Modeling Groundwater Surface by MODFLOW Math Code and Geostatistical Method

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Abstract

Simulation of groundwater flow by mathematical model can be used for developing aquifer balance element analysis scenarios, explaining conditions of droughts, definition of prohibitive extraction policies and analyzing the qualitative models. In this study, the development of a quantitative model in terms of the main parameters affecting on the water surface changes has been performed for the Ardebil plain (located in NW of Iran). Accordingly, a comprehensive processing of raw data sets has been carried-out by means of MODFLOW mathematical model. Also to simulate the groundwater surface changes in the mentioned plain, the geo-statistical method has been used. Results indicate that the mathematical model used in the aquifer balance simulation for the Ardebil plain has approximately 2% relative normal root-mean-square error (NRMSE). This small NRSMSE confirms the model accuracy for the Ardebil plain using the calibration data. Moreover, comparing the results of this method and the ones obtained by mathematical model performed by examining some error criteria like RMSE, Mean, ASE and MS, it is found that the accuracy of the mathematical model is higher than the geostatistical method and the main reason for this is the distribution of uncertainty in a few available piezometric points in the geostatistical method.

Keywords: Ardabil Plain Aquifer; Aquifer Balance; Geostatistics; MODFLOW.

1. Introduction

The irresponsible and uncontrolled use of water and underground resources, reduced precipitations, concentration of consumption in some places (imbalance between water demand and supply potential), inappropriate cultivation patterns, improper irrigation and digging numerous wells and unplanned operation of them in recent decades have made groundwater resources' condition critical in most of the country's plains. Accordingly, groundwater surface has consistently reduced in the majority of aquifers in Iran and the average annual drop in the last 15 years has been estimated to be about 12 meters. Reduced groundwater surface of the plains increases the cost of water extraction and energy consumption reduces water quality and causes land subsidence. As secondary effects, by reducing the amount of extractable underground resources, and increasing the cost of water extraction the price of products increases. Another factor contributing to the criticality of underground water supplies is the changing nature of the conditions and the natural recharge regime of underground aquifers. In some plains, inappropriate hydraulic structures on rivers have been carried out and this reduces the proper recharging of the downstream aquifer [1].

In order to overcome the problems of groundwater resources there is a need for a comprehensive and integrated management of surface and underground water resources, political, cultural and social factors considerations and the

participation of all stakeholders and associations in this all-round management. One of the most influential factors in the technical management of integrated management is the scientific and precise knowledge of the condition of the plains and their water resources; this knowledge is achieved through the production of basic information in the fields of meteorology, hydrology, hydrogeology etc. and its in-depth study. After recognizing the existing condition, several and various scientific, engineering and management scenarios and options have to be tested to establish a correct and balanced management. These scientific studies require a variety of tools, one of which is the mathematical model.

Mathematical models have made it possible to study the changes in existing and future status of aquifers through considering several effective factors in them. Therefore, it is possible to simulate the status of aquifers through collecting information about the inputs and outputs of the groundwater system with the least cost and time. These simulator models can provide surface and underground water interactions in short and long periods one of the most important of which is MODFLOW (Modular Ground Water Flow) model.

Due to the importance of groundwater resources, groundwater studies have a special place, and domestic and foreign researchers have performed useful investigations and are still studying this issue. Yaouti et al. (2008) in the study of Bou-Arega qualifier in the northern plains of Morocco, presented a variety of scenarios adapted to different plain areas using the MODFLOW code in the GMS. In this study, it was concluded that the recharge parameter is more sensitive than other parameters, such as hydraulic conductivity and in transmissivity [2]. Rejani et al. (2008) proposed a 2D quantitative and qualitative groundwater model for Balasore coastal aquifer in India and accordingly they studied the behavior of the aquifer and its salinity variations under different scenarios of pumping [3].

Cho et al. (2009) used a three-dimensional flow model to determine the impact of ground development activities on subsurface flow patterns in the Virginia State Aquifer study. After calibrating the model under continuous conditions, they used three scenarios with eight approaches and finally proposed solutions that a distributive-surface model can dynamically eliminate spatial and temporal contradictions between superficial and subsurface models completely [4]. Yang et al. (2009) developed a conceptual model to analyze the effect of tunneling in southern Taiwan by the GMS software. In this study, which used two MODFLOW and FEMWATER codes simultaneously, it was concluded that the constructed tunnel could not have a negative effect on groundwater resources and hot springs around the tunnel based on the current amount of groundwater extraction [5]. Chenini and Ben Mammou (2010) used GIS capabilities and numerical modeling to determine the proper location of artificial recharge and the development of groundwater resources in Central Tanzania. The result shows that these techniques can be adapted with slight variations in each location especially in semi-arid regions and zoning of groundwater flow recharge can be done by them [6].

Zhang and Hiscock (2010) studied the hydrological land use changes from agriculture to forest by studying the East Midlands aquifer in England. The results of this study showed that a 45 percent decrease in annual recharge rates happens as a result of increased vegetation in the first region with a decrease in summer than winter in both locations [7]. Hu et al (2010) used the MODFLOW model in a study entitled "Agricultural water-saving and sustainable groundwater management in Shijiazhuang Irrigation District" in northern China. The results of this modeling showed that 29.2% reduction in irrigation and 10% reduction in pumping could return its conduction to before agricultural development in 74 years ago in addition to reducing the groundwater aquifer loss [8]. Gaura et al. (2011) presented a new method for assessing groundwater resources using a numerical modeling combination and applying spatial modeling with GIS; this hybrid modeling was used in the Indian Bangana basin. The results showed that increasing extraction from wells would cause exert stress on the aquifer in the area. In this study it was stated that by creating a series of susceptible areas for the extraction of rainwater it would be possible to prevent the reduction in groundwater surface [9]. Lachaal et al. (2012) developed a comprehensive three-dimensional model of the area with the help of MODFLOW and ArcView in the study of the north-eastern Tunisian coastal aquifer. As a result of this research, it is stated that the models can be used as a management tool for other regions with similar geological and hydrological conditions [10]. Page et al. (2012) while describing the necessity of using DSS in managing water resources described and compared the existing software. Simulation of groundwater flow by mathematical model as a prerequisite for the development of qualitative models is an indirect method of study, which can solve existing problems to a great extent with lower costs than direct or field methods. The quantitative models can be used to develop scenarios for analyzing the elements of aquifer balance, describing the conditions of droughts and the corresponding definition of prohibitive extraction policies and analyzing the qualitative models [11]. Singh and Panda (2012) developed a quantitative and qualitative simulator of the aquifer in the region and implemented various management scenarios to investigate the phenomenon of flooding the agricultural land due to the elevation of groundwater levels and the creation of a salinity phenomenon in the Indian state of Haryana State [12]. Cao et al. (2013) proposed an underground water simulation model for the NCP area in China through which they assessed the scenarios resulting in sustainable water resources development in the area [13]. Borsi et al. (2013) examined the modeling of unsaturated and runoff zone flows using a new numerical method. In the new numerical method two LGR (Local Grid Refinement) and VSF (Variable Saturated Flow) methods have been applied as integrated in MODFLOW code. These changes allow the user to solve the three-dimensional Richard's equation only in the specified sections of the zone [14]. Mirzavand and Ghazavi (2014) estimated the groundwater level for Kashan plain in Iran using several time series models. In this research multiple time series methods are combined and provided the best prediction for the next 60 months based on the groundwater level data in the previous months [15]. Chitrakar and Sana (2015) presented a quantitative and qualitative groundwater modeling with MODFLOW and MT3DMS for the Al

Batinah coastal plain in Amman and examined the influence of different scenarios on salt water penetration in the region [16]. Naghibi et al. (2015), using geographic information systems and learning machines zoned groundwaters in Koohrang basin in Chaharmahal and Bakhtiari province of Iran. In this study, they examined the morphological and geological parameters including the slope length, slope level, drainage density, distance to the river, distance to faults and soil type in the region. The results of this study indicate that the model used for the area under study was appropriate [17]. Using the radial basis function network (RBFN) combination with a kind of regression method, Yan and Ma (2016) predicted groundwater level on a monthly basis. The results obtained from this study showed that the combination of the described methods can have better results than using any of the methods individually [18]. Negm and Eltarabily (2016) provided a groundwater quantitative and qualitative simulation model with MODFLOW and MT3DMS for the El-Menoufia- Egypt and investigated the nitrate concentration in the area [19].

In this research, using a comprehensive set of raw data from the regional water authority of Ardabil province, the MODFLOW mathematical and geostatistical models, also the groundwater surface changes in Ardebil plain has been investigated. Calculations have been made in GMS version 10 and MODFLOW-2005 engine [1]. The current study period is 132 months and it has been carefully evaluated by the maximum available data. Here each of these methods is investigated in terms of their accuracy and application in simulating the groundwater surface of Ardabil plain. The following steps have been taken to implement the model: Step 1: Obtaining and applying the results of basic studies; Step 2: Providing the conceptual and mathematical model; Step 3: Converting GIS data to a conceptual model; Step 4: Establishing the network and determining the upper and lower levels of aquifers; Step 5: Applying the boundary and initial conditions of the model; Step 6: Converting conceptual model to network model and determining other network cells' characteristics; Step 7: Running the model for the persistent state; Step 8: Calibrating the model for the persistent state; Step 9: Implementing the model for the non-persistent state; Step 10: Calibrating the model for non-persistent state; Step 11: Validating the results and final step is extracting the final results of the model.

2. Materials and Methods

2.1. General and Geographical Location

Ardebil plain is located in the east of Ardebil province (located at the north west of Iran) and is limited by Guilan province in the east, Khalkhal area in the south, Sarab and Meshkinshahr area in the west and Moghan in the north. This area is located between the geographical latitudes 48°46' -48°41' E and 37°55'-38°48' N. The location of the study area is shown in Figure 1.
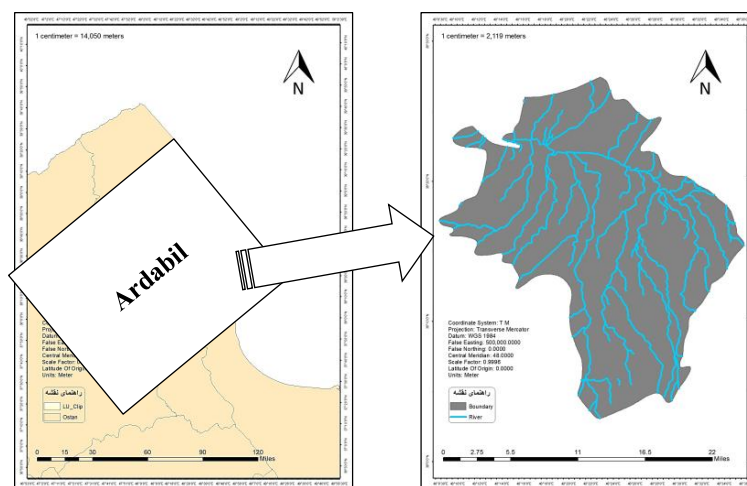

Figure 1. Geographical location of the study area - Ardebil plain and waterways

In terms of topography, the area is divided into two parts. The perimeter of this region is mountainous, while the central part is an alluvial plain. In the mountainous region, the highest elevation is related to Sabalan peak at 4810 meters above the sea surface. The central part extends from a relatively gentle slope from the southeast to the northwest to the outer area of the plain. The average annual rainfall in Ardebil plain is approximately 218.9 mm in the central part and is about 372.6 mm in mountainous areas. In terms of surface water resources, the general trend is the establishment of bedding, waterways and rivers from the heights to the center and north and then to the outer area. Ghoori Chai, Balikhi Chai and Gharaosu rivers are the most important rivers in the Ardebil plain.

2.2. The MODFLOW Mathematical Theory

GMS is one of the most advanced and complete groundwater modeling packets supporting various types of numerical models. The GMS provides the appropriate tools for each of the groundwater simulation stages including describing the studied area, providing a conceptual model, creating a network, geostatistics, calibration, imaging and so on. The GMS

supports both finite difference and a finite element models. In fact, the GMS includes a user interface and a number of analytical codes such as MODFLOW, MODPATH, MT3DMS, FEMWATER, MODAEM, and so on. The program interface is prepared by the Environmental Modeling Research Laboratory in the University of Brigham Young with the participation of U.S Army Engineer Waterways Experiments Station. One of the interesting points of GMS software is its modular design that allows the user to choose the modules as desired and only select the capabilities required for groundwater modeling. As a result, the required parts program can be obtained from separately. The MODFLOW model is actually a finite difference solution for partial differential equations governing groundwater flow. The MODFLOW model is used to simulate groundwater flow in aquifers with specified boundary conditions and assuming the necessary values for hydraulic conductivity and other aquifer parameters. The design of the program parts enables the user to select the modules that are required to study the system in question for specific hydrogeological processes and enable or disable a particular piece. These features have made the MODFLOW model the most efficient and affordable groundwater model. The existence of various packets in this model has led to its usage in the study of various aquifers. This model solves the equations governing groundwater flow in persistent and non- persistent conditions by finite difference method. In a groundwater model a system of algebraic equations (matrices) is solved. This matrix is an approximation of the mathematical model developed by the differential groundwater flow equations [20]. This step involves expressing the conceptual model in the form of mathematical equations. In other words, during this phase, the conceptual model of the physical system is converted into a mathematical system. The mathematical model is usually composed of a set of equations consisting of the governing equations (often in the form of partial differential equations), boundary conditions, and related initial conditions. Generally, the governing equation of groundwater flow in unconfined aquifer is as follows:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = Sy \frac{\partial h}{\partial t} \tag{1}$$

Where, h is the height of the groundwater surface, Sy is the specific yield and K is hydraulic conductivity in the x, y, and z directions of the aquifer.

Matrix equations are solved in the MODFLOW code by the SIP -like processors. In principle, SIP is a method for solving large linear equation system by repeating.

The finite difference equation for a cell with the node ID i, j, k is written in a model network, which indicates the relationships between the water surfaces in the node i, j, k, as well as in all six adjacent cells at the end of the time period. Because each equation may involve seven values and the uncertain value of the water surface in each equation changes to the next value in the network in each equation, the value of the equation entered in the network must be solved as a symmetric mode in each time step. In fact the form of flow equations is considered as a matrix form of Equation (2):

$$[A]\{h\} = \{q\} \tag{2}$$

Where [A] is the matrix of the water surface, {h} is a vector of water surface values, and {q} is equivalent to the vector of the right conditions of the flow equation. It is noteworthy that the matrix [A] is in the form of Sparse. This means that a very small number of its elements are non-zero and all its diameters except seven cases (the diameters of flow equations in the finite difference of the mathematical model) have zero values and accordingly the theory of the possibility of implementing the finite difference of flow equations is possible.

Also, different criteria (shown in relations 3 to 6) are used to evaluate the model. They are related to mean basin error (MBE), mean absolute error (MAE), root-mean-square error (RMSE), normal root-mean-square error (NRMSE) and average standard error (ASE) presented below:

$$MBE = \frac{\sum_{i=1}^{n}[Z_i^* - Z_i]}{n} \tag{3}$$

$$MAE = \frac{\sum_{i=1}^{n}|[Z_i^* - Z_i]|}{n} \tag{4}$$

$$RMSE = \frac{\sum_{i=1}^{n}\sqrt{[Z_i^* - Z_i]^2}}{n} \tag{5}$$

$$NRMSE = \frac{RMSE}{y_{max} - y_{min}} \tag{6}$$

In relations 3 to 5, n is the number of observation periods, Z^* is the simulation values and z is the observational values, and in the relation 6, y_{max} and y_{min} are the length of the range of the head variations. The ASE is also used to estimate the proximity of the average sample to the average population. In other words, the standard error is the standard deviation of a statistical sampling distribution that is used to estimate the standard deviation from a number of samples.

3. Model Calibration and Validation

3.1. Calibration in Persistent Conditions

In the first step after developing a conceptual model based on the graphic interface and pre-processing of the raw data, the model is implemented in a stable and forward form by the MODFLOW 2005 processor in version 10 of the GMS software [1]. In this model, the entire area is divided into smaller pieces resulting in a mesh of cells with a mesh size of 25×25 m. Figure 2 illustrates a schematic of the conceptual model prepared for the Ardabil plain aquifer as well as the meshing performed on it.
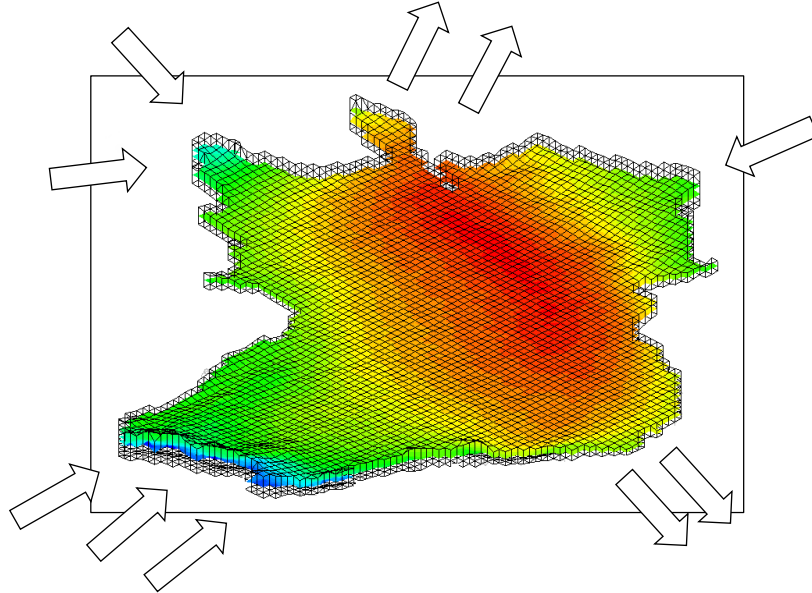
Figure 2- Schematic of the conceptual model and the grid pattern of the aquifer model of Ardabil plain using the GMS software

In order to carry out calibration operations under persistent conditions, a separate pilot number (as a negative number indicating the calibration parameter) is assigned to the conceptual model. Based on this, suitable minimum and maximum ranges of horizontal hydraulic conductivity parameters, horizontal anisotropy of hydraulic conduction and surface recharge are calculated. After four times by implementing the calibration model with 30 trials-errors, the optimal final values of the recharge parameter are calculated and presented in the Table 1. This was performed for both stable and unstable conditions. After the last run, the PCG2 computing engine with a critical limit of convergence changes of 0.01 meters as well as a critical convergence error of 0.01 cubic meters per day is chosen to continue the simulations. In the Table 1, each RCH (recharge parameter) is specified by the number representing the position of the parameter. The recharge parameter number is also presented in Figure 3. The persistent distribution of the recharge parameter from the surface and interpolation of the hydraulic conductivity parameter of observation wells are shown in Figure 4 and 5.

Table 1. Minimum, maximum and optimal parameters of recharge parameter in the stable calibration step

| Recharge parameter	Optimal initial value (M per day)	Minimum (M per day)	Maximum (M per day)
RCH_20	0.0000500641	0.0000500641	0.00500641
RCH_21	0.0000312050	0.0000161918	0.00161918
RCH_22	0.0000161918	0.0000161918	0.00161918
RCH_23	0.0009303737	0.0000161918	0.00161918
RCH_24	0.0000161918	0.0000161918	0.00161918
RCH_25	0.0001253593	0.0000232758	0.00232758
RCH_26	0.0000495357	0.0000232758	0.00232758
RCH_27	0.0003315409	0.0000232758	0.00232758
RCH_28	0.0001802643	0.0000161918	0.00161918
RCH_29	0.0000500641	0.0000500641	0.00500641
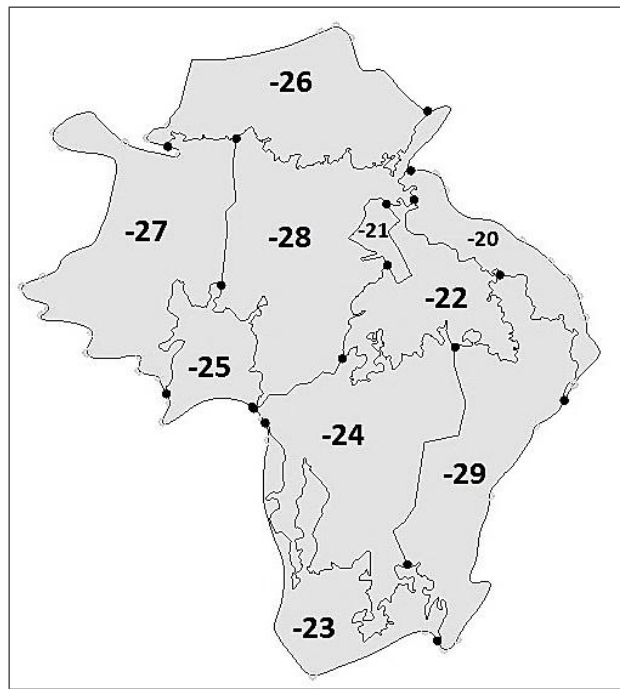
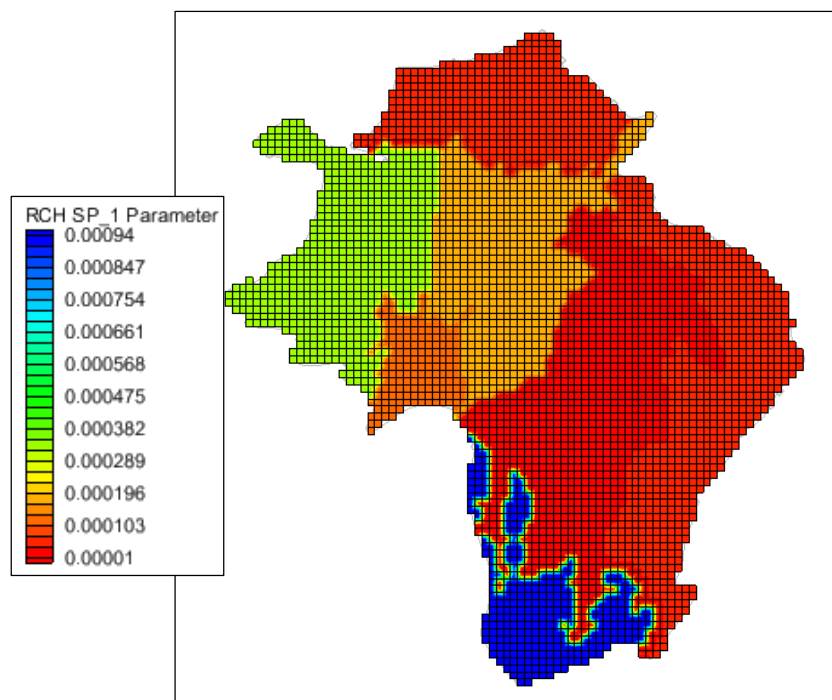**Figure 3. Key numbers of the zoning the recharge parameter from the surface**

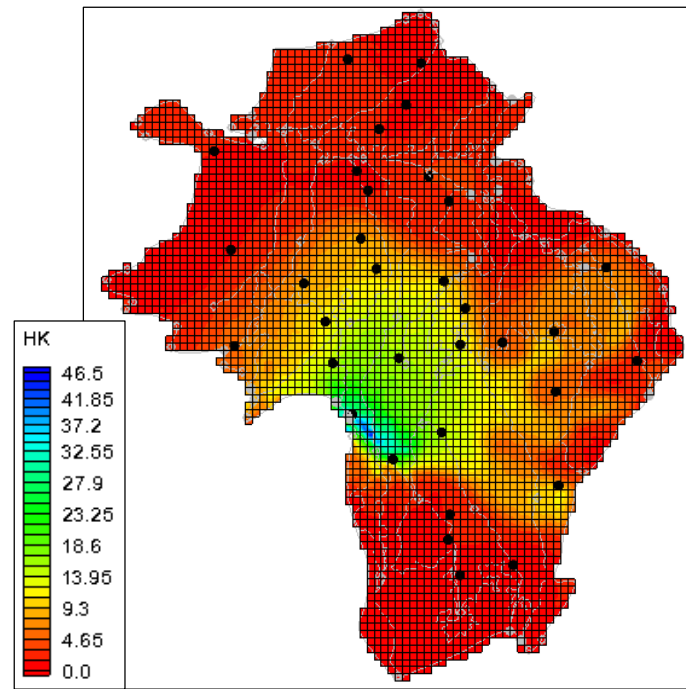**Figure 4. Persistent distribution of the recharge parameter from the surface**

Figure 5. Interpolation of the hydraulic conductivity parameter of observation wells (numbers in meters per day)

3.2. Model Calibration in Non-Persistent Conditions

In the non-persistent step of modeling the calibration operation has been implemented on the specific yield in addition to the horizontal hydraulic conductivity, surface recharge and horizontal anisotropy of hydraulic conduction parameters. Accordingly the minimum error is obtained by introducing the calibration parameters for each modeled 120 months. Finally, the proposed parameters are obtained between two appropriate minimum and maximum values (Table 2). Specific yield pilot interpolation in non- persistent calibration and interpolating horizontal hydraulic conductivity pilot in non- persistent calibration are shown in Figure 6 and 7.

Table 2. Minimum, maximum and optimal parameters of recharge in unstable calibration step

| Recharge parameter	Optimal initial value (M per day)	Minimum (M per day)	Maximum (M per day)
RCH_20	0.0000500641	0.0000500641	0.00500641
RCH_21	0.0000161918	0.0000161918	0.00161918
RCH_22	0.0000161918	0.0000161918	0.00161918
RCH_23	0.0003000439	0.0000161918	0.00161918
RCH_24	0.0000165389	0.0000161918	0.00161918
RCH_25	0.0000831971	0.0000232758	0.00232758
RCH_26	0.0000785209	0.0000232758	0.00232758
RCH_27	0.0001426105	0.0000232758	0.00232758
RCH_28	0.0001098117	0.0000161918	0.00161918
RCH_29	0.0000500641	0.0000500641	0.00500641
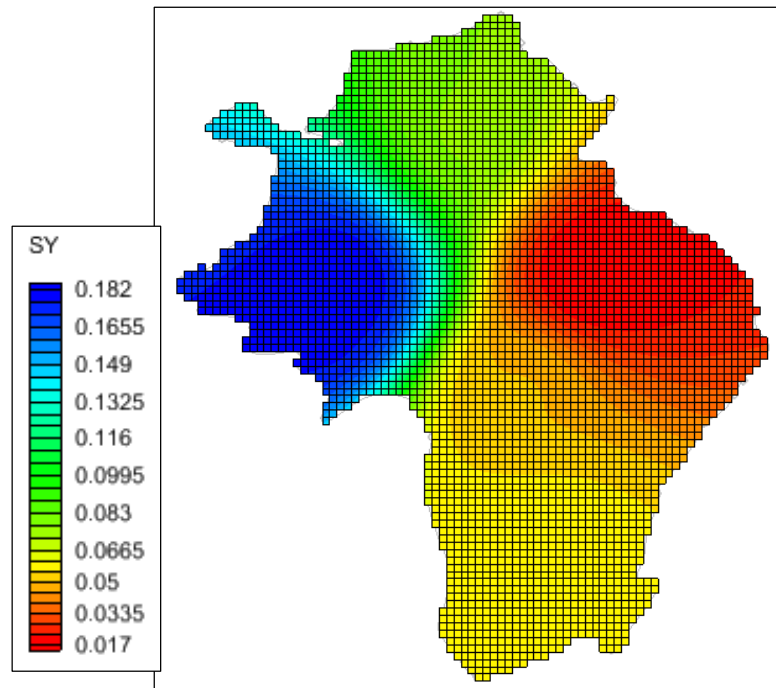
Figure 6. Specific yield pilot interpolation in non- persistent calibration (without unit)
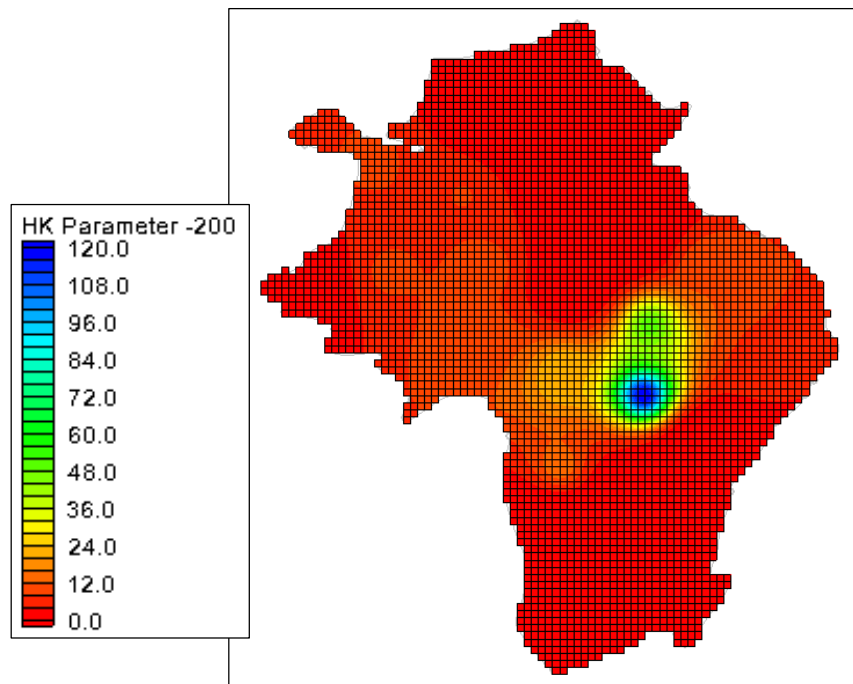

Figure 7. Interpolating horizontal hydraulic conductivity pilot in non- persistent calibration (numbers are in meters per day)

In order to reduce the total error and prevent the major error transmission on a single parameter in the unstable calibration step, the optimal results are achieved with a total of four trials and errors. Accordingly, changes in dynamic hydraulic load boundaries (conceptual model) have reduced the total calibration error in the last step of the quadratic calibration step to the minimum optimal value. In this way, the RMSE is equivalent to 1.25, while the base error value is 3.97. For further evaluation of numerical error values, a normalized error in accordance with Equation (6) has been used. The normalized error value of this study is calculated to be about 2%, which confirms the presented ideal modeling. Figure 8 shows the correlation of observational and computational data in each observation well. The results show that the square of the correlation coefficient ($R^2 = 0.99$) is close to one indicating the optimal simulation after the calibration step.
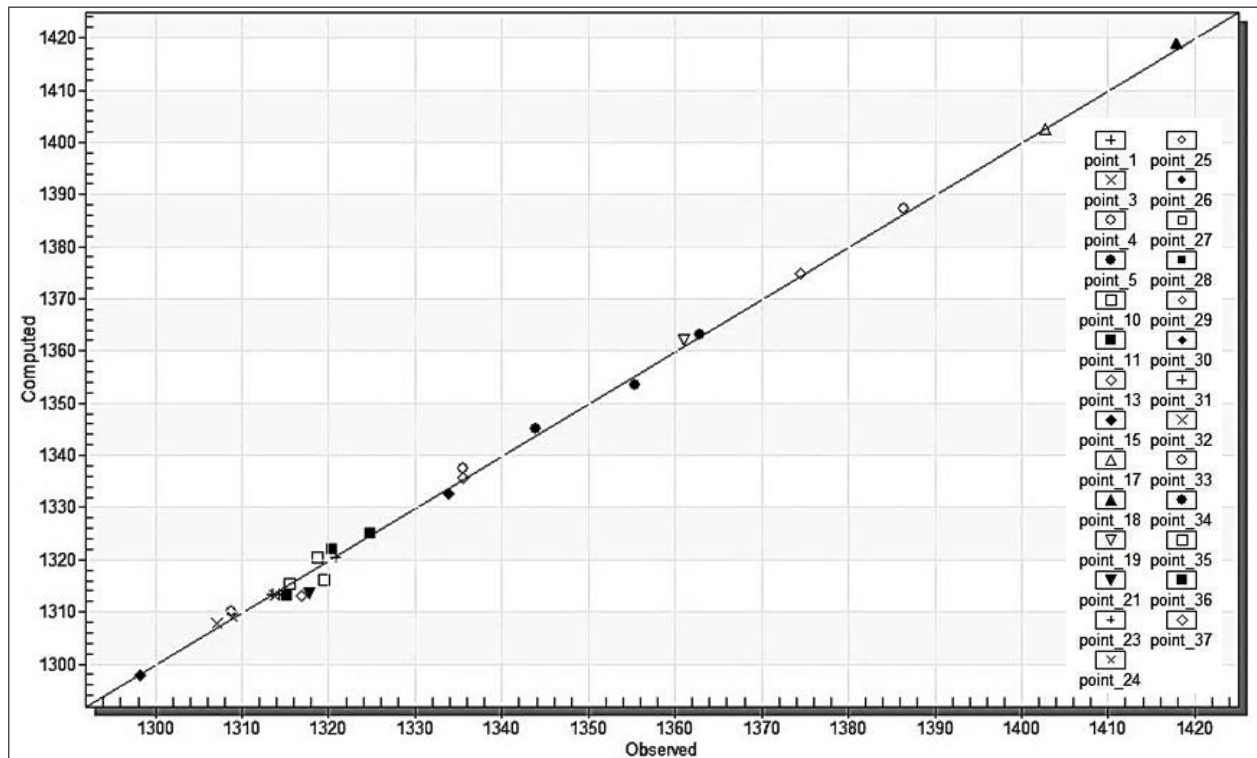
Figure 8. Correlation between observational and computational water surface in the observation wells (first month of modeling)

Figures 9 and 10 represent two random observation wells with a comprehensive dispersion at the plain surface to display the desired calibration results. In these figures, the horizontal axis is the time steps and the vertical axis represents the water surface level. Also, colors represent a schematic location of the calculated water surface level against the level of observation water surface. If the difference between computational and observational amounts is within the acceptable range, it is indicated as green color while the partial and major errors are highlighted with yellow and red colors respectively. As shown in Figures 9 and 10, the errors generated in numerical modeling are very low and acceptable. Figure 11 shows the general schematics of the desirability of the present numerical method.
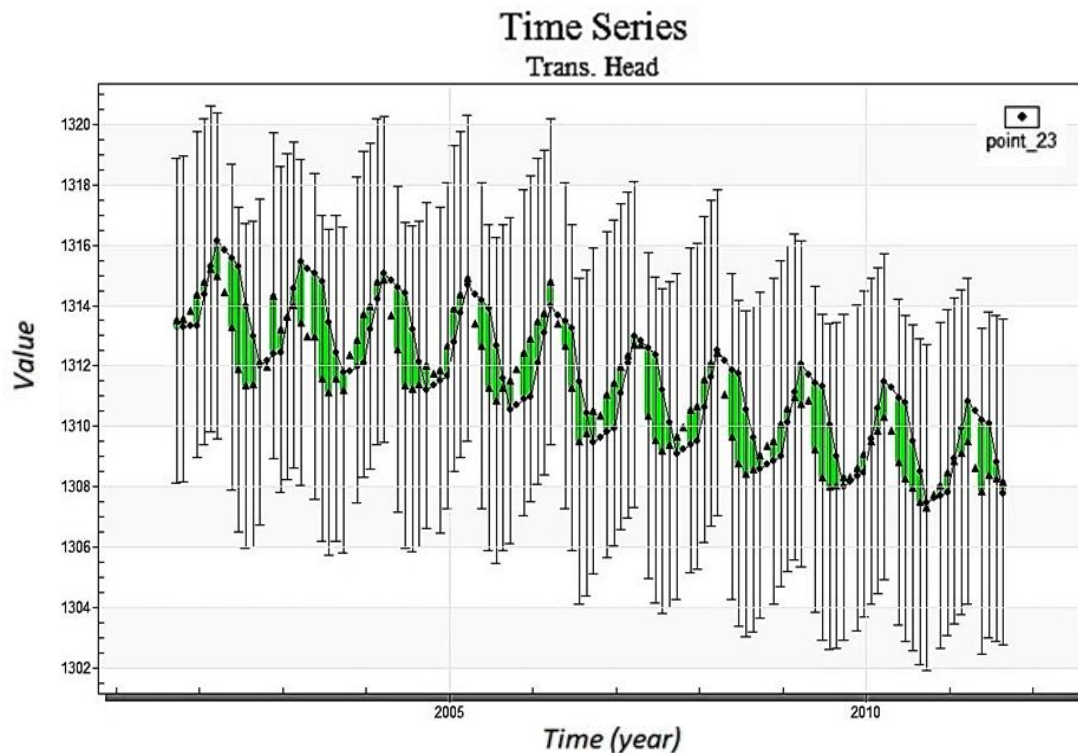

Figure 9. Schematic error of water surface level in observational values versus computational data - Observational well 23
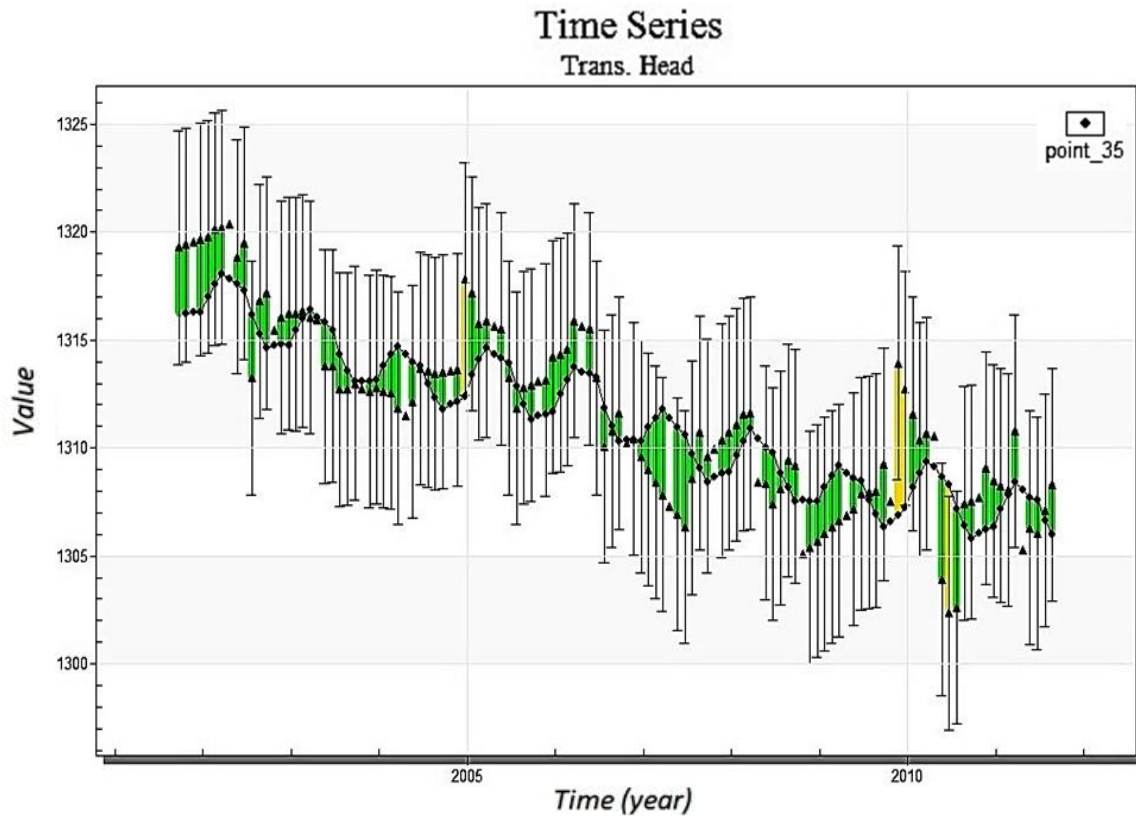
Figure 10. Schematic error of water surface level in observational values versus computational data - Observation well 35
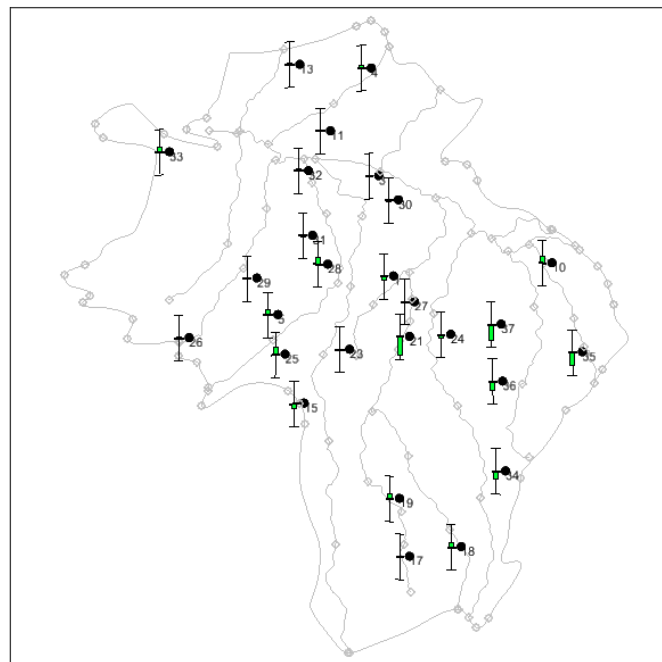

Figure 11. Schematics of the desirability of the present numerical method. Green color of the schematics indicates an error reduction in the position of the observation well selected in Ardabil aquifer

3.3. Validation

The calibration process of model for Ardabil plain aquifer is carried out over a period of 120 months. In order to ensure the results of the model output (considering the 132-months period of the study), the validation step for the next 12 months (from end of the calibration period to the end of the study period) is investigated. The results show a slight error in the verification stage, which confirms the accuracy of the calibration and hydrograph of the math mesh cells. Figure 12 shows the simulation curve of the aquifer surface in the 132-months period for all observation wells with the minimum desirable error. In this figure the horizontal and vertical axes represent the levels of computational and observational water surface level in observation wells.
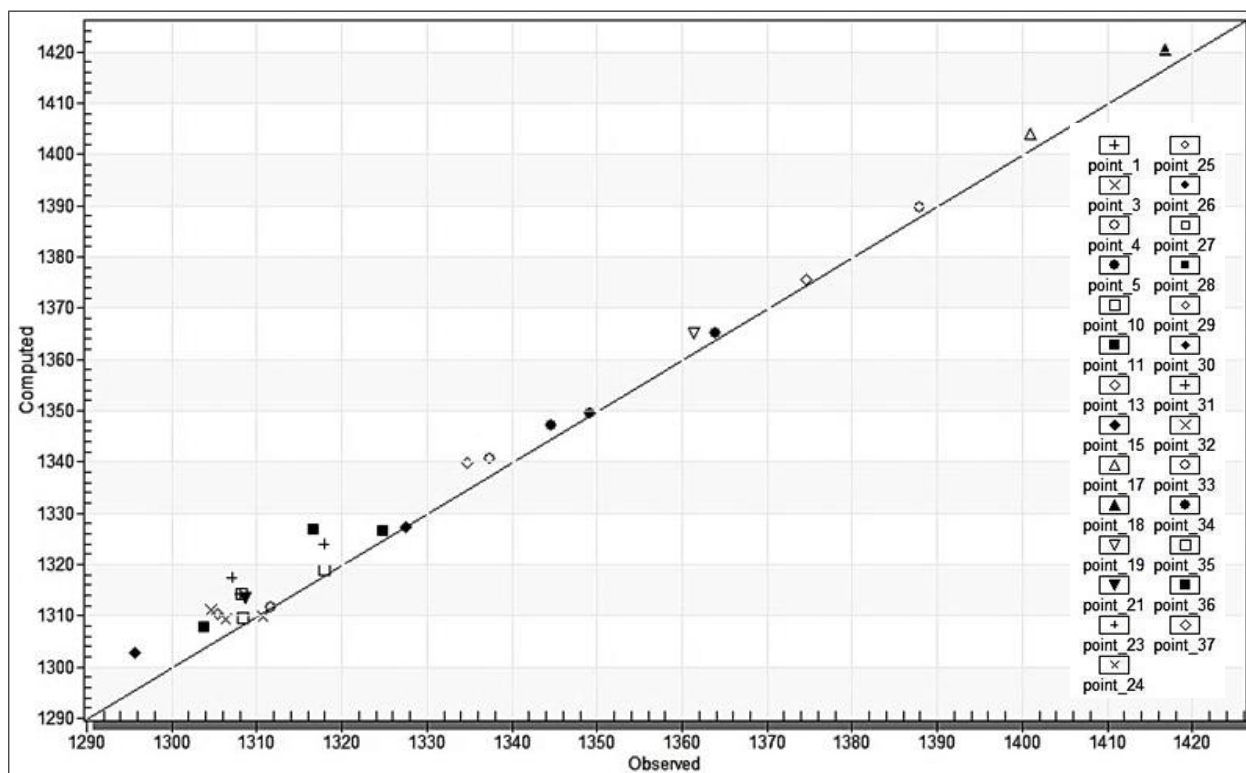
Figure 12. Comparison of computational water surface levels versus observation data at verification stage for observation wells of the aquifer

4. Results and Discussions

4.1. Flow Budget of the Model

Flow budget is a mathematical engine developed by the USGS to calculate the water balance in the MODFLOW model. The extracted numbers by flow budget method show that an average of 31 cubic meters of water per day is reduced from the average aquifer storage in the 132 months simulation period which is equivalent to 113,000 cubic meters during the 10 year period. Figures 13 to 16 show the balance of the aquifer elements. Figure 17 shows the groundwater surface of the Ardabil aquifer in the last 132 month period in this study. These figures indicate that the amount of water entering the aquifer has a direct dependence on the discharge rate of the production wells in addition to its dependence on the physical conditions of the Ardabil aquifer (such as hydrodynamic and gravity coefficients). Therefore, by increasing the extraction of the aquifer, a noticeable change in the exchange flow volume is expected.
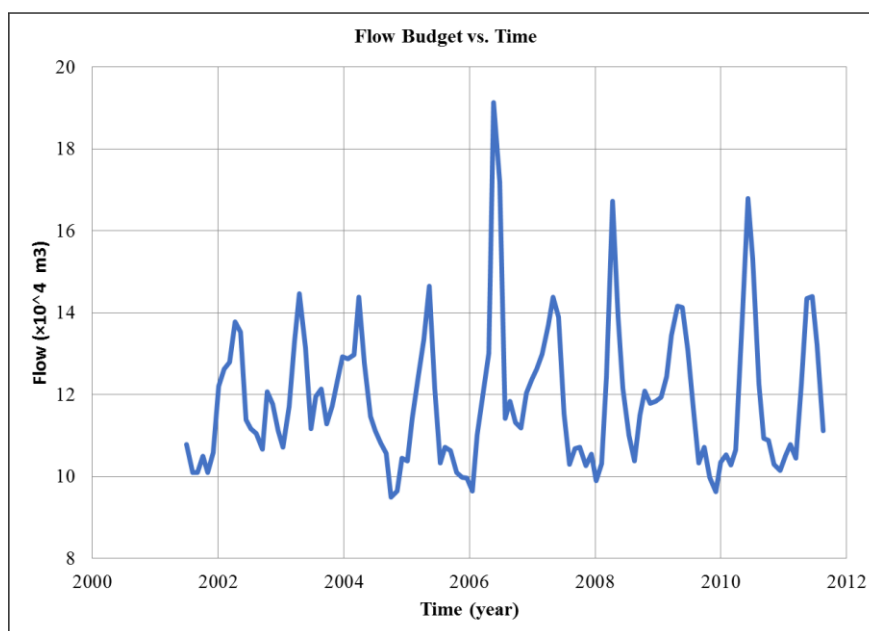

Figure 13. Output flow volume from dynamic load boundaries in Ardabil aquifer (cubic meter per year)
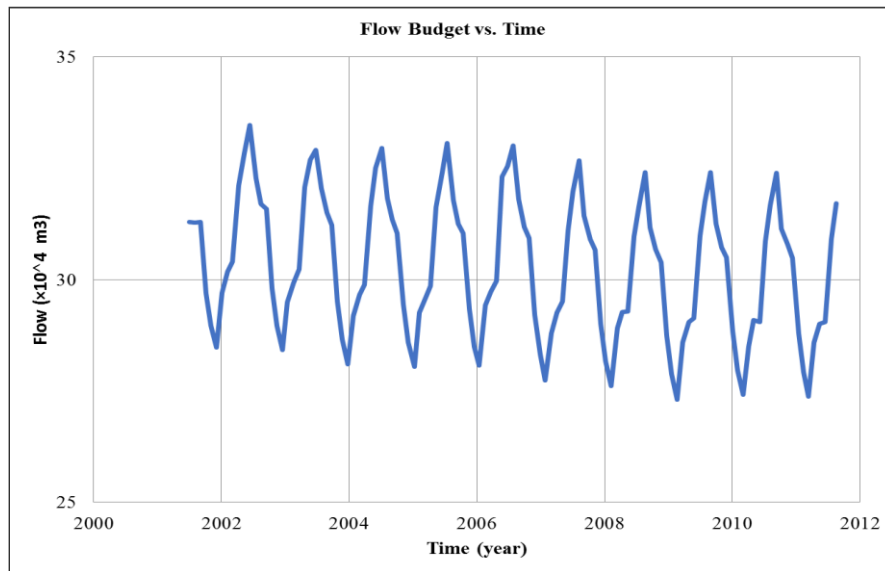
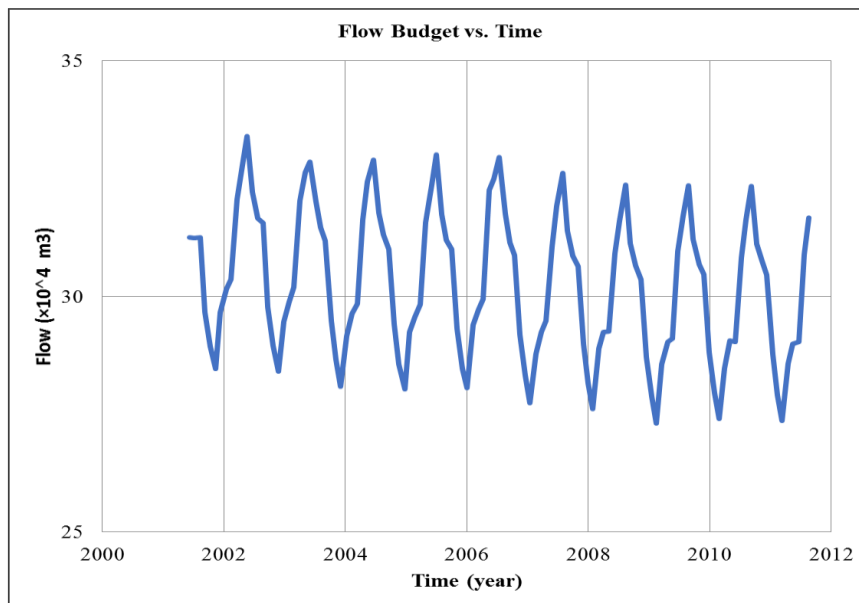**Figure 14. Input flow volume from dynamic load boundaries in Ardabil aquifer (cubic meter per year)**

Figure 15. Volume of recharge flow of the water channel in Ardabil aquifer (m^3 / year)
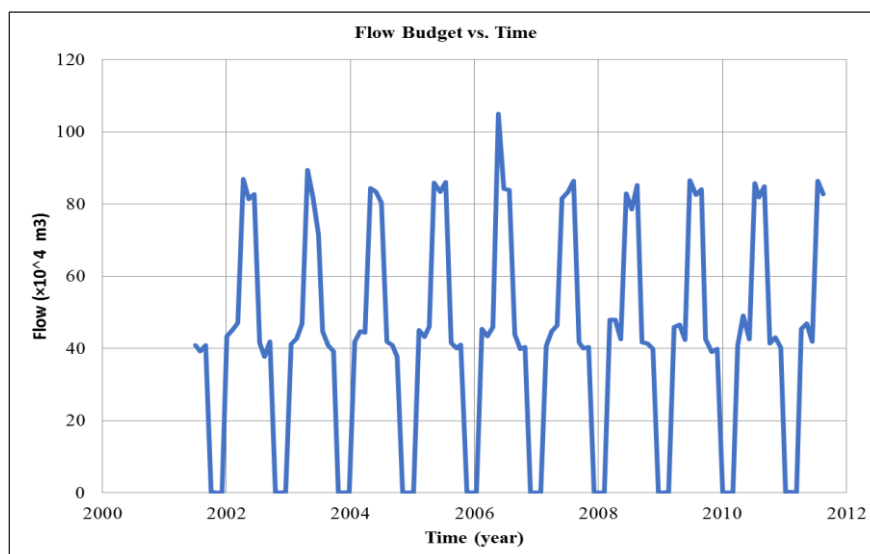

Figure 16. Volume of extracted flow by the operating resources (cubic meters per year)
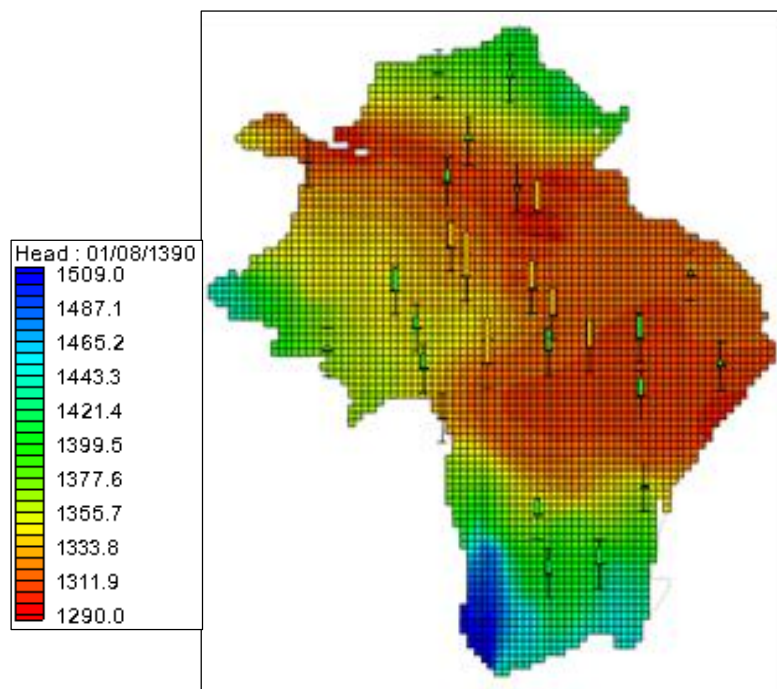
Figure 17. Ardabil aquifer groundwater surface level at the end of the period with the current trend of extracted flow from total wells (numbers are in meter)

4.2. Quantitative Geostatistical Model

In order to develop the geostatistical model, all initial values of the water surface are restored and arranged in the form of a time series. In this research, the Kriging method has been selected as one of the best methods for responding to environmental variations in each piezometric value. The Kriging method is based on the variogram definition. A variogram is used to determine and describe the spatial structure of the data. Variography is the first step in modeling the spatial structure for use in Kriging. The success of the Kriging method depends on selecting the appropriate model or variogram optimum. Three components are used to describe and model the behavior of the variogram in variography including: the influence range (range), threshold or ceiling (Sill) and Nugget Effect. The influence range is the maximum distance after which the variogram reaches a constant value. Nugget effect is due to changes occurring at intervals less than the minimum sampling distance or due to errors in sampling and measurement. When the variogram reaches its constant value, the height of the variogram is equal to the threshold or the sill i.e. equal to the sum of the random variances. In this research, since the environmental data generally have second-order non-random process with an abnormal form, the data are used as normal form using the ArcGIS (add-in application that is used to perform interpolation in GIS) and one of the conversions (LOG conversion in this research). Considering all of these factors, the interpolation error has reached its minimum value.

4.3. Comparative Analysis

Despite the complexity and the time consuming nature of the construction and development of numerical models, a geostatistical model will be superior only when the number of points is much higher than the current standard. Therefore the time and cost of numerical simulations will be less than the cost of drilling, maintaining and storing information of observational points at the aquifer level in the current situations. Generally the reason for the superiority of the mathematical model is the ability of these models to consider different conditions and hidden complexities of the aquifer. Figures 18 through 20 show the comparison between utility criteria in both numerical and statistical models. As it can be observed the quantitative geostatistical model has more error than the numerical model. The reason for the better performance of the numerical model is the lack of appropriate numbers and distribution of raw data in the geostatistical model. In Figure 21, the amount of uncertainty is determined by moving away from the certain points or points that have a low number with small piezometric numbers. In this figure, the areas with dark orange contours illustrate the higher uncertainty.
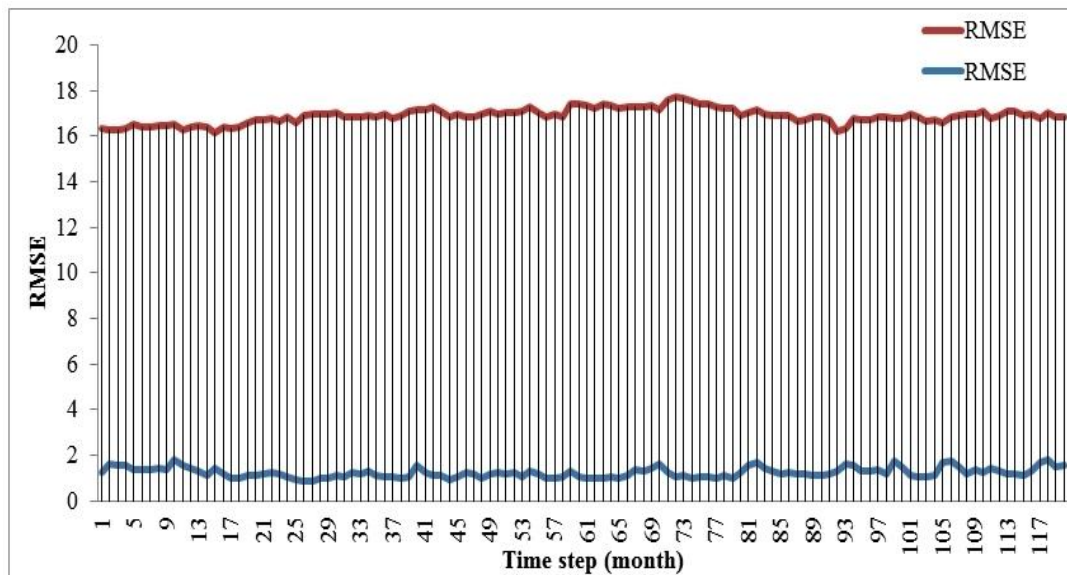
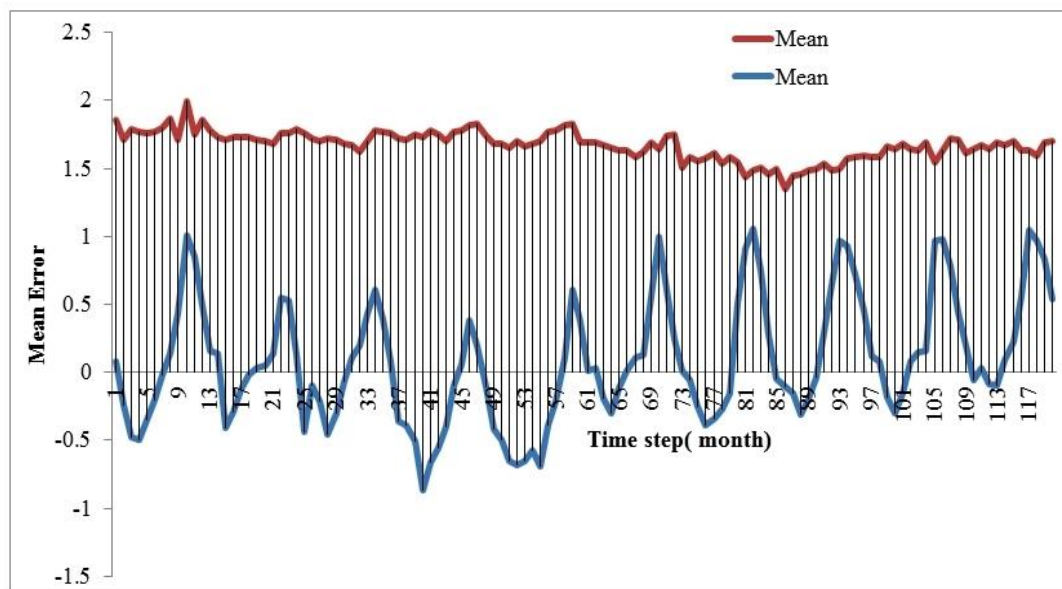**Figure 18. Comparison of the RMSE between the numerical model (blue) and the geostatistical model (red)**

Figure 19. Comparison of the mean error between the numerical model (blue) and the geostatistical model (red)
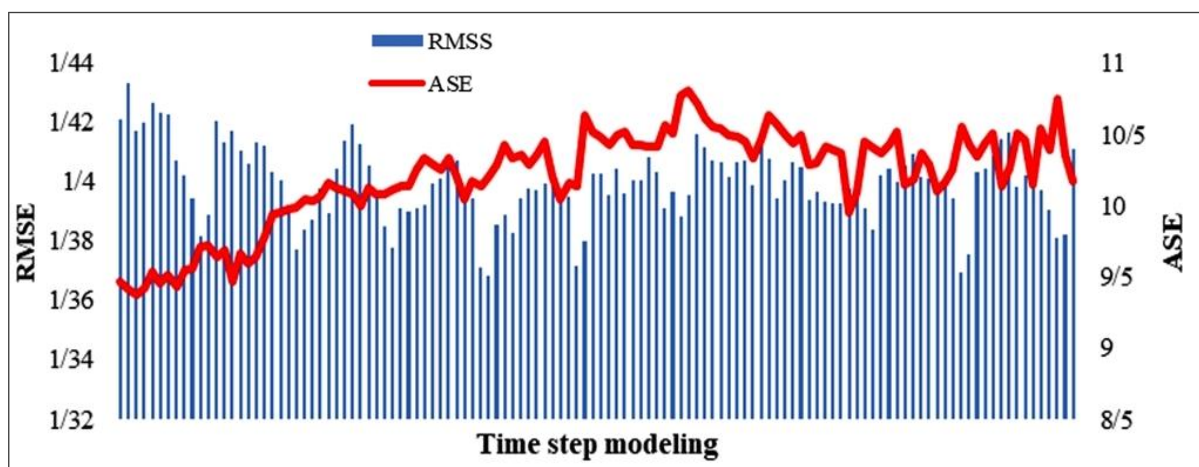

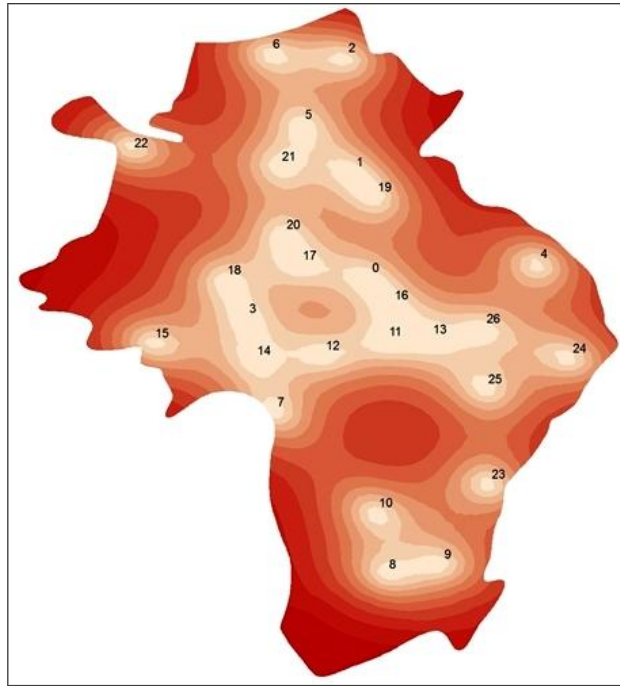**Figure 20. RMSE and ASE error values in the geostatistical model**

Figure 21. Increasing the uncertainty of the geostatistical interpolation by distance from the input points

5. Conclusion

The development of finite difference mathematical model with a favorable calibration condition requires accessing a wide range of hydrogeological and hydrological data for aquifers. In this study, the Ardabil aquifer MODFLOW mathematical model is calibrated for a 120 months period and validated in the 12 interval periods. The total relative error is reduced from 3.97 to 1.25. The normal root-mean-square error (NRMSE) is calculated to be about 2% which confirmed the model accuracy for the Ardebil plain.

By examining the RMSE, mean error, and ASE errors, it is concluded that the accuracy of the mathematical model is better than the geostatistical method and the main reason for this is the distribution of uncertainty in a few available piezometric points in the geostatistical method. Also, the absence of a specific trend in the value of the mean error and RMSE confirms the superiority of the mathematical code method over the geostatistical calculations.

Investigating the conditions of flow balance of the MODFLOW model for the Ardebil plain shows that most of the available flow volume, especially wells, is used for the agriculture purpose. This is one of the main reasons of the fixed water storage reduction in the aquifer. It should however be noted that a large part of the reservoir deficit is recovered in the wet seasons due to the location of the project in the mountainous area.

In general, the important role of the water channel network should be considered with respect to their drainage role. This role is particularly noticeable in areas with lower altitudes in which the flow rate and the flow direction are higher and denser. Nevertheless, the recharge role of channels in the study period is much more highlighted than their drainage role. The groundwater surface in the Ardabil aquifer in the middle and northeastern parts of the country has the lowest water level because of the density of production resources and higher hydrodynamic coefficients while the southwestern and parts of the northern area of this aquifer display the highest water level due to the existence of high recharging areas.

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