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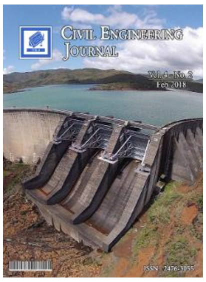
Civil Engineering Journal

Vol. 4, No. 2, February, 2018



Evaluation of Harmony Search Optimization to Predict Local Scour Depth around Complex Bridge Piers

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Received 01 January 2018; Accepted 21 February 2018

Abstract

One of the main causes of bridge collapse may be flood flow scour near piers. Several experimental and local field investigations were carried out to study scour depth. However, existing empirical equations do not commonly provide accurate scour prediction due to the complexity of the scour process. Physical and economic considerations often lead to bridge foundation constructs which included a pier column based on a pile cap supported by an array of piles. Piers with this configuration are referred to as complex piers. A few studies have been done on complex bridge pier scour depth estimation. Such efforts may be classified into theoretical and empirical equations. This paper investigates local scour around complex bridge piers by using harmony search algorithm under clear water conditions. Statistical indices such as the coefficient of determination (R2), root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and bias were used to evaluate the performance of these methods. By designing laboratory tests, 82 experimental data points were measured by authors. Also 615 experimental data sets with the same measured experimental conditions were collected from published literature and used for optimization. The results show that the developed HS model can predict scour depth better than other equations according to statistical indices.

Keywords: Complex Bridge Piers; Local Scour; Empirical Formula; Laboratory Data; Scour Depth Estimation; Harmony Search (HS).

1. Introduction

The flow pattern around a bridge pier may be significantly changed, when a stream is partially obstructed by that. The pier produces an adverse pressure gradient, just upstream to the obstruction. Besides, pier upstream boundary layers may undergo three dimensional separations. The shear stress distribution around the pier drastically changes due to the formation of a horseshoe vortex, which makes a scour hole around the pier, which, in turn, changes the flow pattern and shear stress [1].

Local scour depth estimation around the bridge pier is a vital issue in the design of bridge piers. Various design methods and formula have been developed for estimating local scour depth in the vicinity of bridge piers. Scour depth studies started in the late 50s. Raudkivi [2] described the effects of flow and sediment parameters on the local scour around piers and discussed the functional trends of local scour based on laboratory data. Melville [3] investigated the effective parameters on the pier and abutment scour and presented empirical relations, called K-factors. Ettema et al. [4] discussed the generality of skew factors on scour geometry.

While a substantial amount of knowledge has been accumulated about the scour and flow structures around single piers over the past decade or so, comparatively little knowledge is available about the scour and flow field around pile groups and complex piers. Salim and Jones [5] studied the scour around submerged and un-submerged pile groups and

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doi http://dx.doi.org/10.28991/cej-0309100

presented a relationship to define the effect of pile spacing and attack angle on pile groups. Zhao and Sheppard [6] investigated the effect of flow skew angle on local scour in pile groups. Sumer et al. [7] described scour geometry for pile groups with varying pile spacing. Accordingly, scour around pile groups may be caused by two mechanisms: 1) local scour in individual piles and, 2) global scour (general lowering of the bed) over the entire area of the pile group. Amini et al. [8] evaluated the commonly used equations to estimate the local scour depth in a group of piles for different spacing, arrangements, and submergences.

Circular compound pier and caisson local scour have been investigated experimentally. Based on laboratory data Melville and Raudkivi [9] studied the influence of the ratio of pier width to the maximum scour in a non-uniform pier. Physical and economic considerations often may lead to bridge foundations called complex piers which are constructed of a column founded on a pile cap supported by an array of piles. The details of complex piers are shown in Figure 1. which includes a column, pile cap, and pile groups. Piers of this configuration are referred to as complex piers [10]. Schematic view of complex pier is presented in Figure 1. in which L_c= column length; L_{pc}= pile cap length; b_c= column width; b_{pc}= pile cap width; b_{pg}= pile diameter; S_l= pile spacing in line with flow; S_b= pile spacing normal to the flow; L_u and L_f= extension of the pile cap upstream of and sides of the column, respectively; T= pile cap thickness; Y= pile cap top elevation to the initial bed level.

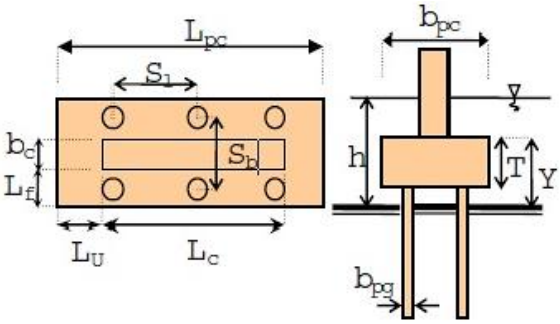


Figure 1. Complex pier geometry characteristics

Knowledge about local scour depth and scour mechanisms around complex piers has been limited to a few investigations. Melville and Coleman [10] proposed a method to predict scour depth in complex piers, which involved equations that were used to characterize different combinations of pier components. This procedure did not consider the existence of the pile group below the pile cap. Therefore, the estimated scour depth increased by increasing the pile cap elevation (Y) relative to the bed level, as the columns were founded on caissons. Coleman [10] revised this procedure by considering five different pile cap elevations (Y) and assuming a linear variation in the scour depth. Ferraro et al. [12] focused on the effect of pile cap thickness on the maximum scour depth around the complex pier. Two test series with different pile cap thicknesses over a wide range of pile cap elevations were considered. A review of the previous studies showed that the existing methods for local scour prediction provide widely differing estimates for different configurations of complex piers. Moreover, the observed scour depths exhibited a considerable degree of data scatter. Whether the data scatter and unrealistic prediction of the scour depth is due to the lack of existing data or inadequacy of the proposed equations has remained to be determined.

Table 1. Summary of experimental data used in evolution of existing methods

Researcher(s)	Flow depth		Sediment median size	Pile cap elevation	Pile cap thickness	Pile group diameter	Column shape	Pile cap shape
	h (cm)	u/u_c	d_{50}	Y (cm)	T(cm)	b_{pg} (cm)	-	-
Parola et al. (1996)	15	0.95	0.58	-15 - 6.9	0.05 -25.23	-	rectangular	rectangular
Melville and Raudkivi (1996)	20	1	0.24, 0.8	-20 - 9	8.37 - 32.27	?	circular	circular
Jones	30.48	1.183	1	-3-15.24	3	?	rectangular	rectangular
Coleman (2005)	33 - 60	0.75-0.85	0.84	-66-21.0007	6-8	2-2.4	rectangular	rectangular
Ataie-Ashtiani et al. (2010)	14 - 60	0.71-0.86	0.6	-3.7 - 2.3	3.2-∞	1.6	rectangular	rectangular
Ferraro et al. (2013)	10	0.92	0.83	-12.4 - 5	0.1, 5	2.5	Rounded rectangular	Rounded rectangular
Oliveto et al. (2004)	10- 20	0.58-0.93	1.7- 2.4	0 - 7.3	4 - 8	2-4	circular	square
Lu et al. (2011)	17.9- 20.4	0.65- 0.9	0.52	-5 - 3	?	?	rectangular	rectangular
Kothyari and Kumar (2012)	16.5	0.75	0.4	0 - 2.1	33- 64	?	circular	?
Martine-Vide et al. (1996)	25.4	0.927	0.65	-25.4	26.4- 40.4	6	rectangular	Circular

Sheppard et al. (2004)	32.6- 33.5	1.5- 3.08	0.84	-22.86- (-10.36)	8	2.5	rectangular	rectangular
Beheshti and Ataie (2010)	0.2853	1	0.71	-6.15	3.36	2.54	rectangular	rectangular
Zhao	21.3- 21.5	0.64- 0.65	0.17	-	-	3.18	-	-
Hannah	14	0.7723	0.75	-	-	3.3	---	-
Present study	19.4- 22.6	0.8- 0.96	0.71	-8 - 2	3	2	rectangular	rectangular

Table 2. Comparison of statistical indices

Equation	**Coefficient of determination**	**Root mean square error**	**Mean absolute error**	**Mean absolute percentage error**	**BIAS**
	R^2	RMSE	MAE	MAPE	-
HEC-18	0.329	0.061	0.044	159.440	0.033
Revised HEC-18	0.515	0.044	0.032	112.548	0.018
Coleman	0.467	0.102	0.065	151.932	0.063
Revised Coleman	0.5799	0.078	0.058	133.072	0.056
Sheppard	0.399	0.071	0.053	180.876	0.049
HS	0.672	0.054	0.034	3.448	0.023

In recent decades, artificial intelligence (AI) approaches such as artificial neural networks (ANNs), adaptive neuro-fuzzy inference system (ANFIS), support vector machine (SVM), and model tree (MT have been applied to predict local scour depth around bridge piers. Bateni and Jeng [13] evaluated the local scour depth at group piles under waves using the ANFIS model. Zounemat-Kermani et al. [14] predicted the scour depth around pile groups due to clear water conditions using ANNs and ANFIS models.in addition, local scour depth at group piles was investigated using the data mining approaches in terms of SVM and MT models [15-19]. These applications indicated that predictive methods can present good validations with minor error for measured datasets compared with empirical equations. Arafa et al. [20] developed harmony search technique to compute the optimum design of steel frames with semi-rigid beam-column connections. They formulated the optimization problem to estimate the minimum steel frame weight. The results showed that nonlinear semi-rigid frames are lighter in same cases and heavier in some others, compared to linear semi-rigid frames, depending on the magnitude of loading and frame configuration. Dai et al. [21] illustrated that HS had better extrapolation and exploitation ability in multi-objective optimization problems. Bashiri-Atrabi et al. [22] applied harmony search algorithm for reservoir operation optimization with respect to flood control. The HS algorithm was used to minimize the water supply deficit and flood damages downstream of a reservoir. They compared the efficacy of HS algorithm with honey-bee mating optimization, and results showed better agreement between HS and global optimum for the objective function. Askarzadeh and Rashedi [23] comprehensively investigated HS implementation for optimization, HS variants and HS applications in detail. They explained the considerable role of HS to solve various optimization problems in different fields like power system, communication, software, civil, water engineering, and pattern recognition. Comparison between the results obtained by HS and other algorithms demonstrated the ability of HS to solve complex optimization problems.

In this study, the HS algorithm is applied to predict scour depth around complex bridge piers under clear water condition. Also, HS the results are compared with those obtained by empirical equations. Statistical indices such as coefficient of determination (R^2), root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and bias were calculated to evaluate the fitness of the HS model. Table 2. indicates the statistical indices in details.

2. Database

In order to investigate the effects of geometric configuration on scour depth around complex piers, 82 experiments were performed (Ghodsi et al. in print). Experiments were executed with six complex pier models to quantify the influence of the pile cap upstream extension (L_u), pile group arrangement ($m \times n$), pile group upstream extension (f_{pm}), and pile cap thickness (T). It was tried to find the relationship between the upper limit of the pile cap undercut elevation and the pile cap thickness. Large quantities of equilibrium local scour data have been contributed by many researches in recent years including data source for non-uniform (column and pile cap) and pile group of 615 data points. 82 experiments were designed so that the contraction effects, sediment size, and flow depth on scour depth become negligible as recommended by Raudkivi and Ettema [24] and Melville and Sutherland [25]. This was true for other data as well. Data collected by these authors were run in clear-water conditions and also some other sources. Measured data in this study have identical spacing between piles in both directions, while 28 data sets have non-uniform spacing. 82 complex pier models were used by authors who had circular piles and also most of the reported experiments data. Furthermore, 82 complex pier models along with published models used rectangular columns and pile caps, except for models applied by Melville and Raudkivi [9], Oliveto et al. [26], and Kothyari and Kumar [27].

These following conditions apply to all data, including 82 author collected data and 615 reported data. The criteria for data selection were done as follows. According to Chiew and Melville [28], the effect of sediment size can be eliminated for $b/d_{50}>50$, in which b is the pier width and d_{50} is the median particle size of sediment bed that holds for all experiments. The dimensions of the column, pile cap, and pile group were chosen so that the width of the channel to the pier width is greater than 6.5, so that channel sides do not have any effect on the maximum scour depth. The ratio of flow depth to pier width should be greater than 3-4; hence, the scour depth is independent of the flow depth. The characteristics of collected data points are summarized in Table 1.

3. Harmony Search Algorithm

Geem et al. [29] first proposed a new meta-heuristic algorithm, i.e., the harmony search (HS) algorithm evaluating the improvisation process of music players, where musician improvise the pitches of their instruments to search for a perfect state of a harmony. That is, the process of searching for a better harmony is analogous to that of seeking better solutions to optimization problem. Like other meta-heuristic algorithm, there is a population, called harmony memory (HM), in HS. Besides, there are some control parameters such as harmony memory size HMS, harmony memory considering rate *HMCR* [0,1] used for determining whether the value of the decision variable is to be chosen from the HM, pitch adjusting rate *PAR* [0,1] used for deciding whether the decision variables are to be further adjusted or not. In other words, the HS algorithm mainly consists of five phases. Figure 2 illustrated HS flowchart including five steps. These steps are initialization of the algorithm, improvisation of a new harmony, and updating for harmony memory as follows:

Step 1: Problem description and initialization of HS parameters

Usually, the optimization problem may be described as follows without loss of generality.

$$min_x \quad f(x.) \tag{1}$$

Where x. = (x_1, x_2, …, x_D) RD, and the feasible solution space or the solution search space is $\Omega = \prod_{j=1}^{p}[L_j, U_j]$, meanwhile, Lj and Uj are the lower and upper bounds of the jth decision variable, respectively.

The HS parameters are initialized at the current step. That is, the parameters *HMS* (harmony memory size), *HMCR* (harmony memory considering rate), *PAR* (pitch adjusting rate), bandwich (*bw*) for controlling the radius of search, and the number of improvisation (*NI*) which correspond to the number of iteration are all initialized.

Step 2: Initialization of harmony memory

In HS, harmony memory is similar to a population of other meta-heuristic algorithms. Likewise, harmony memory size means population size. Thus, a set of *HMS* harmonies, x_{ij} may be generated randomly at this step by the following equation. $x_{ij}= L_j+ (U_j-L_j)$. rand (0,1) where i= 1,2,…, *HMS*, j= 1,2,…, D; L_j and Uj are the lower and upper bounds of the component j, respectively; and each harmony (x_i) representing a solution is a D-dimensional vector. Next, the objective value of each solution is evaluated according to the Eq. (1).

Step 3: Improvise a new harmony

After the above initialization, HS will generate (improvise) a new harmony $x.'=(x_1', x_2',…,x_D')$ from scratch, based on three rules composed of memory consideration, pitch adjustment and random selection.

Step 4: Update harmony memory

If the newly generated harmony vector $x.'$ is better than the worst one $x._w$ in the current HM, then $x.'$ will replace the worst one. In other words, a greedy selection scheme is applied between $x.'$ and $x._w$.

Step 5: Check the termination criterion

At this phase, if the maximum number of iterations is reached, then we stop; otherwise, we go on improvising a new harmony. In other words, in the above mentioned processes, we expect the first two steps to be always repeated in turn, until some specific termination criteria are met [29].

Choosing the suitable HMCR and PAR is the HS limitations. The fixed value of HMCR prevented to achieve globally optimized solution. Kumar et al. [30] proposed a dynamic change in the values of PAR and HMCR consequently modifying the improvisation step of IHS. Initially HS algorithm explores the entire search space, and thereafter a few initial generations, making it confined to a local space. Then the value of HMCR so as to make the algorithm explore each solution no matter it is present in HM or not. The small value of HMCR would help explore to a large extent and is likely to select the solutions from the entire feasible range. The best solutions thus obtained are stored in HM as the algorithm proceeds with the increase in number of generations. And during the final generations, it would like the value of HMCR to increase just to make the search restricted to HM so that the solutions could be obtained from within HM only. Similarly it would like PAR to have high value during earlier generations so as to make the algorithm modify the solutions either stored in HM or from feasible range. The best solutions keep on adding the number of generations is increased and large portion of the best solutions is occupied by HM only and finally the value of PAR is reduced to get better solutions. It has been found that PAR to be directly to the number of generations. PAR to be chosen high initially and it is being allowed to decrease during final generations. The high value of PAR affects the modifications to a greater extent. Towards final generations, the low value of PAR lessons the modification in already modified solutions that leads to the optimal solutions.

4. Prediction of Scour Depth by Harmony Search (HS) Algorithm

Up to here, by using laboratory data, empirical relationships such as HEC-18 [31], FDOT [32], and Coleman [10] have been proposed to estimate the scour depth around complex piers. Ataie-Ashtiani et al. [33], attempted to correct the Coleman equation for semi buried pile cap, by gathering more information. In this study, which focused on finding the pile cap undercut elevation, by using the HS algorithm; efforts have been made to approach the estimation of the experimental equations and the measured values.

Figure 2 computes the coefficient of determination by defining a function and various variables. It defines the linear and nonlinear relationship between the function and the variables. Finally, according to the highest coefficient of determination, the equation between the function and the variable is presented.

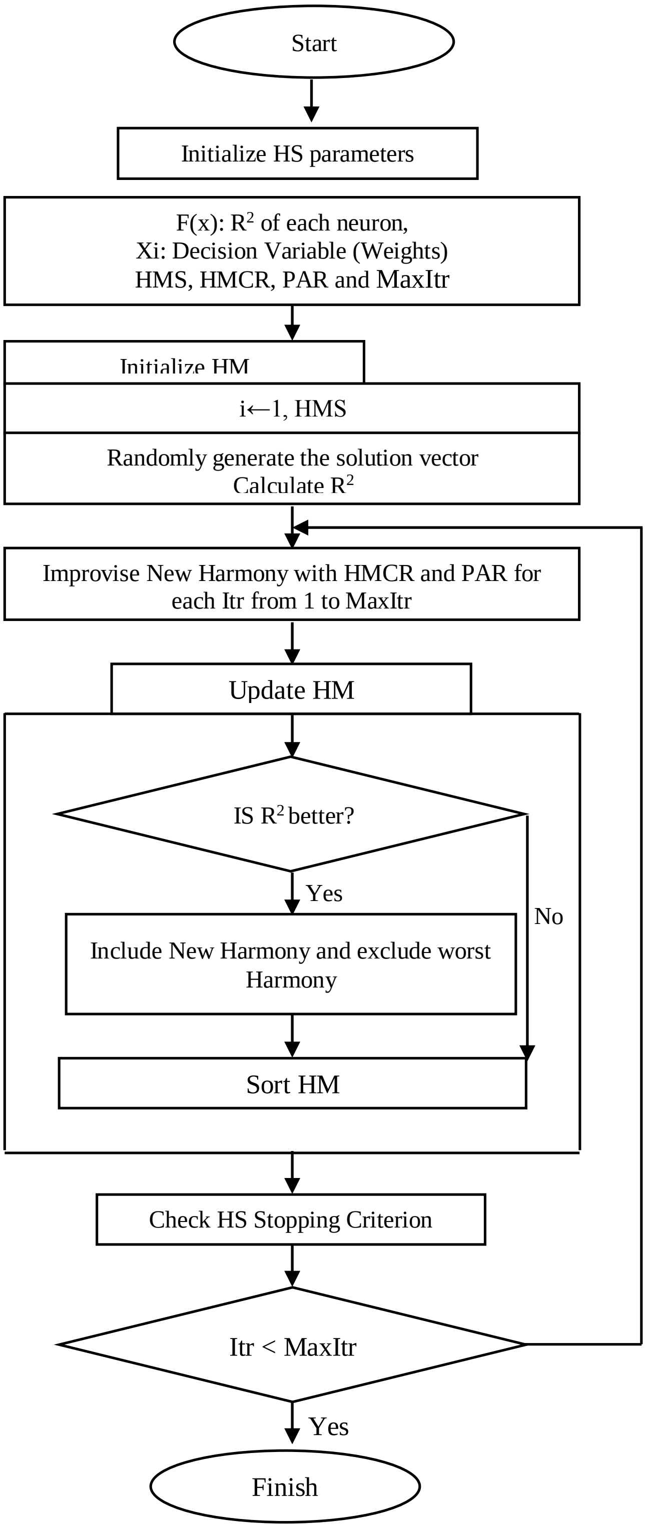


Figure 2. HS algorithm indicating five steps to optimize the problem: problem description and initialization, initialization of harmony memory, improvise a new harmony, update harmony memory, and control the termination criterion

For semi buried pile cap ($Y_T \leq Y < 0$) the equivalent pier diameter was suggested as [9]

$$b_e = b_c\left(\frac{b_c}{b_{pc}}\right)^{\{(bc/bpc)3+0.1-[0.47(0.75-Y/bc)0.5]\}} \qquad for \quad 0 > Y \geq Y_T \tag{2}$$

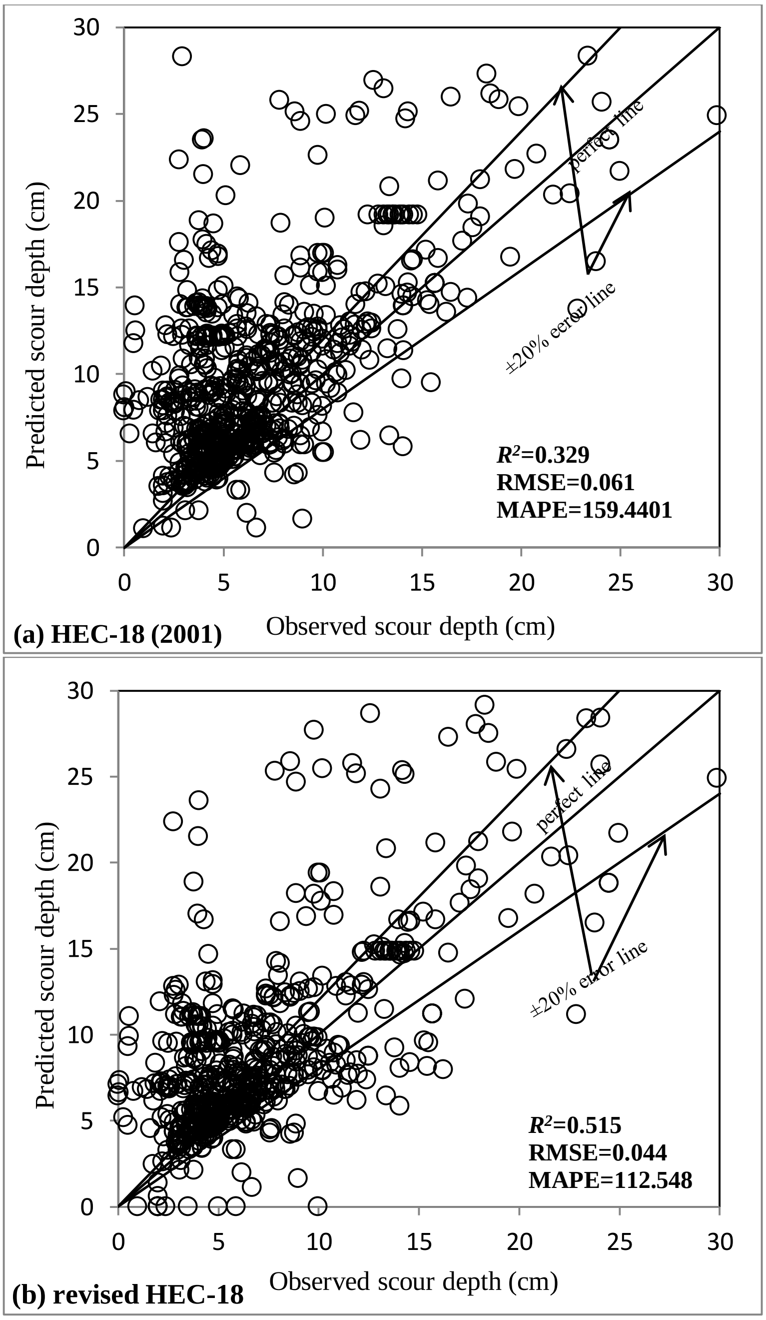
Furthermore, Ataie-Ashtiani et al. [33] revised Equation 2 as

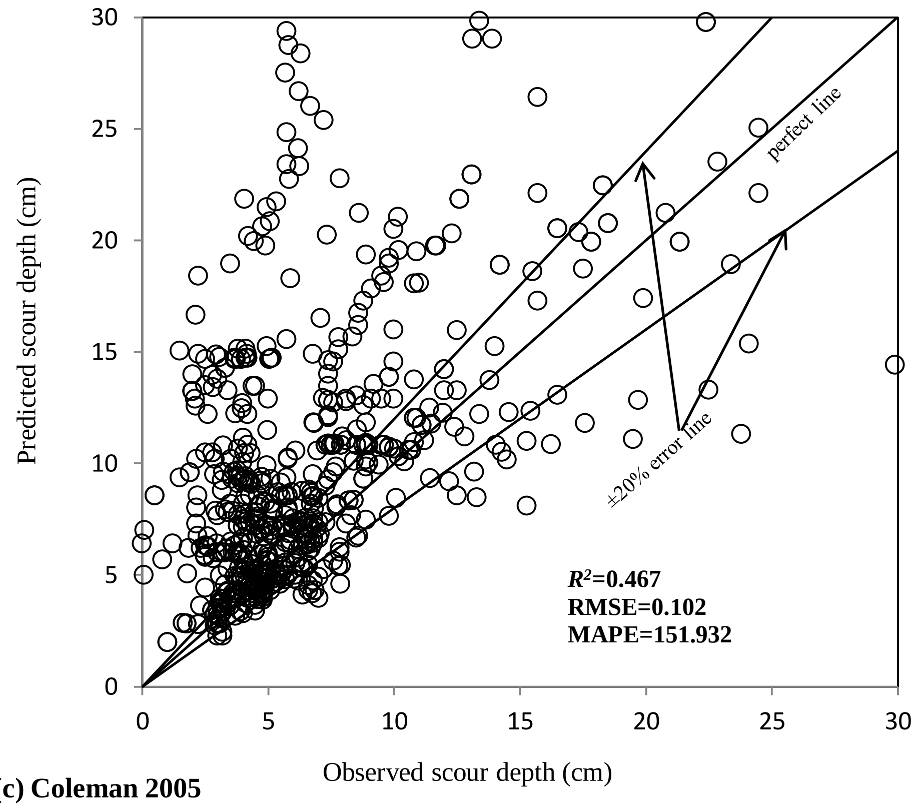
$$b_{eR} = k_{sc}k_{spc}b_e\left(1-0.04\left(\frac{fcu}{fcs}\right)^{1.47}\right)\times\left(\frac{b_c}{b_{pc}}\right)^{0.75\{(bc/bpc)^3+0.1-[0.47(0.75-Y/bc)^{0.5}]\}^{0.66}} \tag{3}$$

Here, by using the HS algorithm, it was tried to improve the scour depth estimation around complex piers. For this purpose, the coefficient of Equation 3 was calibrated to reduce the difference between measured and calculated scour depth. The calibrated equation was presented as follows

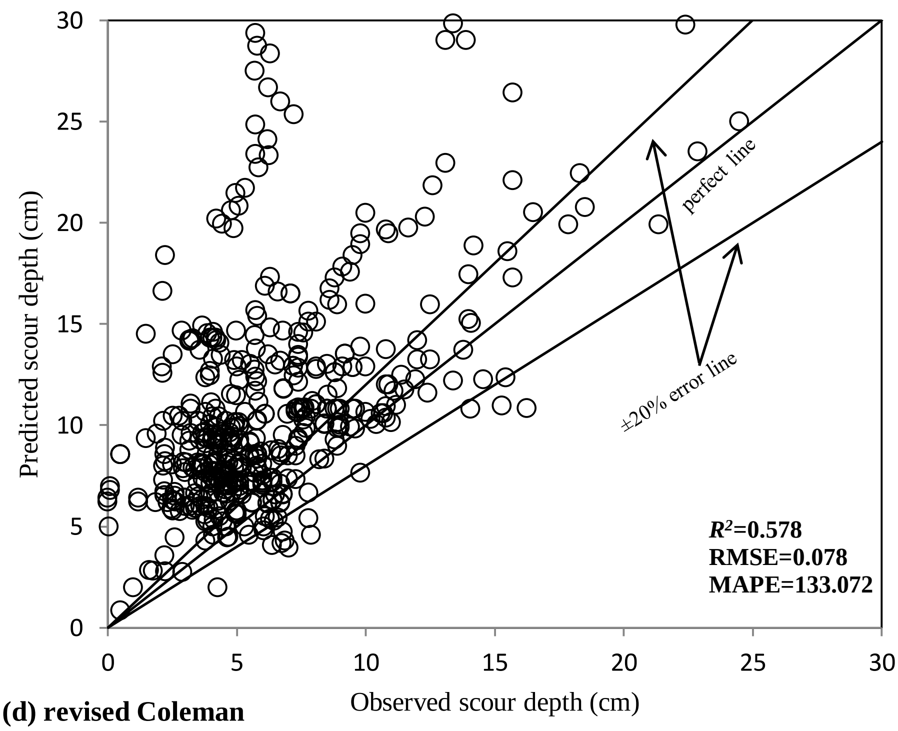
$$b_{eR(HS)} = k_{sc}k_{spc}b_e\left(1-0.3606\left(\frac{fcu}{fcs}\right)^{0.2}\right)\times\left(\frac{b_c}{b_{pc}}\right)^{0.2\{(bc/bpc)^{1.5}+0.2-[0.5595(1.5-Y/bc)^{0.2}]\}^{0.979}} \tag{4}$$

Where $b_{eR(HS)}$= equivalent pier diameter obtained by HS method, b_e= equivalent pier diameter, b_c= column width, b_{pc}= pile cap width, Y= pile cap elevation from original bed, Y_T= pile cap undercut elevation, K_{sc}= column shape factor, K_{spc}= pile cap shape factor, f_{cu}= longitudinal extension of pile cap from column, f_{cs}= transversal extension of pile cap from column.

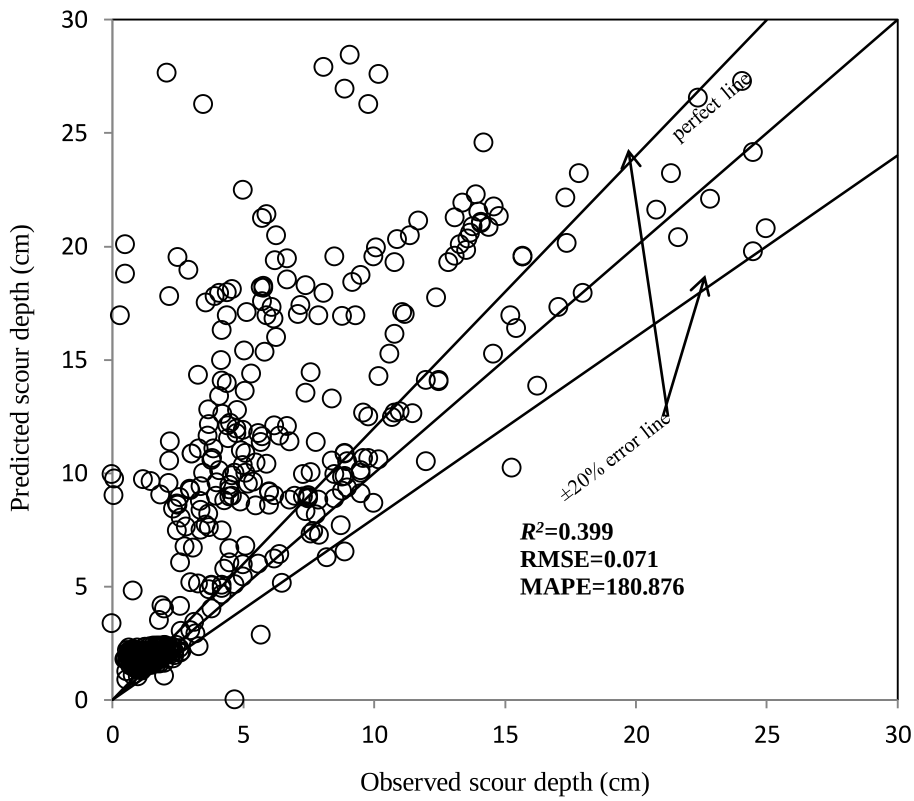




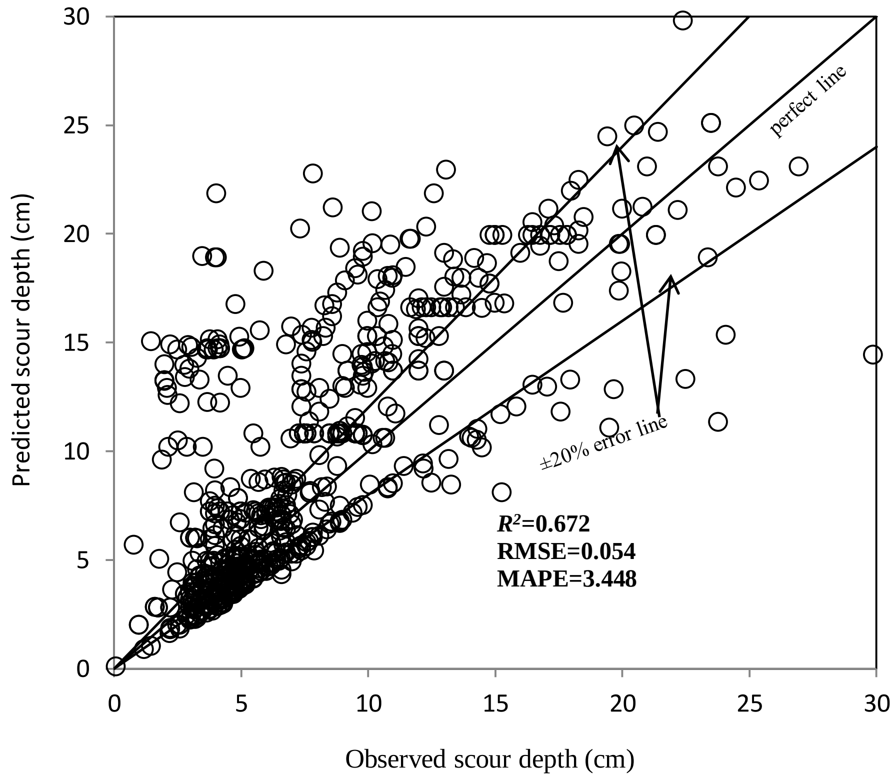
(c) Coleman 2005



(d) revised Coleman



(e) FDOT (2004)



(f) Hs procedure

Figure 3. Comparison of the observed and predicted scour depths

5. Comparison of Scour Depth around Complex Piers

In this study, a calibrated equation for prediction of equilibrium scour depth around complex piers has been proposed. In order to assess the applied method suggested in this research, the performance and outcome of HS algorithm was compared with other empirical equations like HEC-18, FDOT, Coleman equation, revised HEC-18, and the revised Coleman equation; that have been commonly used to estimate scour depth around complex piers. The distribution of 82 data measured by authors and 615 data reported by previous researches is plotted in Figure 3. in which the predicted scour depth obtained by empirical equations were scattered versus observed value reported from experimental works.

The performance of empirical equations along with the HS equation is compared based on reported data. HS efficiency and empirical equations are made based upon the five statistical indices. The root-mean-square error (RMSE), mean absolute error (MAE), and coefficient of determination (R^2), mean absolute percentage error (MAPE), and bias were calculated for this purpose. The values of statistical indices for empirical equations are given in Table 2.

Figure 3a shows results of the measured and predicted scour depths, y_s, using HEC-18 method. The statistical indices R^2, RMSE, MAE, MAPE, and bias were 0.329, 0.061, 0.044, 159.44, and 0.033, respectively. In Figure 3(b), is plotted using the revised HEC-18 method. The statistical indices R^2, RMSE, MAE, MAPE, and bias were 0.515, 0.044, 0.032, 112.548, and 0.018, respectively. Results showed that the revised HEC-18 method exhibited significant improvement with respect to the HEC-18 method. Fig. 3(c) shows results of the measured and predicted scour depths (y_s), using Coleman's [10] procedure. The results showed 0.467, 0.101, 0.066, 151.932, and 0.063 for R^2, RMSE, MAE, MAPE, and bias, respectively. Meanwhile, these quantities were 0.579, 0.078, 0.058, 133.072, and 0.057 for the revised Coleman method (Figure 3(d)). In Figure 3(e) a comparison between the observed and estimated scour depth is shown based on Sheppard et al.'s [27] (FDOT) method. The results showed that R^2, RMSE, MAE, MAPE, and bias were 0.399, 0.071, 0.053, 180.876, and 0.050 respectively. In Figure 3(f) a comparison between the observed and estimated scour depth is shown based on the HS method. In which the estimated scour depths for the cases studied here are in accordance with the observed values. The statistical indices R^2, RMSE, MAE, MAPE, and bias were 0.672, 0.054, 0.035, 3.448, and 0.023 respectively. In this study, six different methods were considered. The best between them was HS method with highest R^2 (R^2 =0.672). The closest Approximation with HS is revised Coleman method, which has a coefficient of determination of 0.578. Comparison of MAPE indicates a significant difference between the obtained values from various methods. The results indicated that performance of HS was more efficient when compared to other empirical equations in predicting scour depth around complex piers.

6. Conclusion

Scour depth at the vicinity of bridge piers may lead to system collapse. Many parameters affect bridge pier scour depth such as pier geometry, sediment texture, and flow characteristics and etc. An artificial intelligence (AI) method which is called Harmony search (HS) has been developed for prediction of scour depth around complex piers. In this study, scour depth around complex piers was investigated. For this purpose 82 laboratory tests were set up and scour depth were measured to see the dominated parameters and to analyse the process. And also 615 reported experimental data around complex bridge pier scour were collected and analysed along with measured data. Empirical equations were applied to evaluate the scour depth prediction. Moreover, the calibrated equation by HS was presented. Five statistical indices such as coefficient of determination (R^2), root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and bias were used to evaluate the performance of the HS optimized method. The comparison between R^2 shows that the HS method with a coefficient of determination equal to 0.672 is more précised than those empirical equations which have been introduced before. Meanwhile, the HS had the second rank RMSE and first place MAPE compared with other methods. Also, the results showed that, HS has better efficiency compared to other models, with respect to empirical equations (MAE= 0.035, MAPE= 3.448, and bias=0.023). Statistical indices are given in Table 2. Between six methods were applied to predict scour depth around complex piers, the HS method has the highest coefficient of determination compared to other methods. The results of this study indicated that the calibrated equation made by the HS method is powerful in estimating scour depth around complex piers, and has 16% more estimation than the existing equations.

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