numbers of judgments decrease to  $n(n-1)/2$  judgments in a full pair-wise comparison. It is necessary that all comparisons in a certain MPC to be based on the same method and same scale. Weighting the criteria are including weighting the main criteria and sub-criteria which consisting a 4x4 matrix for comparison the main criteria .

Therefore just six  $(4(4-1)/2)$  judgments related to upper elements of MPC are required. In order to weight the structural, Operational, economic criteria the dimension of matrices and number of judgments including:

- Structural criteria: A 6x6 matrix and 15 judgments
- Operational criteria: A 5x5 matrix and 10 judgments
- Economic criteria: A 3x3 matrix and 3 judgments

Since the weighting of the criteria can play an important role in final ranking of the alternatives, the MPC are created based on their definition in the part 3.2 and they can be fixed for all of the decision making in the same retrofitting buildings. Fuzzy pair-wise comparisons for weighting the main criteria and the results are shown in Table 2.

**Table 2. Fuzzy-MPC for weighting the Main criteria and results**

	Structural criteria			Operation criteria			Economic criteria			Functional criteria			a1L	a1M	a1U	WiL	WiM	WiU	Wi
Structural	1	1	1	4	5	6	1	2	3	5	6	7	2.115	2.783	3.35	0.537	0.541	0.509	0.529
Operation	1/6	1/5	1/4	1	1	1	1/3	1/2	1	1	1	1	0.486	0.562	0.707	0.123	0.109	0.107	0.113
Economic	1/3	1/2	1	1	2	3	1	1	1	2	3	4	0.903	1.316	1.861	0.229	0.256	0.282	0.256
Functional	1/7	1/6	1/5	1	1	1	1/2	1/3	1	1	1	1	0.435	0.486	0.669	0.110	0.094	0.102	0.102

According to Equation 2 in Section 3.1, in order to derive the weights of main-criteria from the fuzzy pairwise comparison, the following calculation should be used:

$$a_{1L}=(a_{L11} \times a_{L12} \times a_{L13} \times a_{L14})^{1/4} = (1 \times 4 \times 1 \times 5)^{1/4}=2.115$$

$$a_{1M}=(a_{M11} \times a_{M12} \times a_{M13} \times a_{M14})^{1/4} = (1 \times 5 \times 2 \times 6)^{1/4}=2.783$$

$$a_{1U}=(a_{U11} \times a_{U12} \times a_{U13} \times a_{U14})^{1/4} = (1 \times 6 \times 3 \times 7)^{1/4}=3.350$$

By using the above equation, the fuzzy weights for each of the main criterion are obtained.

$$w_{1L} = a_{1L} / (a_{1L} + a_{2L} + a_{3L} + a_{4L}) = 2.115 / (2.115+0.485+0.903+0.435) = 0.537$$

$$w_{1M} = a_{1M} / (a_{1M} + a_{2M} + a_{3M} + a_{4M}) = 2.783 / (2.783+0.562+1.316+0.486) = 0.541$$

$$w_{1U} = a_{1U} / (a_{1U} + a_{2U} + a_{3U} + a_{4U}) = 3.350 / (3.350+0.707+1.861+0.668) = 0.509$$

The fuzzy weights can be defuzzified by one of the available method such as Center of Area (COA) in order to derive crisp weights. The Best Non-fuzzy Performance (BNP) value of the fuzzy number Wi can be found by:

$$BNP_i = [(U_{wi} - L_{wi}) + (M_{wi} - L_{wi})] / 3 + L_{wi} \quad L_{wi} = \text{Lower limit weight's value}$$

$$M_{wi} = \text{Most promising weight's value} \quad U_{wi} = \text{Upper limit weight's value}$$

$$W_s = [(0.509 - 0.537) + (0.541 - 0.537)] / 3 + 0.537 = 0.529 \tag{3}$$

$$\sum_{i=1}^n W_i = 0.529 + 0.113 + 0.256 + 0.102 = 1$$

Based on the description in section 3.2 the other Fuzzy-MPC can be constructed as Table 3 to 5 and according to the above method, the weights of each criterion belong to structural, operational, economic and functional criteria have been calculated by an excel program and put in the last row of each MPC.

**Table 3. Fuzzy-MPC for weighting the Structural criteria**

	Drift			Ductility			Irregularity			Compatibility			Foundation changes			Lateral Strength		
Drift	1	1	1	2	3	4	3	4	5	6	7	8	4	5	6	3	4	5
Ductility	1/4	1/3	1/2	1	1	1	1	2	3	2	3	4	1	2	3	1	1	1
Irregularity	1/5	1/4	1/3	1/3	1/2	1	1	1	1	1	2	3	1	1	1	1	1	1
Compatibility	1/8	1/7	1/6	1/4	1/3	1/2	1/3	1/2	1	1	1	1	1/3	1/2	1	1/3	1/2	1
Foundation changes	1/6	1/5	1/4	1/3	1/2	1	1	1	1	1	2	3	1	1	1	1	1	1
Lateral Strength	1/5	1/4	1/3	1	1	1	1	1	1	1	2	3	1	1	1	1	1	1
Weights	WS1=0.449			WS2=0.166			WS3=0.106			WS4=0.063			WS5=0.102			WS6=0.114		

**Table 4. Fuzzy-MPC for weighting the Operational criteria**

	Availability			Rate of demolition			Quality assurance			Down time			Vulnerability during work		
Availability	1	1	1	6	7	8	4	5	6	4	5	6	2	3	4
Rate of demolition	1/8	1/7	1/6	1	1	1	1	2	3	1	2	3	1/3	1/2	1
Quality assurance	1/6	1/5	1/4	1/3	1/2	1	1	1	1	1	1	1	1/3	1/2	1
Down time	1/6	1/5	1/4	1/3	1/2	1	1	1	1	1	1	1	1/3	1/2	1
Vulnerability during work	1/4	1/3	1/2	1	2	3	1	2	3	1	2	3	1	1	1
Weights	WO1=0.478			WO2=0.095			WO3=0.109			WO4=0.147			WO5=0.171		

**Table 5. Fuzzy-MPC for weighting the Economic criteria**

	Cost of Equipment and materials			Cost of Operation			Cost of Maintenance		
Cost of Equipment and materials	1	1	1	4	5	6	6	7	8
Cost of Operation	1/6	1/5	1/4	1	1	1	1	2	3
Cost of Maintenance	1/8	1/7	1/6	1/3	1/2	1	1	1	1
Weights	WE1=0.732			WE1=0.732			WE1=0.732		

Since the architectural criteria are included just two criteria, it doesn't need to create an MPC and the weights are considered based their definition:  $W_{F1}=0.65$ ,  $W_{F2}=0.35$

**3.4. Scoring the Alternatives and Final Ranking**

Since 16 sub-criteria have been considered in this study, 16 matrices of pair wise comparison should be constructed for scoring the alternatives relative to each criterion. It is obvious that for three nominated alternatives a 3x3 MPC should be created and four  $(3(3-1)/2)$  judgments based on the behavior of each alternative relative to each criterion are required. After solving the each of the matrix similar to the section 3.3 three values of scoring the alternatives are achieved. For example, alternatives' scoring one to three relatives to criterion  $S_1$  can be shown as  $A_1/S_1$ ,  $A_2/S_1$ , and  $A_3/S_1$  respectively. In Table 6, the calculated values for main-criteria weights are listed on the second row and the sub-criteria weights are listed on the fourth row. The alternatives scoring relative to each criterion are calculated on the 5<sup>th</sup> or 7<sup>th</sup> rows and the ranking of each alternatives relative to main-criteria ( $r_{Ai/S}$ ,  $r_{Ai/O}$ ,  $r_{Ai/E}$ ,  $r_{Ai/F}$ ) are calculated on the 8<sup>th</sup> to 10<sup>th</sup> rows. The last column represents the final ranking for each alternative which obtained by the following calculation.

$$\begin{aligned}
 RA_1 &= (r_{A1/S} \times W_S) + (r_{A1/O} \times W_O) + (r_{A1/E} \times W_E) + (r_{A1/F} \times W_F) \\
 RA_2 &= (r_{A2/S} \times W_S) + (r_{A2/O} \times W_O) + (r_{A2/E} \times W_E) + (r_{A2/F} \times W_F) \\
 RA_3 &= (r_{A3/S} \times W_S) + (r_{A3/O} \times W_O) + (r_{A3/E} \times W_E) + (r_{A3/F} \times W_F)
 \end{aligned}
 \tag{4}$$

**Table 6. Alternatives' scoring and final alternatives' ranking**

No.	Structure						Operational					Economic			Functional		Final Ranking	
1	Wi	WS						WO					WE			WF		
2	Criteria	S1	S2	S3	S4	S5	S6	O1	O2	O3	O4	O5	E1	E2	E3	F1		F2
3	wi	WS1	WS2	WS3	WS4	WS5	WS6	WO1	WO2	WO3	WO4	WO5	WE1	WE2	WE3	WF1		WF2
4	SA1	A1/S1	A1/S2	A1/S3	A1/S4	A1/S5	A1/S6	A1/O1	A1/O2	A1/O3	A1/O4	A1/O5	A1/E1	A1/E2	A1/E3	A1/F1	A1/F2	
5	SA2	A2/S1	A2/S2	A2/S3	A2/S4	A2/S5	A2/S6	A2/O1	A2/O2	A2/O3	A2/O4	A2/O5	A2/E1	A2/E2	A2/E3	A2/F1	A2/F2	
6	SA3	A3/S1	A3/S2	A3/S3	A3/S4	A3/S5	A3/S6	A3/O1	A3/O2	A3/O3	A3/O4	A3/O5	A3/E1	A3/E2	A3/E3	A3/F1	A3/F2	
7	rA1	rA1/S= WS1. A1/S1 + ... + WS6. A1/S6						rA1/O=WO1. A1/O1+ ... +WO5. A1/O5					rA1/E=...			rA1/F=...		RA1
8	rA2	rA2/S= WS1. A2/S1+ ... +WS6. A2/S6						rA2/O=WO1. A2/O1+ ... +WO5. A2/O5					rA2/E=...			rA2/F=...		RA2
10	rA3	rA3/S= WS1. A3/S1+ ... +WS6. A3/S6						rA3/O=WO1. A3/O1+ ... +WO5. A3/O5					rA3/E=...			rA3/F=...		RA3

## 4. Case Study

The building that is considered in the case study consists of two basements and three stories above the ground that is located in capital of Iran (Tehran) and was built about 22 years ago. In two directions steel moment frames resist against gravity and earthquake loads, without any bracings and shear walls. After evaluation of building with performing geotechnical and welding tests, analysis of building was done by software SAP 2000(ver.11) and the result of analysis demonstrated that building was needed to be retrofitted. Table 7 shows how the company scored each alternative concerning each criterion through creating a decision matrix. The comparison of the alternatives for selecting the best alternative is based on just five criteria. Regarding the survey result, disruption, down time and functional changes are assessed to cost (negative) criteria, on the contrary, building performance and access to the plan are appraised benefit (positive) criteria. The second alternative got the highest scores in benefit criteria and the least scores in cost criteria; thus, the company logically concluded the second alternative has got the best scores without calculating and considering weighting for the just five criteria. Then the design and retrofitting details were followed by this alternative. It was a rarely a logical comparison for selecting an appropriate alternative, because most of retrofit designers just proposed one alternative to clients without any document for comparison among alternatives in low rise buildings. Although different alternatives considering a retrofitting code can improve the structural behavior of the building, some criteria such as economic, operational, and functional criteria may be missed.

**Table 7. Decision matrix for comparison the alternatives used by the company**

Criteria Alternatives	Disruption During work	Down time	Building performance	Access to plans	Architectural changes
1-Strengthening of Moment frames	high	3-4 months	good	good	fair
2-Braced Frame(EBF)	low	2-3 months	good	good	low
3-Shear wall	fair	2.5-3.5 months	good	fair	high

### 4.1. Weighting the Criteria and Scoring the Alternatives

The weights that have been determined in part 3.3 considered in this case study. In order to score the alternatives respects to criteria, specification of three retrofitting alternatives (similar the nominated alternatives by company) are being surveyed. Scoring the alternatives have been done based on their behavior and properties concerning to each criteria.

#### 4.1.1. Strengthening of Moment Frames

Frame members that are inadequate to resist the seismic demands are strengthened with cover plates or by adding side plates to create box sections. This reduces axial and flexural stresses in beams and columns and could also be used to increase the shear strengths of these members. According to FEMA 547 [1] boxing a column decreases its slenderness and also increases its axial and flexural capacities. Whether the areas of the new plates can be directly included in computing these capacities depend on their continuity and detailing at a beam-column joint. Except for one side of the exterior columns, beams framing into the columns at each floor will disrupt the continuity of the new plates. The flexural strength of a beam can be improved by welding cover plates to the bottom if there is a composite slab present. If there is only a bare metal deck, a cover plate on only one side of the beam may not be very effective. However, it could be useful for strengthening beams with large axial forces, primarily collector members. Where cover plates are added to the columns at their base, a reevaluation of the foundation system is warranted. It is not uncommon for frame columns to develop plastic hinges at their bases and thus, the increase in demand on the foundation may be greater than intended. Schemes that involve slab removal, work around a connection, and foundation work are costly. As typical with seismic upgrades, cost and disruption are minimized when schemes are kept simple.

#### 4.1.2. Adding Eccentric Braced Frames (EBF)

According to FEMA 547 [1] the seismic performance of a building may be improved by adding braces to existing steel moment frames. Braces can be added without considerably increasing the mass of the building. Various Concentrically Braced Frame (CBF) configurations should be considered, though some tend to perform much better than others in earthquakes. In addition, systems that meet the provisions for Special Concentrically Braced Frame (SCBF) are expected to exhibit stable and ductile behavior in great earthquakes. Moment frames are not normally converted to Eccentrically Braced Frame (EBF) because of complicated design and detailing issues that would be encountered. The addition of braces to an existing structure changes the architectural function and character of the building. Braces in exterior frames will be visible in buildings with clear glazing. At interior bays, braces have to be arranged to avoid obstruction of existing parking, entrance ways, and other building systems. Braces are also commonly exposed and incorporated into the interior architecture. If the braced frames are buried in partition walls, the designer should be alert that these kinds of walls will be thicker than typical walls. Beams that are increased in size and new

collectors may affect nonstructural components by reducing clear floor heights. These components typically include suspended ceilings, conduits, and ducts. Coordination with the architect and other trades should not be neglected or under estimated. This system could be cost-effective when compared to other alternatives for upgrading steel moment frame buildings. Costs will be less when existing moment frames are converted into braced frames to take advantage of the existing strength and stiffness of the frame members, connections, and foundations. This alternative has good compatibility with steel moment frames and also has good availability of materials, skilled worker, and equipment.

**4.1.3. Adding Shear Walls**

Addition of shear walls as mentioned in Section 2.3.5 is a common method of adding significant strength and stiffness to the structure, therefore shear walls can improve lateral stability and drift. Torsion and irregularities should be considered in arrangements based on Section 12.4.2 FEMA 547 [1]. The existing building is not regular; however, a good arrangement of shear walls can decrease this criterion. In terms of structural criteria this alternative is similar to the addition of bracing while the strength and stiffness of shear walls are greater, hence fewer bays should be filled rather than bracings and depends on arrangements, can usually impact to the architectural and functional use of the buildings. Walls can be exposed and incorporated into the interior architecture or hidden in partition walls. This alternative has enough compatibility with steel moment frames; however, connection between walls and columns should be meet current codes. New foundations are almost always required for new walls and could be extremely costly if deep foundations, such as drilled piers, are added. Shotcrete walls are typically cheaper and faster to construct than conventional concrete walls due to the savings in materials and labor associated with formwork. Cost savings can be even greater if shotcrete is applied against an existing wall at a stair or elevator and mechanical shafts.

Based on above definition, properties, advantages, and disadvantages of each alternatives respect to each criterion, fuzzy MPC has been assigned in Table 8. According to the part 3.3 with the excel program, the scoring have been calculated and put in the last row of each MPC. It is obvious that sixteen MPC should be created for each of sixteen considered criteria; however, some of the MPC are similar to each other.

**Table 8. Fuzzy MPC for scoring the alternatives respect to each criterion**

MPC S1=S6=O3										MPC O1										
A1			A2			A3				A1			A2			A3				
A1	1	1	1	1/3	1/2	1	1/3	1/2	1	A1	1	1	1	1/6	1/5	1/4	2	3	4	
A2	1	2	3	1	1	1	1	1	1	A2	4	5	6	1	1	1	1	2	3	
A3	1	2	3	1	1	1	1	1	1	A3	1/4	1/3	1/2	1/3	1/2	1	1	1	1	
Scores	0.18			0.41			0.41				Scores	0.23			0.59			0.16		
MPC S2										MPC O2=O5										
A1			A2			A3				A1			A2			A3				
A1	1	1	1	1	2	3	1	2	3	A1	1	1	1	1/4	1/3	1/2	1	2	3	
A2	1/3	1/2	1	1	1	1	1	1	1	A2	2	3	4	1	1	1	5	6	7	
A3	1/3	1/2	1	1	1	1	1	1	1	A3	1/3	1/2	1	1/7	1/6	1/5	1	1	1	
Scores	0.489			0.255			0.255				Scores	0.21			0.61			0.12		
MPC S3=E1										MPC O4=E3										
A1			A2			A3				A1			A2			A3				
A1	1	1	1	1	1	1	2	3	4	A1	1	1	1	1	1	1	1/4	1/3	1/2	
A2	1	1	1	1	1	1	2	3	4	A2	1	1	1	1	1	1	1/4	1/3	1/2	
A3	1/4	1/3	1/2	1/4	1/3	1/2	1	1	1	A3	2	3	4	2	3	4	1	1	1	
Scores	0.439			0.439			0.121				Scores	0.201			0.201			0.597		
MPC S4										MPC E2										
A1			A2			A3				A1			A2			A3				
A1	1	1	1	1	1	1	1	2	3	A1	1	1	1	1/6	1/5	1/4	1/4	1/3	1/2	
A2	1	1	1	1	1	1	1	2	3	A2	4	5	6	1	1	1	1	2	3	
A3	1/3	1/2	1	1/3	1/2	1	1	1	1	A3	2	3	4	1/3	1/2	1	1	1	1	
Scores	0.386			0.386			0.227				Scores	0.11			0.566			0.323		
MPC S5										MPC F1=F2										

	A1			A2			A3			A1			A2			A3			
A1	1	1	1	1	1	1	3	4	5	A1	1	1	1	2	3	4	1	2	3
A2	1	1	1	1	1	1	3	4	5	A2	1/4	1/3	1/2	1	1	1	1	2	3
A3	1/5	1/4	1/3	1/5	1/4	1/3	1	1	1	A3	1/3	1/2	1	1/3	1/2	1	1	1	1
Scores	0.442			0.442			0.115			Scores	0.528			0.261			0.210		

**Table 9. Alternatives' scoring with respect to Main criteria and final alternatives' ranking**

No.		Structure					Operational					Economic			Functional		Final Ranking		
1	Wi	WS =0.524					WO =0.111					WE =0.264			WF =0.100				
2	Criteria	S1	S2	S3	S4	S5	S6	O1	O2	O3	O4	O5	E1	E2	E3	F1		F2	
3	wi	0.449	0.166	0.106	0.063	0.102	0.114	0.478	0.095	0.109	0.147	0.171	0.732	0.168	0.10	0.65		0.35	
4	SA1	0.18	0.442	0.386	0.439	0.489	0.18	0.23	0.201	0.18	0.23	0.23	0.201	0.11	0.439	0.528		0.528	
5	SA2	0.41	0.442	0.386	0.439	0.255	0.41	0.61	0.201	0.41	0.61	0.59	0.201	0.566	0.439	0.261		0.261	
6	SA3	0.41	0.115	0.227	0.121	0.255	0.41	0.12	0.597	0.41	0.12	0.16	0.597	0.323	0.121	0.210		0.210	
7	rA1	rA1/S= 0.298					rA1/O=0.220					rA1/E=0.359			rA1/F=0.528			RA1=0.329	
8	rA2	rA2/S= 0.389					rA2/O=0.519					rA2/E=0.438			rA2/F=0.261			RA2=0.405	
9	rA3	rA3/S= 0.312					rA3/O=0.241					rA3/E=0.203			rA3/F=0.21			RA3=0.266	

### 5. Results and Discussions

The final fuzzy AHP ranking is also determined that the second alternative (Eccentric Braced Frame) is the best alternative same as the company, although no longer the first alternative can probably be a competitor for the second alternative. It is clear that the second alternative has got the highest score in structural, operational, and economic criteria, however, the first alternative in functional criteria could get the highest score. If the building had an especial usage such as official, important commercial, art gallery etc., consequently, operational criteria (especially down time and rate of demolition) and functional criteria had a higher weight and probably the first alternative got the better scores. It is obvious that the weights of criteria have an important role on final raking of the alternatives as structural criteria and economic criteria have had 0.524 and 0.264 respectively of total of weights. On the other hand, drift (structural criteria), availability (operational criteria) and costs of material (economic criteria) have had 0.45, 0.48, and 0.73 of weights respectively. Therefore, it seems the weights of main criteria and sub-criteria have an important influence on the results and change by different decision maker's judgements. The accuracy the weights of criteria can be enhanced by group decision at least by three decision maker's judgments and getting average of them. On the other hand, the usage of building can be affected on the weights of main and sub criteria. For example in the historical buildings, functional and operational criteria are more important than structural and economic criteria. Scoring the alternatives are easier than weighing the criteria because the decision makers encounter a 3x3 matrix and only 3 judgments are required, while for weighting the criteria larger matrices and judgments are needed. Secondly, each alternative has its own technical characteristic with respect to each criterion and therefore the rate of error in scoring the alternatives will decrease among decision maker. However, the accuracy of the scoring can also be enhanced by using of three judgements. This proposed algorithm can process using more than three alternatives. Applying three alternatives in the case study have just been used for comparison the results with the company. Buckling Restrained Brace (BRB) and Rotational Friction Damper could also be considered as the fourth and fifth alternatives for retrofitting of the case study building.

### 6. Conclusion

Most of the criteria in seismic retrofitting such as availability, vulnerability during performance, and possibility of phased work have fuzzy nature and other criteria such as drift, irregularity, cost of operation and maintenance can be evaluated by fuzzy algorithms from first phase proposals of the nominated alternatives without any designing and estimating of the alternatives. In this study, the effective criteria are categorized with an AHP pattern that consists of four main criteria; structural, operational, economic and functional criteria, and for each of main criteria, related sub-criteria are extracted from codes and articles that probably satisfy the entire involved groups in retrofitting of a building. Increasing the number of main criteria, sub-criteria and expanding sub criteria through sub-sub criteria for different types of buildings through a hierarchical pattern can enhance the accuracy of the results. On the other hand, increasing the number of sub-criteria can raise the number of judgments in a MPC and complicate the process of weighing the

criteria, as Satty confirmed that human beings can simultaneously compare up to seven criteria. Group decision by engineering societies and companies can enhance the accuracy of weights of criteria for different type of usage of buildings and these can be fixed and used for several decisions making. Since usually three nominated alternatives proposed for selecting the best one, three judgments are easily can be done by retrofit designers, however two or three decision makers can increase the accuracy of results. Fuzzy Importance scale with Triangular Fuzzy Numbers (TFN), Normalization of the Geometric Mean (NGM) and Center Of Area (COA) have been chosen as logical and user friendly methods for comparison the criteria, computing weights and scoring, and defuzzification to determine the Best Non-fuzzy Performance (BNP) values of weights, however the other methods can be checked for the accuracy of results.

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